Identity-Based Multi-Signcryption with Public Verifiability

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**Abstract:** Multi-signcryption can meet the requirement of message signcryption with multi-participant. Since the existing identity-based multi-signcryption scheme cannot offer the function of public verifiability, based on identity and bilinear pairing on the Elliptic Curve, a new scheme with public verifiability is proposed. In the scheme, with the steps which is comparatively independent to the signcryption process, it can provide the public verification of each signcryption in need. Therefore, our scheme efficiently achieves the cryptographic functions of multi-signcryption.

1 Introduction

Identity based cryptosystems were introduced by Shamir in 1984 ([1]). The idea was to get rid of public key certificates by allowing the user's public key to be the binary sequence corresponding to an information identifying him in a non ambiguous way (e-mail address, IP address combined to a user name, social security number,...). Since Boneh and Franklin gave a practical ID-Based encryption scheme [2] from Weil pairing in 2001, a large number of papers have been published in this area. The concept of public key signcryption schemes was found by Zheng in 1997 ([3]). The idea of this kind of primitive is to perform encryption and signature in a single logical step in order to obtain confidentiality, integrity, authentication and non-repudiation more efficiently than the sign-then-encrypt approach. Many schemes have been designed as the extension of signcryption such as proxy signcryption, Multi-proxy-signcryption, ID-based signcryption, etc, and many research have been proposed[4-8].

With the continue growth of the internet, user sends and forwards an original message to other users. Through this process, the message may be modified, improved and added a convenient feature by many users. But we must detect the malicious attackers and prevent the malicious code from damaging the receiver or prevent the attackers from obtaining the private messages. Therefore the concept of multi-signcryption was proposed in[9] which can meet with the requirement of multi-signers performing together the signcryption operation on messages and a specific scheme called Seo-Lee scheme was proposed in [10]. It efficiently provides message flexibility, order flexibility, message verifiability, order verifiability, message confidentiality, message unforgeability, non-repudiation and robustness. Based on Seo-Lee scheme, a Multi-signcryption scheme using identity and bilinear pairing was proposed. It greatly decreases the cost of building and managing public key infrastructures; the expense of the users’ management of public-key and their certificates is avoided. Up to the present, various studies on ID-Based multi-signcryption have been proposed[12,13].

Most of the existing ID-Based multi-signcryption scheme don’t provide the public verifiability though it is a very important property in many practical application. Even there are some schemes can provide the verification, they need the plaintext or the private key of verifier. In this paper, we propose a new ID-Based multi-signcryption that can provide public verification together with other security properties fulfilled and we also give the analysis of the scheme.
2 Preliminary Works
2.1 Bilinear Pairings

We consider two groups \( G_1 \) (additive) and \( G_2 \) (multiplicative) of the same prime order \( q \). We need bilinear maps \( e : G_1 \times G_1 \rightarrow G_2 \) satisfying the following properties:

1. Bilinearity: \( \forall P, Q \in G_1, \forall a, b \in \mathbb{F}_q^* \), we have \( e(aP, bQ) = e(P, Q)^{ab} \).

2. Non-degeneracy: The map does not send all pairs in \( G_1 \times G_1 \) to the identity in \( G_2 \). Observe that since \( G_1, G_2 \) are groups of prime order this implies that if \( P \) is a generator of \( G_1 \) then \( e(P, P) \) is a generator of \( G_2 \).

3. Computability: there exists an efficient algorithm to compute \( e(P, Q) \forall P, Q \in G_1 \).

There are two problems that our scheme base on as follows:

**DBDHP**: Given two groups \( G_1 \) and \( G_2 \) of the same prime order \( q \), a bilinear map \( e : G_1 \times G_1 \rightarrow G_2 \) and a generator \( P \) of \( G_1 \), the Decisional Bilinear Diffie-Hellman problem (DBDHP) in \((G_1, G_2, e)\) is to decide whether \( h = e(p, p)^{ab} \) given \((P, aP, bP, cP)\) and an element \( h \in G_2 \).

**CBDHP**: Given two groups \( G_1 \) and \( G_2 \) of the same prime order \( q \), a bilinear map \( e : G_1 \times G_1 \rightarrow G_2 \) and a generator \( P \) of \( G_1 \), the Computational Bilinear Diffie-Hellman problem (CBDHP) in \((G_1, G_2, e)\) is to compute \( h = e(p, p)^{ab} \) given \((P, aP, bP, cP)\).

No algorithm is known to be able to solve any of them so far, though \textbf{DBDHP} is no harder than \textbf{CBDHP}.

2.2 Security properties

Due to the identity-based nature of singncryption, and the combined requirements on confidentiality and non-repudiation, the security requirements are multifaceted and quite stringent. We assume Alice is the recipient, \( I_i \) are the signers and Charlie is a third party. The properties a multi-signcryption should meets are as follows:

**Confidentiality**: It is impossible for the attacker to compute the secret messages \( m_1, m_2, \ldots, m_n \), or compute the private information of Alice by the singncryption.

**Unforgeability**: It is impossible for any attacker to forge a valid multi-signcryption even any one of \( I_i \) or Alice.

**Non-repudiation**: Charlie can judge the validity of a signcryption when dispute occurs for sender and recipient.

3 a new ID-Based multi-signcryption

This section proposes a new ID-Based multi-signcryption schemes with flexibility and verifiability for both message and order.

**Setup**

The PKG chooses the system parameters that include two groups \((G_i, +)\) and \((G_2, \cdot)\), a bilinear map \( e : G_1 \times G_1 \rightarrow G_2 \) between these groups, a generator \( P \) of \( G_1 \), a master secret \( s \in \mathbb{Z}_q^* \), and a public key \( P_{pub} = sP \in G_1 \). It also chooses a secure symmetric scheme \((E, D)\) and hash
functions, $H_0 : \{0,1\}^* \rightarrow Z_q^*$, $H_1 : \{0,1\}^* \rightarrow G_1$ and $H_2 : \{0,1\}^* \times \{0,1\}^* \times G_1 \rightarrow Z_q^*$, $n$ is the length of plaintext. The public key and private key of Alice is $Q_a = H_0(ID_a)$, $S_a = sQ_a$, the keys of $I_i (1 \leq i \leq n)$ is $Q_i = H_0(ID_i)$, $S_i = s Q_i$. PKG keeps the $s$, the public parameters are $G_1$, $G_2$, $P$, $P_{pub}$, $e$, $H_0$, $H_1$, $H_2$.

**Signcrypt**

We assume that Alice wants $n$ signers $I_i (1 \leq i \leq n)$ to generate a signcryption on a fixed message $M$ according to order fixed beforehand. First Alice send the original message $m$ and the public key $Q_a = H_0(ID_a)$ to all the signers $I_i (1 \leq i \leq n)$, each signer changes the message $m$ into $m_i$ which includes the secret information for Alice.

Assume $I_1$ is the first signer and $I_{i+1}$ is the next one to $I_i$.

$I_1$: choose $k_1 \in Z_q^*$ in random, compute

$$K_1 = e(P_{pub}, Q_a)^{k_1}, r_1 = H_1(m_1 \| Q_1 \| K_1), c_1 = E_{k_1}(m_1 \| Q_1), s_1 = (k_1 P_{pub} - r_1 S_1), X_1 = k_1 P, R_1 = H_2(ID_1, c_1, X_1)P_{pub}, Z_1 = S_1 + k_1 R_1$$

$I_1$ sends $(Q_1, s_1, r_1, c_1, X_1)$ to $I_2$, and sends $Z_1$ to Alice.

According to each signer $I_i$, the signcryption process is as follows:

$I_i$: receive $(Q_{i-1}, s_{i-1}, r_{i-1}, c_{i-1}, X_{i-1})$, choose $k_i \in Z_q^*$ in random, compute:

$$K_i = e(P_{pub}, Q_a)^{k_i}, r_i = H_1(m_i \| Q_i \| K_i \cdot r_{i-1}), c_i = E_{k_i}(m_i \| Q_i \| Q_{i-1} \| s_{i-1} \| c_{i-1}), s_i = (k_i P_{pub} - r_i S_i) \in G_1, X_i = H_i(c_i, Q_i, K_i)X_{i-1}$$

And sends $(Q_i, s_i, r_i, c_i, X_i)$ to the next signer $I_{i+1}$. Until the last signer $I_n$ sends $(Q_n, s_n, r_n, c_n, X_n)$ to Alice, Alice take the steps as follow to unsigncrypt the multi-signcryption and verify.

**Unsigncrypt**

Alice:

Receives $(Q_n, s_n, r_n, c_n, X_n)$, with the private key $S_a$ to compute the session key with

$I_n: K_n = e(s_a, Q_a)e(Q_n, S_a)r_n$,

With $m_n \| Q_n \| Q_{n-1} \| s_{n-1} \| c_{n-1} = D_{K_n}(c_n)$, get message $m_n$, $Q_n$, $Q_{n-1}$, $s_{n-1}$, $c_{n-1}$; from $r_{n-1} = H_1(m_n \| Q_n \| K_n)^{-1} r_n$ to recover $r_{n-1}$, from $X_{n-1} = H_1(c_n \| Q_n \| K_n)^{-1} X_n$ to recover $X_{n-1}$. Then Alice computes each session key $K_j = e(s_j, Q_a)e(Q_j, S_a)r_i$ with $I_i, 1 \leq i \leq n$, get all the part signcryption $(Q_i, s_i, r_i, c_i, X_i)$ and message $m_i$ together with the order of signer.
Do the same steps until Alice compute \( X_1 = H_1(c_2 \parallel Q_2 \parallel K_2)^{-1} X_2 \), \( r_1 = H(m_2 \parallel Q_2 \parallel K_2)^{-1} r_2 \), and unsigncrypt \( m_1, Q_1 \) through \( K_1 \).

Finally, Alice accepts the signcryption if \( r_1 = H(m_1 \parallel Q_1 \parallel K_1) \) and \( e(P, Z_1) = e(P_{pub}, Q_1)e(X_1, R_1) \) are established, else refuses it.

**Public verify:**

If a signer \( I_i \) denies the signcryption, Alice sends \((c_1, X_1, Z_1)\) to the third party to arbitrate.

The third party compute \( R_i = H_2(ID_1,c_1,X_1)P_{pub} \) judge the signcryption is true if \( e(P, Z_1) = e(P_{pub}, Q_1)e(X_1, R_1) \) is established.

Proof of correctness

\[
e(P, Z_1) = e(P, S_1 + k_1 R_1) = e(P, sQ_1 + k_1 R_1) = e(P, sQ_1) e(k_1 P, R_1) = e(P_{pub}, Q_1)e(X_1, R_1)
\]

4 validity and security analysis

**Message flexibility:** A message does not need to be fixed beforehand. Therefore each signer can modify an original message.

**Order flexibility:** Neither order of signers nor signers themselves need to be designated beforehand. Therefore we can easily change order of signers, add a new signer and exclude a signer.

**Confidentiality:** It is impossible for the attacker to compute the secret messages \( m_1, m_2, ..., m_n \), from the signcryption \((Q_i, s_i, r_i, c_i, X_i)\) without \( k_1, S_1 \) and \( S_a \). Base on the problem of discrete logarithm, it is also computational impossible for attacker to compute \( k_i \) from \( K_i = e(P_{pub}, Q_a)\), or \( S_i \) from \( s_i = (k_i P_{pub} - r_i S_i) \). Therefore it is impossible to get \( m_i \).

**Unforgeability:** It is impossible for any attacker to forge a valid signcryption without \( S_2 \), the private key of \( I_i \) even any one of \( I_i \) or Alice.

**Non-repudiation:** Since the signcryption of each \( I_i \) is unforgeable, once the multi-signcryption is generated, it cannot be denied.

**Public verification:** When dispute occurs for sender and recipient, the recipient can send \((c_1, X_1, Z_1)\) to the judge. The judge can verify the validity of the signcryption without the private key of recipient. And the information still kept secret since the verify uses cipher text. The judge first compute \( R_i = H_2(ID_1,c_1,X_1)P_{pub} \), then judge the signcryption is true if \( e(P, Z_1) = e(P_{pub}, Q_1)e(X_1, R_1) \) is established.

Assume that an attacker or Alice choose \( k_1' \in Z_q^* \) in random, compute \( X_1' = k_1' P \), \( R_1' = H_2(ID_1,c_1',X_1')P_{pub} \), \( Z_1' = S_1' + k_1' R_1' \), forge a message \((c_1', X_1', Z_1')\), if \( e(P, Z_1') = e(P_{pub}, Q_1)e(X_1', R_1') \) established, namely
\[ e(P,Z'_1) = e(P,S'_1 + k'_1 R'_1) = e(P,s'_1 Q_1 + k'_1 R'_1) = e(P\cdot s Q_1) = e(k'_1 P, R'_1) = e(P_{pub}, Q_1) e(X_1, R_1) \]

Because the attacker doesn’t know value of \( s \), if \( e(P, s'_1 Q_1 + k'_1 R'_1) = e(P, s Q_1) e(k'_1 P, R'_1) \) is satisfied, the problems of ECDL and BDH are solvable.

**Verification flexibility:** The public verification is alternative and calculates of \( X_1 \) can be omitted. Therefore it will be computed only when we need public verification.

### 5 Conclusions

This paper proposed a new ID-Based multi-signcryption with its analysis. The scheme can provides the public verification of signcryption and the signer. It also satisfies the security properties efficiently with short cipher text. It can be applied in e-commerce or e-voting.

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