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Optimal coverage model in Clifford 3-connected wireless sensor networks

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Abstract: While moderate loss of coverage can be tolerated by WSN applications, loss of connectivity can be fatal. Moreover, since sensors are subject to unanticipated failures after deployment, it is not sufficient for a wireless sensor network to just be connected, it should be Clifford 3-connected . In this dissertation, we propose optimal deployment patterns to achieve both full coverage and Cliford 3-connectivity, and analyses their optimality for all values of $\left|r_c(\mathrm{e1^\wedge e2})\right|/\left|r_s(\mathrm{e1^\wedge e2})\right|$, where $\left|r_c(\mathrm{e1^\wedge e2})\right|$ is the communication radius and $\left|r_s(\mathrm{e1^\wedge e2})\right|$ is the sensing radius.

1. Introduction

Mechanisms that optimize sensor energy utilization have a great impact on extending network lifetime. Most existing works concentrate on scheduling sensors between sleep and active modes [2-3] or adjusting sensing range [4-5] to maximize network lifetime while maintaining target/area coverage and network connectivity. In this paper, we analyze the energy efficiency of Clifford sensor network [1] and present an algorithm to extending network lifetime. In this approach, we deploy the sensor network globally with the Clifford sensor network connection graph, and optimize each connecting coverage path based on the angle information of the work node relative to the central node.

On the basis of ref [1,6], which proposed a method for analyzing space sensor network coverage with Clifford algebra, In this dissertation, we propose optimal deployment patterns to achieve both full coverage and 3-connectivity, and analyses their optimality for all values of $\left|r_c(\mathrm{e1^\wedge e2})\right|/\left|r_s(\mathrm{e1^\wedge e2})\right|$, where $\left|r_c(\mathrm{e1^\wedge e2})\right|$ is the communication radius and $\left|r_s(\mathrm{e1^\wedge e2})\right|$ is the sensing radius.

2. The mathematical model of Clifford 3-connected wireless sensor networks

Using the Clifford 3-connected wireless sensor networks coverage theory, each sensor node can track and monitor the multi-type targets in sensor networks. In this paper, the Clifford algebra defined the mathematical model of hybrid-type target in sensor networks. The definition of entity was also introduced in the targets, and used to the universal description of the targets.

In $\mathcal{G}_n$ space, the range of field A is constructed by the 2^n dimensional $\mathcal{G}_n$ basis of Clifford algebra:

$$\{1, \mathop{\wedge}_{i_1=1}^{n} e_{i_1}, \mathop{\wedge}_{i_1,i_2=1}^{n} e_{i_1 i_2}, ..., \mathop{\wedge}_{i_1,i_2,...,i_n=1}^{n} e_{i_1 i_2 ... i_n}\}, i_1, i_2, ..., i_n \in [1,2,...,n], i_1 \neq i_2 \neq \cdots \neq i_n \tag{1}$$

where 1 denotes the unit vector on real axis (0-grade algebra), $\overset{n}{\underset{i_1=1}{\wedge}} e_{i_1}$ denotes the orthographic basis in $\mathcal{G}_n$ space (1-grade algebra), $\overset{n}{\underset{i_1,i_2=1}{\wedge}} e_{i_1 i_2} = \overset{n}{\underset{i_1,i_2=1}{\wedge}} e_{i_1} e_{i_2}$ denotes the dual vector (2-grade algebra), … , $\overset{n}{\underset{i_1,i_2,\dots,i_n=1}{\wedge}} e_{i_1 i_2,\dots i_n} = \overset{n}{\underset{i_1,i_2,\dots,i_n=1}{\wedge}} e_{i_1} e_{i_2} \dots e_{i_n}$ denotes the highest grade blade in $\mathcal{G}_n$ space (n-grade algebra).

Because every element in $\mathcal{G}_n$ space can be considered as an entity, the entity t in field A can be defined as:

$$t = \sum_A f_A(t) e_A \quad = \sum_{k=0}^{n} \sum_{i_1 < \dots < i_k} \xi_{i_1 \dots i_k} e_{i_1} \wedge \dots \wedge e_{i_k} \tag{2}$$

In our optimal pattern study, we assume the disc model for sensing and communication.

Assumption 2.1. [Disc-based sensing] We assume a disc-based sensing model where each active sensor has a sensing radius of rs; any object within the disc of radius rs centered at an active senso e1,e2 r is reliably detected by it. The sensing disk of a sensor located at location u is denoted by $D_{r_s}(u) = r_s\left(\text{e1\^{}e2}\right)$.

Assumption 2.2. [Disk-based communication] We assume a disc-based radio model where each active sensor has a communication range of rc; two active sensors at a distance of rc or less can communicate reliably. The communication disk of a sensor located at location u is denoted by Drc(u).

Assumption 2.0.3. [Homogeneous sensing and communication range] We assume that the sensing range of all sensors are the same, as are their communication range.

Assumption 2.0.4. [Bounded value of rs/rc] We also assume that $\lim_{|r_s(e_1 \wedge e_2)| \to 0} \left|(r_s/r_c) e_1 \wedge e_2\right| < M$, for some M > 0. The limit $\lim\left|r_s\left(e_1 \wedge e_2\right)\right| \to 0$ signifies that we need an increasing number of sensors to cover a given region, which is needed for the asymptotic analysis.

Definition 2.1. [Globally Optimal Pattern] A deployment pattern is called globally optimal if it needs the minimum number of sensors to achieve a given coverage and connectivity requirement, among all patterns.

To define γ-optimality, we need to provide the following definitions.

Definition 2.2. [Communication Graph] A communication graph, denoted by Gc = (Vc,Ec), is a graph that is subject to the following conditions: 1) the elements of its vertex set $V = v_1 e_1 + v_2 e_2$, where $v_1 \in R^+, v_2 \in R^+$ are sensors, and 2) the elements of its edge set E=(V^v)/v+τv ($\tau \in R$) are straight line segments connecting all pairs of vertices whose Euclidean distance is no larger than rc.

Definition 2.3. [Deployment Graph] A deployment graph, denoted by G =(V,E), is a planar subgraph of Gc with $V = V_c, E \subseteq E_c$.

There exists a communication graph, Gc, for any given sensor deployment, from which we can obtain multiple deployment graphs. We write Gk = (Vk,Ek) to denote a deployment graph that achieves k-connectivity.

Definition 2.4. [Direct Neighbor] In a deployment graph G = (V,E), if there is an edge between two vertices $x = x_1 e_1 + x_2 e_2$, $y = y_1 e_1 + y_2 e_2$, then x and y are direct neighbors of each other.

Definition 2.5. [Angular Distance] The angular distance between two vertices x and y as measured from a given vertex $z = z_1 e_1 + z_2 e_2$, is the size of the angle $\tan\theta_z = \dfrac{rej(x,y)}{proj(x,y)} = \dfrac{x \wedge y}{x \bullet y}$, $rej(x,y), proj(x,y)$ is projective and rejective of z., If a vertex x has k_x direct neighbors, then we say that the degree of x in G is k_x.We denote its k_x neighbors by $n_1, n_2, \cdots, n_{k_x}$ in some order. We further denote the angular distance measured from x between n_1, n_2 by α_{1x} ,

that is $\alpha_{1_x} = |rej(n_1, n_2)| = |(n_1 - (n_1 \bullet n_2)/n_2)| = |((n_1 \wedge n_2)/n_2)|$ between n_2, n_3 by $\alpha_{2_x} = |rej(n_2, n_3)| = |(n_2 - (n_2 \bullet n_3)/n_3)| = |((n_2 \wedge n_3)/n_3)|$ · · · , between n_{k_x}, n_1 by $\alpha_{k_x} = |rej(n_{k_x}, n_1)| = |(n_{k_x} - (n_{k_x} \bullet n_1)/n_1)| = |((n_{k_x} \wedge n_1)/n_1)|$. Now, we are ready to introduce the definition of regular deployment.

Definition 2.6. [Regular Deployment] A sensor deployment is called regular if it has a deployment graph G where for any two vertices x and y in G , $k_x = k_y$ and $\alpha_{1x} = \alpha_{1y}$, $\alpha_{2x} = \alpha 2y$, · · · , $\alpha_{kx} = \alpha_{ky}$ following the same order.

Definition 2.7. [γ-Optimal Pattern] A regular deployment pattern is called γ-optimal if it needs the minimum number of sensors to achieve a given coverage and connectivity requirement, among all regular patterns.

3. Optimal Patterns for Full-coverage and 3-Connectivity

Figure 1 shows how optimal patterns that achieve full coverage and 3-connectivity evolve as $|r_c(\text{e1^e2})|/|r_s(\text{e1^e2})|$ decreases. The pattern shown in Figure1(a) is globally optimal and patterns shown in Figure 1(b)-(h) . We present pattern description for Po as follows. The distance between two connected sensors is denoted by d.

- $\sqrt{3} \le |r_c(\text{e1^e2})|/|r_s(\text{e1^e2})| : \{6*(60)\frac{e_1 \wedge e_2}{e_1 \cdot e_2}.6*(120)\frac{e_1 \wedge e_2}{e_1 \cdot e_2}.6*(180)\frac{e_1 \wedge e_2}{e_1 \cdot e_2}\}$, shown in Figure 1(a). $d = \sqrt{3}|r_s(\text{e1^e2})|$.
- $\sqrt{2} \le |r_c(\text{e1^e2})|/|r_s(\text{e1^e2})| < \sqrt{3} : \{6*(\theta_1)\frac{e_1 \wedge e_2}{e_1 \cdot e_2}.6*(\theta_2)\frac{e_1 \wedge e_2}{e_1 \cdot e_2}.6*(180)\frac{e_1 \wedge e_2}{e_1 \cdot e_2}\}$ √2, shown in Figure 1(b), where

$$\theta_1 = 2\arccos\left(2|r_c(\text{e1^e2})|/|r_s(\text{e1^e2})|\right), \theta_2 = 180 - \theta_1, d = |r_c(\text{e1^e2})|$$

- $1.0459 \le |r_c(\text{e1^e2})|/|r_s(\text{e1^e2})| < \sqrt{2} : \{6*(\theta_1)\frac{e_1 \wedge e_2}{e_1 \cdot e_2}.6*(\theta_2)\frac{e_1 \wedge e_2}{e_1 \cdot e_2}.6*(\theta_2)\frac{e_1 \wedge e_2}{e_1 \cdot e_2}\}$ shown in Figure 1(c), where $\theta_1 = 4\arcsin(|r_s(\text{e1^e2})|/2|r_c(\text{e1^e2})|), \theta_2 = (360 - \theta_1)/2, d = |r_c(\text{e1^e2})|\ \theta$
- $0.9462 \le |r_c(\text{e1^e2})|/|r_s(\text{e1^e2})| < 1.0459 : \{6\frac{e_1 \wedge e_2}{e_1 \cdot e_2}.8*(90)\frac{e_1 \wedge e_2}{e_1 \cdot e_2}.8*(150)\frac{e_1 \wedge e_2}{e_1 \cdot e_2}\}$ (e.g., the sensor at position A) and $\{6.4.8*(150)\}\}$ (e.g., the sensor at position B), shown in Figure 1(d). $d = |r_c(\text{e1^e2})|$
- $0.7405 \le |r_c(\text{e1^e2})|/|r_s(\text{e1^e2})| < 0.9462 : \{4\frac{e_1 \wedge e_2}{e_1 \cdot e_2}.8*(\theta_1)\frac{e_1 \wedge e_2}{e_1 \cdot e_2}.8*(\theta_2)\frac{e_1 \wedge e_2}{e_1 \cdot e_2}\}$, shown in Figure 1(e). When $0.7655 \le |r_c(\text{e1^e2})|/|r_s(\text{e1^e2})| < 0.9462$,

$$\theta_1 = 2\arcsin(\sqrt{2}|r_s(\text{e1^e2})|/2|r_c(\text{e1^e2})|), \theta_2 = 270 - \theta_1, d = |r_c(\text{e1^e2})|$$

When $0.7405 \le |r_c(\text{e1^e2})|/|r_s(\text{e1^e2})| < 0.7655$, $\theta_1 = \theta_2 = 135, d = |r_c(\text{e1^e2})|$,

- $0.7254 \le |r_c(\mathrm{e1}\wedge\mathrm{e2})|/|r_s(\mathrm{e1}\wedge\mathrm{e2})| < 0.7405 : \{12*(90)\frac{e_1\wedge e_2}{e_1\cdot e_2}.12*(162)\frac{e_1\wedge e_2}{e_1\cdot e_2}.5\frac{e_1\wedge e_2}{e_1\cdot e_2}\}$ (e.g., the sensor at position A) and $\{4\frac{e_1\wedge e_2}{e_1\cdot e_2}.5\frac{e_1\wedge e_2}{e_1\cdot e_2}.12*(162)\frac{e_1\wedge e_2}{e_1\cdot e_2}\}$ (e.g., the sensor at position B), shown in Figure 1(f).

When $0.7403 \le |r_c(\mathrm{e1}\wedge\mathrm{e2})|/|r_s(\mathrm{e1}\wedge\mathrm{e2})| < 0.7405$, $d = |r_c(\mathrm{e1}\wedge\mathrm{e2})|$.

When $0.7403 \le |r_c(\mathrm{e1}\wedge\mathrm{e2})|/|r_s(\mathrm{e1}\wedge\mathrm{e2})| < 0.7405, d = 0.7403|r_c(\mathrm{e1}\wedge\mathrm{e2})|$

- $0.4291 \le |r_c(\mathrm{e1}\wedge\mathrm{e2})|/|r_s(\mathrm{e1}\wedge\mathrm{e2})| < 0.7254 : \{12*(\theta_1)\frac{e_1\wedge e_2}{e_1\cdot e_2}.12*(\theta_2)\frac{e_1\wedge e_2}{e_1\cdot e_2}.3\frac{e_1\wedge e_2}{e_1\cdot e_2}\}$, shown in Figure 1(g) where θ_1 and θ_2 are denoted by " ∘ " and " × ", respectively. When $0.5176 \le |r_c(\mathrm{e1}\wedge\mathrm{e2})|/|r_s(\mathrm{e1}\wedge\mathrm{e2})| < 0.7254$, $\theta_1 = 2\arcsin(|r_s(\mathrm{e1}\wedge\mathrm{e2})|/2|r_c(\mathrm{e1}\wedge\mathrm{e2})|), \theta_2 = 300-\theta_1$.

When $0.4291 \le |r_c(\mathrm{e1}\wedge\mathrm{e2})|/|r_s(\mathrm{e1}\wedge\mathrm{e2})| < 0.5176$, $\theta_1 = \theta_2 = 150, d = |r_c(\mathrm{e1}\wedge\mathrm{e2})|$.

- $|r_c(\mathrm{e1}\wedge\mathrm{e2})|/|r_s(\mathrm{e1}\wedge\mathrm{e2})| < 0.4291 : \{3\frac{e_1\wedge e_2}{e_1\cdot e_2}.16*(135)\frac{e_1\wedge e_2}{e_1\cdot e_2}.16*(165)\frac{e_1\wedge e_2}{e_1\cdot e_2}\}$

(e.g., the sensor at position A) and {3. 8. 16∗(165)} (e.g., the sensor at position B), shown in Figure 1(h). When $0.4118 \le |r_c(\mathrm{e1}\wedge\mathrm{e2})|/|r_s(\mathrm{e1}\wedge\mathrm{e2})| < 0.4291$. $d = 0.4118|r_s(\mathrm{e1}\wedge\mathrm{e2})|$

When $|r_c(\mathrm{e1}\wedge\mathrm{e2})|/|r_s(\mathrm{e1}\wedge\mathrm{e2})| < 0.4118$, $d = |r_c(\mathrm{e1}\wedge\mathrm{e2})|$.

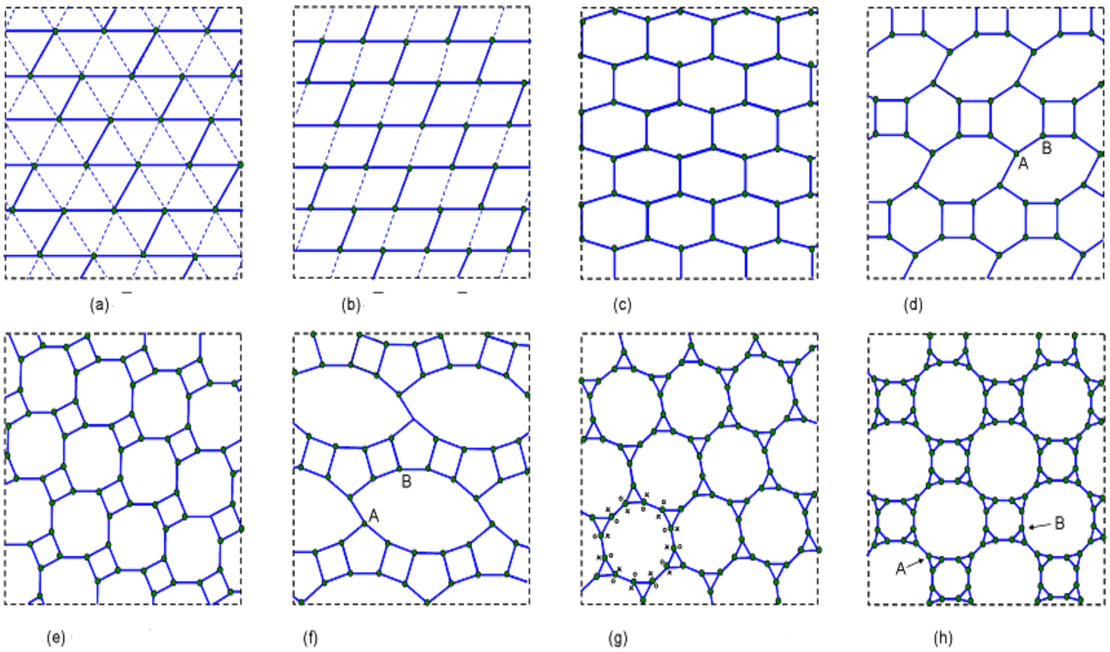

Fig 1: Optimal deployment patterns to achieve full-coverage and 3-connectivity for full range of $|r_c(\mathrm{e1}\wedge\mathrm{e2})|/|r_s(\mathrm{e1}\wedge\mathrm{e2})|$.

4. Conclusion

We built the Clifford 3-conncected sensor network model on the basis of ref [1]. Then we analyzed the metric relation in this model. We farther study WSN Clifford algebra k-connection graph. At last, we verify the rationality of our model and algorithm.

Clifford sensor network aims at monitoring and tracking moving targets in practice. So, the connecting coverage of Clifford sensor network in dynamic is the main objective of further study.

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