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Application of P1-nonconforming Element for Shell Structure of Incompressible Materiel

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Abstract. This research focused on solving volumetric locking problem of shell structure of incompressible material. Degenerated solid-shell elements are widely applied on curved structure. But, volumetric locking will take place when the structure is made of incompressible material, such as rubber. Due to Poisson's locking free property of P1-nonconforming element, it is employed to solve volumetric locking problem of shell structure. Furthermore, the study on shell structure is extended to topology optimization design. To verify the volumetric locking free of P1-nonconforming element on shell structure of incompressible material, some structures are studied by different elements. Comparing with the utilization of high order elements to solve volumetric locking problems, P1-nonconforming elements can save calculation time and reduce the numerical cost.

1. Introduction

P1 quadrilateral nonconforming element [1] is a kind of discontinuous and Poisson locking-free element. It could solve volumetric locking and membrane locking problems caused by inextension of incompressible materials. Comparison with other locking-free triangular or quadrilateral nonconforming elements [2, 3], P1-nonconforming element has linear shape functions and reduced numerical cost could be expected. According to displacement-based formulation, P1-nonconforming element is applied on degenerated solid-shell elements problems.

This study is extended to topology optimization design. Topology optimization is a mathematical approach for optimizing material layout [4, 5]. It could provide best concept design that meets the design requirements for engineers. Thus, it is widely used in structural mechanics because it could reduce product development time and overall cost while improving design performance. Some numerical examples are analyzed and compared with reference results presented in other papers.

2. Theory Formulation

2.1 P1- nonconforming Element

The distinctive characteristic of P1- nonconforming element compared to conforming element is that the nodes of P1 element are located at the middle points of the edges, such as m_1, m_2, m_3 and m_4 as shown in Fig. 1.

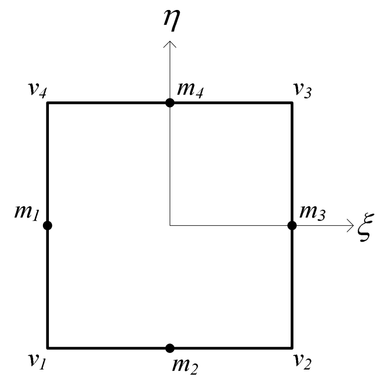


Fig. 1. P1-nonconforming element

This property would lead to the model discretized by P1 elements to be continuous only at the middle points of each element. The shape functions of P1- nonconforming element are

$$\widetilde{\varphi}_1 = \frac{1}{2}(1-\xi-\eta),\quad \widetilde{\varphi}_2 = \frac{1}{2}(1+\xi-\eta),\quad \widetilde{\varphi}_3 = \frac{1}{2}(1+\xi+\eta),\quad \widetilde{\varphi}_4 = \frac{1}{2}(1-\xi+\eta) \tag{1}$$

The displacement of P1 element could be expressed as

$$u = u(m_1)\widetilde{\varphi}_1 + \{u(m_2) - u(m_1)\}\widetilde{\varphi}_2 + \{u(m_3) - u(m_2) + u(m_1)\}\widetilde{\varphi}_3 \tag{2}$$

The middle points could also be calculated by vertex v_i (i=1,2,3,4). Then displacement u could be expressed by

$$u = \frac{\widetilde{\varphi}_1}{2}u(v_1) + \frac{\widetilde{\varphi}_2}{2}u(v_2) + \frac{\widetilde{\varphi}_3}{2}u(v_3) + \frac{\widetilde{\varphi}_4}{2}u(v_4) \tag{3}$$

2.2 Basic Equations of Degenerated Solid-Shell Element

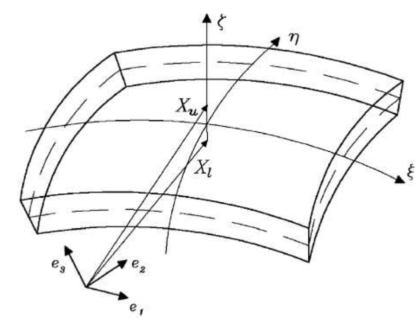


Fig. 2. Geometry of degenerated solid-shell element

According to the coordinate of Fig. 2, each point of solid-shell element could be described as following relations:

$$\boldsymbol{x}(\xi,\eta,\zeta) = \frac{1}{2}[(1+\zeta)\boldsymbol{X}_u(\xi,\eta) + (1-\zeta)\boldsymbol{X}_l(\xi,\eta)] \tag{4}$$

where $\boldsymbol{X}_u(\xi,\eta)$ and $\boldsymbol{X}_l(\xi,\eta)$ are responding vectors on top surface and bottom surface.

By using the strain-displacement transformation, the strain vectors $\{\varepsilon\}$ may be written as

$$\{\varepsilon\}=\sum_{a=1}^{n_{en}}\boldsymbol{B}_a\begin{Bmatrix}u_a\\ v_a\\ w_a\\ \theta_{a1}\\ \theta_{a2}\end{Bmatrix} \tag{5}$$

where n_{en} is the number of nodes for each solid-shell element and a is the order of each node. Vectors $\{u_a, v_a, w_a\}$ is the displacements of node a in lamina coordinate system. The quantities θ_{a1} and θ_{a2} represent the rotations of fiber about the basis vectors. $\boldsymbol{B}_a$ is the strain-displacement matrix and could be obtained by calculating displacement gradients.

By principle of minimum potential energy, the relationship of load vectors $\boldsymbol{F}$ and displacement vectors $\boldsymbol{U}$ can be expressed as

$$\boldsymbol{KU}=\boldsymbol{F} \tag{6}$$

where $\boldsymbol{K}$ is the global stiffness matrix and can be expressed as

$$\boldsymbol{K}=\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1}\boldsymbol{B}^{T}\boldsymbol{DB}\,|J|\,d\zeta\, d\xi\, d\eta \tag{7}$$

where $|J|$ is the determinant of Jacobi matrix and can be expressed as

$$|J|=\det\begin{bmatrix}x_{1,\xi} & x_{1,\eta} & x_{1,\zeta}\\ x_{2,\xi} & x_{2,\eta} & x_{2,\zeta}\\ x_{3,\xi} & x_{3,\eta} & x_{3,\zeta}\end{bmatrix} \tag{8}$$

2.3 Topology Optimization of Shell Element

Following Sigmund and Clausen [6], the Young's modulus, E, for element e are parameterized with respect to its corresponding design variable, ρ_e, as

$$E(\rho_e)=E_e=E_{void}+\rho_e^{p}(E_{solid}-E_{void}) \tag{9}$$

where the subscripts, solid and void, indicate the state of the material. In Eq. 9, $0\le\rho_e\le 1$, and p is the penalty exponent used to push the optimization solution toward a discrete 0-1 design.

A mean compliance minimization problem by using the P1-nonconforming element is formulated as following equations:

$$\underset{\rho\in\Re^{N_D}}{\text{Minimize}}\ : c(\rho)=F^{T}U \tag{10a}$$

$$\text{Subject to}:\sum_{e=1}^{N_D}\rho_e v_e\le V \tag{10b}$$

$0 \le \rho_e \le 1$, e=1, 2, 3, …N_D

where $\boldsymbol{U}$ and $\boldsymbol{F}$ are the displacement solution vector and force vector, respectively. v_e is the element volume, V is the predefined volume limit and N_D is the number of design variables. Thus, the new stiffness matrix will be expressed by

$$K_{new} = \sum_{e=1}^{N_D} k_e = \sum_{e=1}^{N_D} \int_{\Omega_e} B^T D_e B \, d\Omega \tag{11}$$

where k_e denotes the element stiffness matrix of element e . In Eq. 12, D_e is the matrix representation of the elasticity tensor of element e:

$$D_e = \frac{\rho^p E}{(1-v^2)} \begin{bmatrix} 1 & v & 0 & 0 & 0 \\ v & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{1-v}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{k(1-v)}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{k(1-v)}{2} \end{bmatrix} \tag{12}$$

where $\boldsymbol{E}$ is Young's modulus and v is Poisson's ratio. Considering the deformation of cross-sectional area, shear correction factor k is introduced.

3. Numerical Examples

3.1 Application of P1-nonconforing Element for Shell Structure

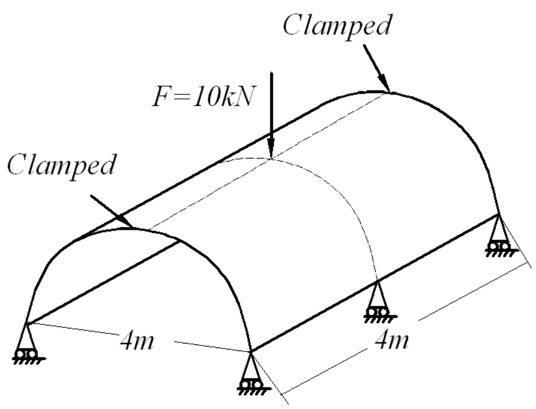
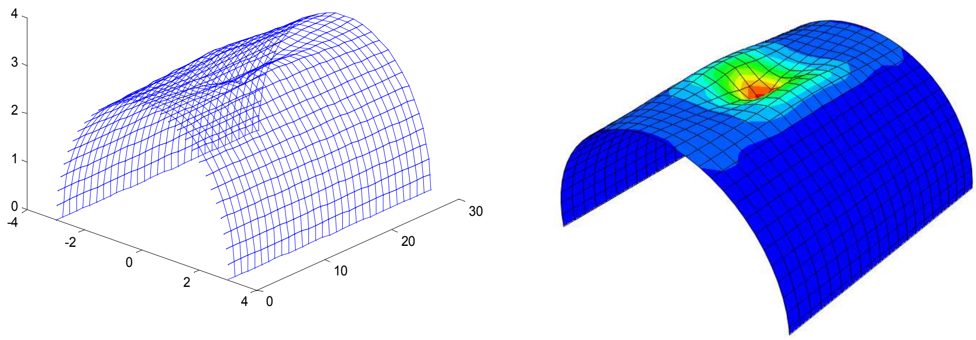


Fig. 3. Half of cylindrical shell structure with clamped ends

As shown in Fig.3, there is a half of cylindrical shell structure which is made of incompressible material. The radius is 2m and the thickness is 0.01m. Young's modulus E is 300 GPa and Poisson's ratio v is 0.5. There is a concentrated load applied at the middle point and both ends of the structure are clamped.

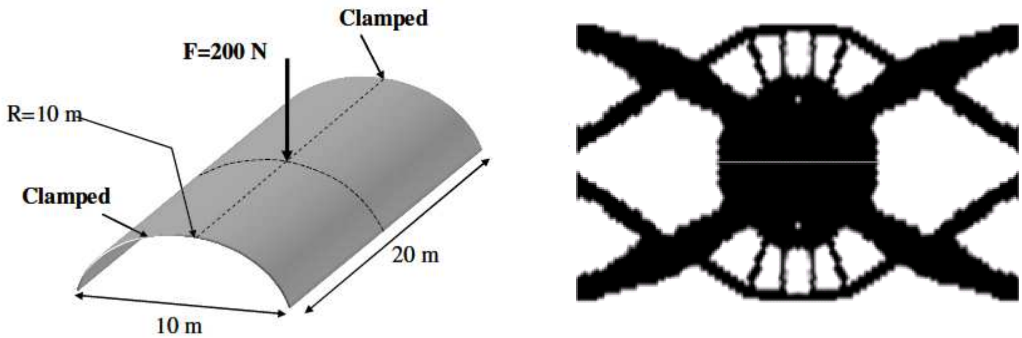
To avoid volumetric locking problem, P1-nonconforming element is employed. Fig. 4(a) illustrates the deformation of clamped ends shell structure which is plotted by Matlab with calculated results. The maximum displacement at the center of the structure is 0.87 mm. To verify the result presented above, a common FEM software ABAQUS [7] is used to simulate the deformation of model as shown in Fig. 4(b). The accurate result which is obtained from ABAQUS is 0.849 mm. Comparing with the deformation presented by different methods, it is observed P1-nonconforming could be applied to analyze shell strucuture of incompressible material and get reasnonable results.



(a) (b)

Fig. 4. Deformation of shell structure calculated by (a) P1-nonconforming element and (b) ABAQUS

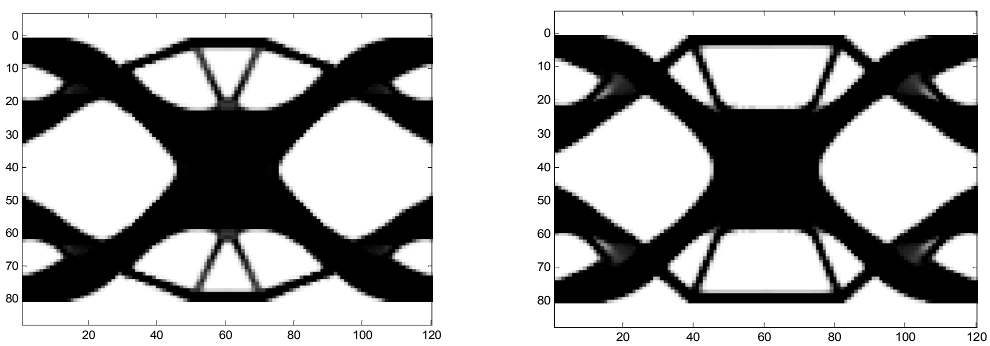
3.2 Optimal Shape of Shell Structure



(a) (b)

Fig. 5. (a) Ends clamped roof structure and (b) reference topology optimal shape [8]

As shown in Fig. 5 (a), the design domain and boundary conditions of ends clamped roof structure problem are illustrated. A concentrated load F is applied at the middle point of the roof structure. The full domain is discretized with 120×80 degenerated solid-shell elements. The thickness of the roof is t=0.1m. The material properties used in this example are elastic modulus E =210 GPa and Poisson's ratio v=0.3. The volume used for the reinforced region is constrained below 50% of the design domain.



(a) (b)

Fig. 6. Topology optimal shape obtained by (a) P1-nonconforming and (b) Q4 conforming element

Figures 6(a) and 6(b) show the optimized enforced structures of P1-nonconforming element and Q4 conforming element, whose main parts are almost the same with reference topology optimal shape of Fig. 5(b). It could be observed there are some small differences between the optimal shapes of conforming and nonconforming degenerated solid-shell elements. Comparison of objective function values between two kinds of elements, it can be found that the mean compliance of P1-nonconforming element will smaller than that of conforming element. Thus, it could be considered the enforced structures of P1-nonconforming element are more reasonable than that of conforming element.

4. Discussions and Conclusions

The main purpose of this paper is to apply P1-nonconforminge element to solve shell structure problem. From numerical example presented above, it could be observed P1-nonconforming element could solve volumetric locking problem of shell structure completely due to its Poisson's locking free property. Comparing with the utilization of high order elements to solve volumetric locking problems, P1-nonconforming elements could save calculation time and reduce the numerical cost.

The optimal topology of the shell structure presented above is reliable and helpful for other researches. Even though this structure still could not work as a final optimal design because it was only analyzed as a static model, it can be useful and could work as initial model for other analysis.

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