Advanced Engineering Forum *Online: 2011-09-09*
ISSN: 2234-991X, Vol. 1, pp 9-15
doi:10.4028/www.scientific.net/AEF.1.9

Multi-objective Particle Swarm Optimization with dynamic Crowding Entropy-based Diversity Measure

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Keywords: particle swarm optimization (PSO); multi-objective optimization problems; elitist archive; dynamic crowding entropy

Abstract. A multi-objective particle swarm optimization with dynamic crowding entropy-based diversity measure is proposed in this paper. Firstly, the elitist strategy is used in external archive in order to improve the convergence of this algorithm. Then the new diversity strategy called dynamic crowding entropy strategy and the global optimization update strategy are used to ensure sufficient diversity and uniform distribution amongst the solution of the non-dominated fronts. The results show that the proposed algorithm is able to find better spread of solutions with the better convergence to the Pareto front and preserve diversity of Pareto optimal solutions the more efficiently.

Introduction

Evolutionary algorithms have been successfully applied to multi-objective optimization problems (EMO). In 1985, Schaffer proposed vector evaluated genetic algorithms (VEGA) [1], it is seen as the pioneering work for solving multi-objective optimization by evolutionary algorithm. Scholars from various countries developed different evolutionary multi-objective optimization algorithms after 1990.Fonseca and Fleming proposed Multi-objective Genetic Algorithm (MOGA) [2], Sriniva and Deb proposed Non-dominated Sorting Genetic Algorithm (NSGA) [3], and Horn and Nafpliotis proposed Niched Pareto Genetic Algorithm (NPGA) [4] respectively in 1993.The corresponding improved versions with keeping elitists appeared soon after, i.e. NPGA2[5],NSGA-II[6]and so on.
Scholars have proposed multi-objective algorithms based on new pattern evolutionary algorithms in recent years.Multi-objective particle swarm optimization algorithm based on dynamic crowding distance and its application [7], and multi-objective self-adaptive differential evolution with elitist archive and crowding entropy-based diversity measure[8].A multi-objective particle swarm optimization with dynamic crowding entropy strategy (MOPSO-DCE),which combines the elitist archive strategy, dynamic crowding entropy strategy and the update of global optimal strategy, is introduced in this paper.

Multi-objective optimization problem and related concepts

Owing to that minimization and maximization are essentially the same optimization problems, we only consider the minimization problem.
Definition 1[9] (Multi-objective optimization problem,MOP) A general multi-objective optimization problem with k conflicting objectives can be described as follows:

$$\begin{cases} \min & \mathbf{y} = \mathbf{f(x)} = (f_1(\mathbf{x}), f_2(\mathbf{x}), ..., f_k(\mathbf{x})), \\ \text{s.t.} & \mathbf{x} = (x_1, x_2, ..., x_n) \in X, \\ & \mathbf{y} = (y_1, y_2, ..., y_k) \in Y. \end{cases} \quad (1)$$

where $\mathbf{x}$ is decision vector and X is the decision space, $\mathbf{y}$ is the objective vector and Y is the objective space.

Definition 2[9] (Pareto dominance)A vector $\mathbf{u} = (u_1, u_2, ..., u_k)$ is said to dominate another vector $\mathbf{v} = (v_1, v_2, ..., v_k)$ (denoted by $\mathbf{u} \prec \mathbf{v}$), if and only if $\mathbf{u}$ is partially less than $\mathbf{v}$,i.e. $\forall i \in (1, 2, ..., k)$, $(u_i \le v_i) \wedge (\exists i \in \{1, 2, ..., k\} : u_i < v_i)$.

Definition 3[9] (Pareto optimality) A solution $\mathbf{z} = (z_1, z_2, ..., z_n)$ is said to be Pareto optimal with respect to feasible areas if and if only there is no $\mathbf{w} = (w_1, w_2, ..., w_n)$ for which $\mathbf{f(w)}$ dominates $\mathbf{f(z)}$.

Definition 4[9] (Pareto-optimal set) The Pareto optimal set PS is defined as the set of all Pareto optimal solutions.

Definition 5[9] (Pareto front) The Pareto optimal front PF is defined as the set of all objective functions values corresponding to the solutions in PS .

Definition 6[10] (Convergence Metric γ) This metric is defined as: $\gamma = \frac{\sum_{i=1}^{n} d_i}{n}$ where n is the number of non-dominated solutions found so far, d_i is the Euclidean distance between the i th solution of the n obtained solutions and its nearest neighbor on the true Pareto optimal front.

Definition7[10](Spread Metric Δ)This metric is defined as: $\Delta = \frac{d_f + d_l + \sum_{i=1}^{n-1} \left| d_i - \bar{d} \right|}{d_f + d_l + (n-1)\bar{d}}$ Where n is the number of non-dominated solutions found so far. The parameter d_i is the Euclidean distance between neighboring solutions in the obtained non-dominated solutions set and $\bar{d}$ is the mean of all d_i . The parameter d_f and d_l are the Euclidean distance between the extreme solutions and the boundary solutions of the obtained non-dominated set.

Multi-Objective Particle Swarm Optimization with Dynamic Crowding Entropy Strategy

Basic PSO

The velocity of particle and its new position will be assigned according to the following two equations[11]:

$$v_i(t+1) = wv_i(t) + c_1 r_1 (p_i(t) - x_i(t)) + c_2 r_2 ((p_g(t) - x_i(t)), \quad (2)$$

$$x_i(t+1) = x_i(t) + v_i(t), \quad (3)$$

where the superscript t denotes the t^{th} iteration; c_1 and c_2 are positive constants, called the cognitive and social parameter respectively, r_1 and r_2 are random numbers uniformly distributed in the range (0,1). This paper adapts linearly decreasing inertia weight[12] w.

External elitist archive strategy

An external elitist archive is used to store non-dominated solutions found so far in the whole evolution process. Initially, An external elitist archiveis empty. Table 1 will give the pseudo-code of external elitist archive strategy. Where $\mathbf{A}$ is the set of non-dominate solutions in the current archive; $\mathbf{x}$ is new non-dominate solutions.

Table 1 The pseudo-code of external elitist archive strategy

If $\mathbf{x}$ is dominated by any member of $\mathbf{A}$ in external elitist archive discard $\mathbf{x}$
Else if $\mathbf{x}$ dominates a set of member $\mathbf{Y(A)}$, $\mathbf{A}=\mathbf{A}/\mathbf{Y(A)}$
Else if $\mathbf{x}$ and $\mathbf{A}$ are non-dominated each other $\mathbf{A}=\mathbf{A}\cup\{\mathbf{x}\}$

The size of external elitist archive increase gradually as the evolution process, and its computational complexity is $O(ktN^2)$,where t is iteration; k is the number of objective ; N is the population size. As the evolution process, if there is no control of external elitist archive, computational complexity will greatly increase. Therefore, when the external archived population reaches its maximum capacity, the crowding entropy measure is used in [8]. In this paper, the dynamic crowding entropy strategy based on crowding entropy measure is proposed in next section.

Dynamic Crowding Entropy

In this paper, we present a dynamic crowding entropy strategy to remain the size of external elitist archive, which can assure the spread of solutions with the better convergence to the Pareto front and the uniformity of Pareto optimal solutions. We give the definition of crowding entropy [8]as following:

$$CE_i=\sum_j^k c_{ij}E_{ij}\Big/(f_j^{\max}-f_j^{\min}) \tag{5}$$

where $E_{ij}=-[pl_{ij}\log(pl_{ij})+pu_{ij}\log(pu_{ij})]$, $pl_{ij}=\dfrac{dl_{ij}}{c_{ij}}$, $pu_{ij}=\dfrac{du_{ij}}{c_{ij}}$, $c_{ij}=dl_{ij}+du_{ij}$

where the parameters $f_j^{\max}$ and $f_j^{\min}$ are the maximum and minimum values of the j th objective function and k is the number of objective functions, dl_{ij} and du_{ij} are the distances of the i th solution to its lower and upper adjacent solution along the j th objective function.

In the external archived maintenance process, if the size of elitist external archive M does not reach its maximum capacity N , then the new generated non-dominated solutions store in the external elitist archive; otherwise, we will adopt the dynamic crowding entropy strategy to removing $N-M$ individuals one by one from the elitist external archive, its update strategy is as follows:1) Calculate the crowding entropy CE_i of each individual in external elitist archive by Eq.(5); 2) Sort the crowding entropy CE_i ;3) Remove the minimum individual of the crowding entropy from external elitist archive;4) If the size of elitist external archive $M\le N$,then stop; Otherwise returns 1).

Dynamic crowding entropy strategy has one important characteristic in maintenance external elitist archived: remove only one individual of external elitist archive at a time; then recalculate the crowding entropy CE_i of each individual in external elitist archive, this method can prevent the removal more than one at a time of a region caused by the phenomenon of missing individuals and obtain more uniformly distributed Pareto front.

Update of global optimal strategy

In generally, archive strategy was used in multi-objective particle swarm optimization. Firstly, the non-dominated solutions generated in iterative process were stored in an external archive, then randomly selected a particle from the external archive as the global optimal position, this selection strategy lose the opportunity to get non-dominated solutions in dense regions so that the population loss diversity. We should make particles in scattered region search relative, in order to ensure that the diversity of population and the uniformly distribution of Pareto front. Therefore, we use the following strategy to update the global optimal:

1) If the crowding entropy value of each individual is infinite in the elitist external archive, which includes only a small number of boundary individual, then randomly select one as $\mathbf{p_g}$.
2) If the crowding entropy value of each individual is not infinite in the elitist external archive, roulette wheel selection method is used, namely the greater probability selected the individual as $\mathbf{p_g}$. Computation formula is:

$$p(x_i) = \frac{CE_i}{\sum_{i=1}^{M} CE_i}$$

where $p(x_i)$ is selected probability of i, CE_i is the crowding entropy of i, M is the size of the elitist external archive. Notice, the individual with infinite crowding entropy results failure of roulette wheel selection method. So we define the crowding entropy of the individuals, the crowding entropy of which are infinite or boundary, are the median of rest individuals.

Description of MOPSO-DCE algorithm

Step 1 Initialization populations. The maximum iteration t_{max}, let the size of internal population $\mathbf{x}$ is N, and generate randomly the position of each particle $\mathbf{x_i}$ in feasible decision space, let the initial velocity $\mathbf{v_i}$ of each particle is 0, and the optimal of each individual $\mathbf{p_i}$, the size of external population $\mathbf{A}$ is M ,and $\mathbf{A} = [\ ]$,then calculate the fitness of each particle.

Step 2 Update the external elitist archive $\mathbf{A}$ according to Table 1and calculate the crowding entropy of each particle in the external elitist archive $\mathbf{A}$.

Step 3 According to the update of global optimal value strategy to update new $\mathbf{p_g}$.

Step 4 The velocity and position of the internal population are updated according to Eq.(2) and Eq.(3). The extreme of each individual is updated according to the domination.

Step 5 Update the external elitist archive $\mathbf{A}$.

Step 6 If the maximum iteration is reached, stop and output Pareto optimal solution set, otherwise return **Step3**.

Experimental results

To validate our approach, we adopted the test problems[8](ZDT1,ZDT2,ZDT3,ZDT6) and the methodology normally adopted in the evolutionary multi-objective optimization literature, where the convergence metric γ and diversity metric Δ proposed in [10] are applied.

The Results of Corresponding Comparision

In order to know how competitive the proposed approach was, it is compared with the NSGA-II, one of the most classical evolutionary multi-objective algorithms are given as follows:

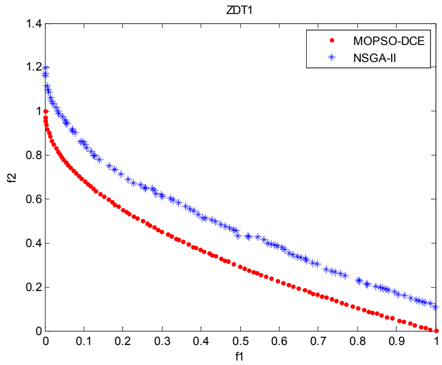

Fig 1

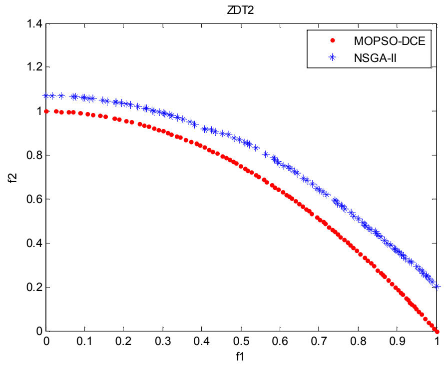

Fig 2

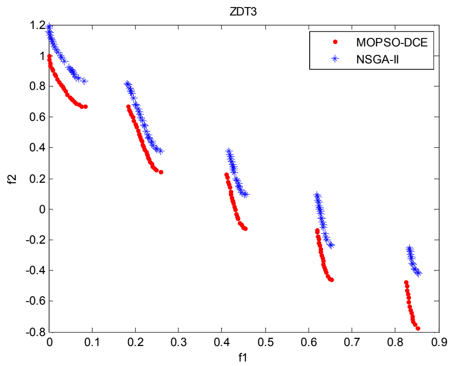

Fig 3

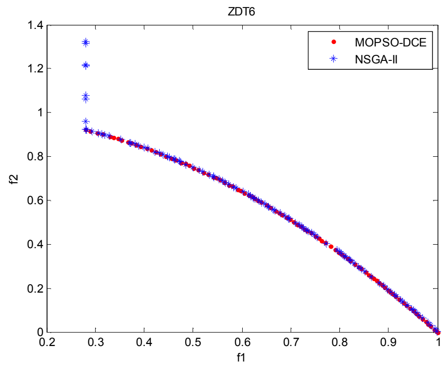

Fig 4

Fig 1-Fig 4 gives the Pareto Front of all test functions in objective space using different algorithms. Clearly, the Pareto Fronts of MOPSO-DCE are below the Pareto Front of NSGA - II.

To further confirm the efficiency and feasibility of algorithm, we will compared the result of MOPSO-DCE algorithm and the six classical algorithms[8,13-14].

Table 2 Statistics of Results on ZDT1

Algorithm	Convergence Metric γ	Spread Metric Δ
NSGA-II	0.033482 ± 0.004750	0.390307 ± 0.001876
SPEA	0.001799 ± 0.000001	0.463292 ± 0.041622
PAES	0.082085 ± 0.008679	1.229794 ± 0.000742
PDEA	N/A	0.298567 ± 0.000742
MODE	0.005800 ± 0.000000	N/A
MOPSO	$0.018577 \pm 7.23e-5$	$0.580741 \pm 3.65e-3$
MOPSO-DCE	$0.0011 \pm 6.9417e-9$	0.2133 ± 0.0058

Table 3 Statistics of Results on ZDT2

Algorithm	Convergence Metric γ	Spread Metric Δ
NSGA-II	0.072391 ± 0.031689	0.430776 ± 0.004721
SPEA	0.001339 ± 0.000000	0.755784 ± 0.004521
PAES	0.126276 ± 0.036877	1.165942 ± 0.007682
PDEA	N/A	0.317958 ± 0.001389
MODE	0.005500 ± 0.000000	N/A
MOPSO	$0.0017045 \pm 5.92e-4$	$0.650889 \pm 7.97e-2$
MOPSO-DCE	$7.8199e-4 \pm 3.2004e-9$	$0.1762 \pm 3.2237e-4$

Table 4 Statistics of Results on ZDT3

Algorithm	Convergence Metric γ	Spread Metric Δ
NSGA-II	0.114500 ± 0.004940	0.738540 ± 0.019706
SPEA	0.047517 ± 0.000047	0.672938 ± 0.003587
PAES	0.023872 ± 0.000010	0.789920 ± 0.001653
PDEA	N/A	0.623812 ± 0.000225
MODE	0.021560 ± 0.000000	N/A
MOPSO	0.130567 ± 5.54e-5	0.543900 ± 1.88e-3
MOPSO-DCE	0.0044 ± 1.3287e-5	0.4477 ± 2.5163e-4

Table 5 Statistics of Results on ZDT6

Algorithm	Convergence Metric γ	Spread Metric Δ
NSGA-II	0.296564 ± 0.013135	0.668025 ± 0.009923
SPEA	0.221138 ± 0.000449	0.849389 ± 0.002713
PAES	0.085469 ± 0.006664	1.153052 ± 0.003916
PDEA	N/A	0.473074 ± 0.021721
MODE	0.026230 ± 0.000861	N/A
MOPSO	0.330672 ± 7.73e-1	0.963582 ± 5.22e-4
MOPSO-DCE	0.0013 ± 9.6519e-7	0.2189 ± 0.0123

As can be seen from Table 2-5, whether from the Convergence degree of Pareto-optimal set or the uniform of the Pareto-optimal set, the new algorithm is better than other algorithms. It shows that the proposed MOPSO-DCE is feasible and effective, and it can be used for solving multi-objective optimization problem.

Conclusion

This paper proposed MOPSO-DCE, which combines the elitist archive strategy, dynamic crowding entropy strategy and the update of global optimal strategy. The results show that the proposed algorithm generally outperforms in convergence and diversity performance..

The work is supported by the National Natural Science Foundation of China (60962006).

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