Online Multi-attribute Reverse Auction Mechanism Design Based On Utility

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Abstract. A good auction mechanism is able to release the information who owns to the other side who lacks of. In order to ensure a positive flow of information and to improve the active participation of suppliers, negotiation mechanism is introduced, and suppliers can participate in the design of auction mechanism. Several rounds of English auction is applied to determining the final winner. Taking into account that the buyer has a preference on the property, we use AHP to calculate final weights, and use multi-attribute utility function to calculate the greatest utility as the winner.

0 Introduction

A reverse auction is a type of auction in which the roles of buyers and sellers are reversed. In a regular auction, buyers compete to obtain a good or service, and the price typically increases over time. In a reverse auction, the buyer advertises need for an item or service, sellers compete to obtain business, and prices typically decrease over time. Because it is widely used in purchase of goods in large quantity, besides price, there are several attributes should be considered, such as supplier’s credit, quality and date of delivery. Scholars both at home and abroad have studied on multi-attribute reverse auction: Che firstly studied design competition in government procurement by developing a model of two-dimensional auctions, where firms bid on both price and quality, and bids are evaluated by a scoring rule designed by a buyer[1]; Bichler conducted experiments on several sorts of auction, he found the utility scores achieved in multi-attribute auctions were significantly higher than those of single-attribute auctions[2]. Wu Gang thought setting weights should be based on the importance of attributes, and proposed a new approach called hierarchical interactive collaborative group decision-making[3]. Xie Anshi proposed a decision making methods for multi-attribute online auction based on fuzzy rough set[4].

This paper carries Bichler’s model of multi-attribute weighted summation, introduces consultation mechanism, uses root method in AHP to determine the ultimate weights, and bids several times to make competition sufficient, at last sums the normalized attribute values with weights. Highest score wins.

1 Mechanism design

1.1 Basic assumptions

Before designing mechanism, there are some basic hypothesis of this study as follow: a purchaser and several suppliers are available; only one product is involved, there are not any products combinatorial auctions; information asymmetry exists between the purchaser and suppliers, so it
can constitute the game of incomplete information, however, purchaser has its own preferences; the purchaser and suppliers are assumed to be risk neutral; in this study, we supposes that there is no transaction cost, the supplier’s decision and submitting bidding costs are neglected; finally we assumed that suppliers are the equal relations.

1.2 Process design

In purchasing auction, purchaser is relative lack of information. Meanwhile, suppliers have more information. How to design a good mechanism which releases the suppliers’ information to the purchaser? This paper introduces consultation mechanism. There is the flow chart of the whole auction, as shown in figure 1.

![Multi-attribute reverse auction Flowchart](image)

**Fig. 1** Multi-attribute reverse auction Flowchart

1.3 Consultation stage

At first, purchaser issues a tender announcement through online auction platform, the content combined the purchaser’s demand includes: the project background, purchasing product quantity and specific purchase requirements. Initial attribute requirements should be setting in the purchase requirements, including attribute set $A^{(r)}$ and specific attribute index $a_j$, such as:

$$A^{(r)} = \{a_j|j = 1,2,\cdots,n^{(r)}\}$$  \hspace{1cm} (1)
In the formula, \( r \) is the round number of the auction. When \( r=0 \), it means preparation period before the first round bid. \( a_j \) means the \( j^{th} \) attribute, and you can assign a new value or a threshold to it. Some cost attributes, such as the price attribute, marked \( a_1 \), purchaser should define a reserve price, the price supplied offered should not exceed it. Other benefit attributes, such as the speed attribute, marked \( a_2 \), the speed supplied offered should be greater than or equal to threshold. \( n^{(r)} \) means the attribute number in the \( r^{th} \) round of attribute set. In the preparation period, the purchaser gives the attribute number called \( n^{(0)} \), and sets the deadline in this period, called \( T^{(0)} \).

If the suppliers can meet the requirement, they can apply for auction on line. After qualification examination, the suppliers offer bidding documents. The purchaser determines the set of suppliers in the preparation period.

\[
Q^{(0)} = \{q_i | i = 1, 2, \ldots, m^{(0)} \} 
\]

This can be described that there are \( m^{(0)} \) qualified suppliers \( q \) in the preparation period. Every bidding document is equivalent to a solution, forming the following set

\[
S^{(0)} = \{s_i^{(0)}(a_{ij}) | i = 1, 2, \ldots, m^{(0)}; j = 1, 2, \ldots, n_i^{(0)} \}
\]

\( s_i^{(0)} \) means the supplier \( q_i \) offers the solution, the \( j^{th} \) attribute value is \( a_{ij} \), \( n_i^{(0)} \) means the supplier \( q_i \) gives the number of attributes in the preparation period. We can conclude that \( n_i^{(0)} > n^{(0)} \), because some suppliers would like to raise some new attributes which represent their brand advantages. For example, one purchaser wants to buy engines, and the procurement plan mentions two attributes—price and date of delivery. Given the engine been ready for family car, one supplier combined their own advantages proposes a new attribute—fuel consumption, and gives a higher weight, so that they can highlight their advantage and be in a good position. This can encourage suppliers to release their real information to the purchaser who lacks information, also can make the purchaser fully understands the attribute demand of the purchased products and formulate better auction strategy.

Purchaser firstly will take the new attributes into account, then combine their own needs, at last decide the attribute set \( A \). There are \( n \) attributes in it, and every \( a_j \) is endowed with specific attribute values, as follows:

\[
A = [a_j | j = 1, 2, \ldots, n] \quad (4)
\]

For each attribute, the purchaser has its own preferences. How to describe the preferences accurately? In fact, this is the process of the purchaser setting weight. This paper uses 1-9 scale method to get the attributes’ relative importance \( a_{ij} \), then can get judgment matrix \( A = (a_{ij})_{n \times n} \), and use the root method in AHP to figure out the final weight. \( a_{ij} \) means the relative importance of
attribute $a_i$ to attribute $a_j$, the value of $a_{ij}$ can be expressed with number form one to nine and their reciprocal. $a_{ii}=1$, $a_{ij}=a_{ji}^{-1}$. Then calculate the product of each line of elements in the judgment matrix

$$M_i = \prod_{j=1}^{n} a_{ij} \quad i = 1, 2, \ldots, n \quad (5)$$

$$\omega_i = \frac{1}{\sqrt{M_i}} \quad i = 1, 2, \ldots, n \quad (6)$$

Next, standardizes the vector $\overrightarrow{W} = [\omega_1, \omega_2, \ldots, \omega_n]^T$

$$\omega_i = \frac{\omega_i}{\sum_{j=1}^{n} \omega_j} \quad (7)$$

Vector $\overrightarrow{W} = [\omega_1, \omega_2, \ldots, \omega_n]^T$ is the weight we want, but we should work out the maximum eigenvalue of the judgment matrix $\lambda_{\text{max}}$, and then do the coincidence test.

$$\lambda_{\text{max}} = \sum_{i=1}^{n} \frac{(\overrightarrow{AW})_i}{\text{No}_i} \quad (8)$$

$(\overrightarrow{AW})_i$ means the $i^{th}$ element of the product of vector $\overrightarrow{A}$ and vector $\overrightarrow{W}$. We can use the formula below to do the coincidence test.

$$\text{CR} = \frac{\text{CI}}{\text{RI}} \quad (9)$$

$$\text{CI} = \frac{\lambda_{\text{max}} - n}{n-1}$$, CI is the coincident indicator of matrix $\overrightarrow{A}$, RI the average random coincident indicator, Saaty gives the value of RI.

When $\text{CR} < 0.1$, we consider the estimation matrix is consistency, and $\overrightarrow{W} = [\omega_1, \omega_2, \ldots, \omega_n]^T$ is the final weight, otherwise we need to regulate the estimation matrix to have good consistency, and work out the final weight in accordance with the above method.

$$\overrightarrow{W} = \{\omega_i\} = 1, 2, \ldots, n \quad (10)$$
Equation 4 and 10 is the internet negotiation result between the purchaser and suppliers, and this result will not change in the later auction process. The purchaser releases the information of the final attributes, the ranking of attributes and \( T^{(2)} \) the deadline of the first round auction to the suppliers, but the purchaser won’t release the value of attribute and the weight. This is the end of consultation stage.

1.4 Winner wins stage

Suppliers according to bidding information of this round decide whether to eventually participate in this bid. Now we introduce the individual rationality constraint (IR). It means that rational suppliers will take part in the bidding only if the gap between the prospective earnings and the cost is greater than reservation utility. So the new supplier set is shown in the following equation 11, and \( m^{(1)} \leq m^{(0)} \).

\[
Q^{(1)} = \{q_i| i = 1,2,\ldots, m^{(1)}\}
\]

Suppliers offer the first round bidding documents, forming the following set

\[
S^{(2)} = \{s_i^{(1)}(s_{ij})| i = 1,2,\ldots, m^{(1)}; j = 1,2,\ldots, n\}
\]

Because each attribute has different unit, dimension and the order of magnitude, before making decision, we should normalize the attribute. The usual method is range transformation and linear scale transformation. The first method will convert the worst value to zero, thus the weight can’t take effect, so we use the second method. Now we put the attributes from purchaser and the attributes from suppliers in the first round together, and get attributes matrix \( R_{m^{(2)}\times n} \)

\[
R = \begin{pmatrix}
a_{11} & \cdots & a_{1m} \\
\vdots & \ddots & \vdots \\
a_{m1} & \cdots & a_{mn}
\end{pmatrix}
\]

Set \( f^b_j = \max_{1 \leq i \leq m^{(2)}} a_{ij} > 0 \), \( f^c_j = \min_{1 \leq i \leq m^{(2)}} a_{ij} > 0 \)

To the benefit attribute, we make

\[
r_{ij} = \frac{a_{ij}}{f^b_j}
\]

To the cost attribute, we make

\[
r_{ij} = \frac{f^c_j}{a_{ij}}
\]
$f^j_1$ means the maximal attribute value in j attributes, $f^j_n$ means the minimal attribute value in j attributes, $1 \leq j \leq n$. Now we can get normalized matrix $R^{'}_{(m \times n)}$:

$$R^{'} = \left( \begin{array}{cccc}
    r^{11}_{1} & \cdots & r^{1n}_{1} \\
    \vdots & \ddots & \vdots \\
    r^{m1}_{n} & \cdots & r^{mn}_{n}
\end{array} \right)$$

Next we calculate the product of $R^{'}$ and W

$$R^{'}W = \left( \begin{array}{cccc}
    r^{11}_{1} & \cdots & r^{1n}_{1} \\
    \vdots & \ddots & \vdots \\
    r^{m1}_{n} & \cdots & r^{mn}_{n}
\end{array} \right) \left( \begin{array}{c}
    \omega_1 \\
    \vdots \\
    \omega_n
\end{array} \right)$$

(15)

The results are the suppliers’ actual utility to the purchaser. We can get the ranking of the suppliers according to the results. The highest score is the winner of this round. The purchaser releases the winner’s attributes value of this round and their ranking to each supplier. Asking whether anyone bid, if no one want to go on bid, the winner of this round is the final winner, and the bid ends; if more than one supplier go on bid, then the purchase announces the deadline $T^{(2)}$ in the next round. The suppliers optimize their attributes value and offer new tenders. This can encourage sufficiently competition between suppliers, and constantly towards their bottom line. Until no suppliers re-bid the auction is over. After several rounds of competition, the supplier offers the maximal practical utility is the ultimate winner.

$$WIN_{q_{1}} = \max \left( \sum_{i=1}^{m} \sum_{j=1}^{n} r_{ij}\omega_{j} \right)$$

(16)

2 Conclusion

This consultative mechanism introduced by this paper can increase the suppliers’ enthusiasm of taking part in and make a positive flow of information. In addition, the proposed multi-round English auction will be applied to the multi-attribute reverse auction method, not only expand the only price bidding way, but also make the only one round bid to several rounds. We can get full competition, and the purchaser obtains the utility is higher than the original utility which uses the original bidding way.

References