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Adaptive State Feedback Control of Electromagnetic Levitation System for Uncertain Ball Mass

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Abstract. For the last several decades, many results have been presented for controlling nonlinear systems that have parameter uncertainty. In this paper, we propose an adaptive state feedback controller based on input-output feedback linearization for electromagnetic levitation system(EMS) with unknown ball mass. We analytically show the regulation of the controlled electromagnetic levitation system by the proposed adaptive state feedback controller. We show the experiment results of electromagnetic levitation system and where there is uncertain ball mass.

Introduction

In recent years, many contributions have been presented in control literature that solve the control design problem for a class of nonlinear systems. Input-output feedback linearization is one of the well known popular nonlinear control schemes to deal with nonlinear systems. For the exactly known system dynamics, the stability, regulation, and tracking problem have been successfully solved in the literature of [1],[3],[5],[7]. However, in real practice, since there exists parameter uncertainty, it is difficult to use exact feedback linearization directly. To overcome this limitation, many adaptive and robust schemes have been developed [2],[4],[6],[9]. In this paper, we propose an adaptive control under input-output feedback linearization for electromagnetic levitation system where there is uncertain ball mass. We provide the stability analysis of the controlled system. We use the Quanser's electromagnetic levitation system to show experimental results to verify the validity of our proposed method. The proposed adaptive controller is derived based on the feedback linearization method and adaptive function is engaged in order to deal with the uncertain ball mass.

Modeling

The EMS(Quanser) can be modeled as [10]

$$V = RI + L(\frac{d}{dt}I)$$

$$\ddot{x} = -\frac{1}{2}\frac{K_m I^2}{Mx^2} + g \tag{1}$$

where x is the position of steel ball, R is the coil resistance, I is the coil current, L is the coil inductance, M is the mass of ball and g is the gravitational constant. We define the system states as follows

$$x = x_1, \quad \dot{x} = x_2, \quad I = x_3, \quad V = u, \quad y = x_1 \tag{2}$$

where u is the system input and y is the system output. Then, the state equation is given by

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = g - \frac{1}{2}\frac{K_m x_3^2}{M x_1^2}$$
$$\dot{x}_3 = -\frac{R}{L}x_3 + \frac{1}{L}u$$
$$y = x_1 \tag{3}$$

Since the ball mass is not known exactly, we let

$$M = M_o + \Delta M \tag{4}$$

where M_o is the nominal ball mass and ΔM is the uncertain ball mass. We obtain following the equation from the equation (3).

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = g - \frac{1}{2}\frac{K_m x_3^2}{(M_o + \Delta M)x_1^2} = g - \frac{1}{2}\frac{K_m x_3^2}{x_1^2}\frac{1}{(M_o + \Delta M)} = g - \frac{1}{2}\frac{K_m x_3^2}{x_1^2 M_o} + \frac{1}{2}\frac{K_m x_3^2}{x_1^2}\underbrace{\frac{\Delta M}{M_o(M_o + \Delta M)}}_{\theta}$$
$$\dot{x}_3 = -\frac{R}{L_c}x_3 + \frac{1}{L_c u}$$
$$y = x_1 \tag{5}$$

Adaptive controller design

In this section, we propose an adaptive control for the EMS where is uncertain ball mass. We let $y = z_1 = x_1$.

$$z_1 = x_1$$
$$z_2 = x_2$$
$$z_3 = g - \frac{1}{2}\frac{K_m x_3^2}{x_1^2 M_o} + \frac{1}{2}\frac{K_m x_3^2}{x_1^2}\theta \tag{6}$$

Then through the input-output feedback linearization, we obtain

$$\dot{z}_1 = \dot{x}_1 = x_2 = z_2$$
$$\dot{z}_2 = \dot{x}_2 = z_3$$
$$\dot{z}_3 = \left(\frac{x_2 K_m}{x_1} + \frac{RK_m}{L}\right)\left(\frac{x_3}{x_1}\right)^2\left(\frac{1}{M_O} - \theta\right) - \frac{K_m}{L}\frac{x_3}{x_1^2}\left(\frac{1}{M_O} - \theta\right)u$$
$$= \alpha(x) + \beta(x)u \tag{7}$$

where $\alpha(x)$ and $\beta(x)$ is given by

$$\alpha(x,\theta) = \left(\frac{x_2 K_m}{x_1} + \frac{RK_m}{L}\right)\left(\frac{x_3}{x_1}\right)^2\left(\frac{1}{M_O} - \theta\right),\ \beta(x) = -\frac{K_m}{L}\frac{x_3}{x_1^2}\left(\frac{1}{M_O} - \theta\right)u \tag{8}$$

Hence, the state equation is

$$\dot{z} = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}}_{A} z + \underbrace{\begin{bmatrix} 0 \\ 0 \\ \beta(x) \end{bmatrix}}_{B} u + \underbrace{\begin{bmatrix} 0 \\ 0 \\ \alpha(x,\theta) \end{bmatrix}}_{\delta(x,\theta)}$$
$$y = \underbrace{\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}}_{C} \tag{9}$$

where u is a control input to the EMS. Hence

$$u = \frac{1}{\beta(x)}(-\alpha(x) + v) \tag{10}$$

and v is given by

$$v = \frac{k_1}{\varepsilon^3} z_1 + \frac{k_2}{\varepsilon^2} z_2 + \frac{k_3}{\varepsilon} z_3 \tag{11}$$

Note that z_3 contains the uncertain parameter θ. We set $\hat{z}_3$ as

$$\hat{z}_3 = g - \frac{1}{2}\frac{K_m x_3^2}{x_1^2}\frac{1}{M_O} + \frac{1}{2}\frac{K_m x_3^2}{x_1^2}\hat{\theta} \tag{12}$$

where $\hat{\theta}$ is the estimate of θ, we let

$$\xi_3 = \int_0^t \hat{\theta} d\tau \tag{13}$$

Based on (13), an adaptation law of $\hat{\theta}$ is defined by

$$\hat{\theta} = \frac{\eta \xi_3}{\Omega_3 + c} \tag{14}$$

where η is an adaptive to be chosen gain and $\Omega_3 = \dfrac{\partial \theta(x, M_O)}{\partial z_3}$.

Convergence of $\hat{\theta}$

We let $\psi(x,\theta) = z$, where z includes the uncertain ball mass (θ) such that $\dfrac{\partial \hat{z}(x,\theta_0)}{\partial \hat{\theta}}$. In order to estimate θ, we define

$$\xi_i = \int_0^t \psi_i(x,\hat{\theta}) d\tau, \qquad 1 \le i \le m \tag{15}$$

where $\hat{\theta}$ is the estimate of θ. Based on (15), an adaptation law of $\hat{\theta}$ is defined by

$$\hat{\theta} = \theta_O + \sum_{i=1}^{m} \phi_i \quad \text{with} \quad \phi_i = -\frac{\eta_i}{\Omega_i}\xi_i \tag{16}$$

where η_i is an adaptive gain and $\Omega_i = \dfrac{\partial \psi_i(x,\theta_O)}{\partial \hat{\theta}}$. To preceed futher, we provide an intuition of the adaptation law (16) by considering the case when $m = 1$ for simplicity. Then, we note that from (16) if $\lim_{t\to\infty} \phi_1 = \theta - \theta_O$ is achieved, η_1 also converges to $\eta_1^{eq} = -\dfrac{(\theta - \theta_O)\Omega_1}{\xi_1}$ as $t \to \infty$. Tayor series expansion of (15) about $\hat{\theta} = \theta_O$ gives

$$\begin{aligned}\dot{\xi}_1 &= \psi(x,\theta_O) + \frac{\partial \psi_1(x,\theta_O)}{\partial \hat{\theta}}(\hat{\theta} - \theta_O) + o(\hat{\theta} - \theta_O) \\ &= \psi(x,\theta_O) + \Omega_1 \phi_1 + o(\phi_1) \\ &= \psi(x,\theta_O) - \eta_1 \xi_1 + o(\phi_1)\end{aligned} \tag{17}$$

where $o(\phi_1)$ is

$$o(\phi_1) = \psi_1(x,\hat{\theta}) - \psi_1(x,\theta_O) - \frac{\partial \psi_1(x,\theta_O)}{\partial \hat{\theta}}(\phi_1) \tag{18}$$

Since $\psi_1(x,\theta_O)-\eta_1\xi_1+o(\hat{\theta}-\theta_O)=0$ at the equilibrium point, $\xi_1=\xi_1^{eq}$ at $t\to\infty$ from (18), by choosing a sufficiently large η_1, it can be derived that

$$\psi_1(x,\theta_O)-\psi_1\xi_1+o(\phi_1)\begin{cases} >0 & \text{if} \quad \xi_1<\xi_1^{eq} \\ =0 & \text{if} \quad \xi_1=\xi_1^{eq} \\ <0 & \text{if} \quad \xi_1>\xi_1^{eq} \end{cases} \tag{19}$$

Thus, $\lim_{t\to\infty}\phi_1=\theta-\theta_O$ is achieved from $\lim_{t\to\infty}\xi_1=\xi_1^{eq}$.

Fig. 1 shows the general nature of convergence by (19). If the condition $\dfrac{\partial\hat{z}(x,\theta_O)}{\partial\theta}\neq 0$ does not hold, we cannot apply the adaptation law in (16) directly. So, in that case, we modify our estimator slightly and still make the whole system asymptotically stable. Instead of our previous choice, we choose the parameter estimator as

$$\hat{\theta}-\theta_O=\Delta\hat{\theta}=\sum_{i=1}^{m}\phi_i \quad \text{with} \quad \hat{\phi}_i=-\frac{\eta_1}{\Omega_i+c_i}\xi_i \tag{20}$$

where $\Omega_i+c_i\neq 0$. We only consider the case $m=1$. Then the adaptation dynamic becomes

$$\dot{\xi}_1=\psi_1(x,\theta_O)-\overline{\eta}_1\xi_1+o(\phi_1) \quad \text{with} \quad \overline{\eta}_1=\frac{\Omega\eta_1}{\Omega_1+c_1} \tag{21}$$

As shown in Fig. 2, ξ_i^{eq} is shifted to $\xi_i^{eq'}$ but the steady-state still remains as a constant. Therefore $\dot{\xi}_i=z_i\to 0$ again.

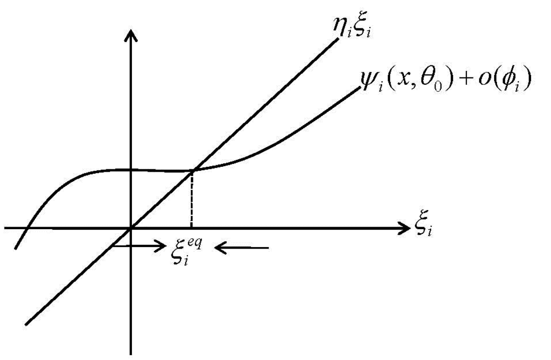

Fig. 1. Determination of ξ_i^{eq} by setting $\eta_i\xi_i$

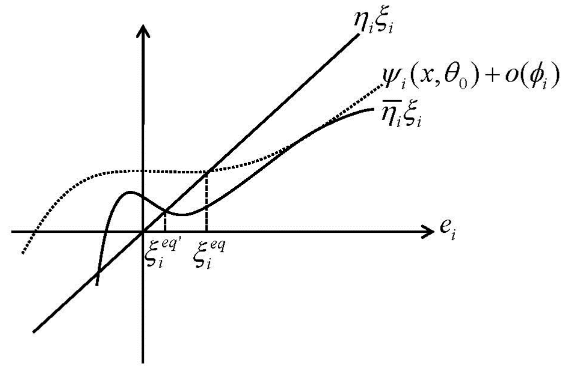

Fig. 2. Convergence by modified estimator with c_i

Analysis of system stability

The closed-loop dynamics is

$$\dot{z}=A(\varepsilon)z+\delta(x,\theta) \tag{22}$$

where $A(\varepsilon)=A+BK(\varepsilon)$, we denote $E(\varepsilon)=diag[1,\varepsilon,\varepsilon^2]$.

$$\begin{aligned} &\varepsilon A(\varepsilon)=E(\varepsilon)^{-1}AE(\varepsilon) \\ &\qquad A=\varepsilon E(\varepsilon)A(\varepsilon)E(\varepsilon)^{-1},\quad A^T=\varepsilon E(\varepsilon)^{-1}A^T(\varepsilon)E(\varepsilon) \end{aligned} \tag{23}$$

By Lyapunov equation $A^TP+PA=-I$, then we have the following equation

$$\begin{aligned} &A^T(\varepsilon)P(\varepsilon)+P(\varepsilon)A(\varepsilon)=-\varepsilon^{-1}E(\varepsilon)^2 \\ &P(\varepsilon)=E(\varepsilon)PE(\varepsilon) \end{aligned} \tag{24}$$

We set $V_c(z)=z^TP(\varepsilon)z$ for (22). Then, along the trajectory of (22)

$$
\begin{aligned}
\dot{V}_c(z) &= \dot{z}^T P(\varepsilon)z + z^T P(\varepsilon)\dot{z} \\
&= [A(\varepsilon)z + \delta(x,\theta)]^T P(\varepsilon)z + z^T P(\varepsilon)[A(\varepsilon)z + \delta(x,\theta)] \\
&= z^T [A^T(\varepsilon)P(\varepsilon) + P(\varepsilon)A(\varepsilon)]z + 2z^T P(\varepsilon)[\delta(x,\theta)] \\
&\le -\varepsilon^{-1} \| E(\varepsilon)z \|^2 + 2z^T P(\varepsilon)[\delta(x,\theta)] \\
&\le -\varepsilon^{-1} \| E(\varepsilon)z \|^2 + 2\| P \| \| E(\varepsilon)z \| \| E(\varepsilon)\delta(x,\theta) \|
\end{aligned} \quad (25)
$$

We note that $\| E(\varepsilon)\delta(x,\theta) \| \le \gamma(\varepsilon) \| E(\varepsilon)z \|$ where $\gamma(\varepsilon) = 2\| P \| \alpha(x,\theta)$.

$$\dot{V}_c(z) \le -\varepsilon^{-1} \| E(\varepsilon)z \|^2 + 2\| P \| \| E(\varepsilon)z \| \| E(\varepsilon)\delta(x,\theta) \| \le -(\varepsilon^{-1} - \gamma(\varepsilon)) \| E(\varepsilon)z \|^2 \quad (26)$$

$\dot{V}_c(z)$ is negative definite when $\varepsilon^{-1} - \gamma(\varepsilon) > 0$.

Experimental results

The values of the parameters of the EMS are listed in Table 1. The experiment is carried out on the Quanser's EMS. The composition of equipment is shown Fig 3. Also, the power module used is the Quanser VoltPAQ-X1 with ± 24 and $5A$ output.

Parameter	Description	Value
L	Coil Inductance	412.5mH
R	Resistance	11 Ω
K_m	Electromagnet Force Constant	$6.5308 \times 10^{-5} N \cdot m^2 / A^2$
M_O	Nominal Mass of Ball	0.064kg
ΔM	Uncertain Mass of Ball	$\Delta M < \pm 0.3Kg$
T_b	Air gap	0.02m
g	Gravitational Constant	9.81m/s²

Table. 1 EMS parameter specifications

Fig. 3 Experimental set of EMS

To evaluate the performance, experimental comparison is made between the proposed controller and a classical controller. The most popular classical design of the controllers for EMS systems is a feedback linearizing control. In [8], the classical feedback controller provides a linearized model about a nominal operating point and design procedures. The classical feedback controller takes the form

$$u = \frac{1}{\sigma(x)}\left(v - \rho(x)\right) \quad (27)$$

where $\sigma(x) = -\frac{K_m x_3}{L M_O x_1^2}$, $\rho(x) = \frac{K_m}{M_O}\left(\frac{x_2 x_3^2}{x_1^3} + \frac{R}{L}\frac{x_3^2}{x_1^2}\right)$, $v = k_1 z_1 + k_2 z_2 + k_3 \tilde{z}_3$, and $\tilde{z}_3 = g - \frac{1}{2}\frac{K_m x_3^2}{x_1^2 M_o}$ [8].

Here, we choose $K = [-125050, -7500, -150]$. According to the design procedure given previously, we select $K = [-421950, -16875, -225]$, $\eta = 1$, $c = 0.01$ in the proposed controller and our control goal is to regulate the ball at $8\,mm$. As shown in Fig. 4(a)-(b), we show that regulates the ball both methods where there is no mass uncertainty.

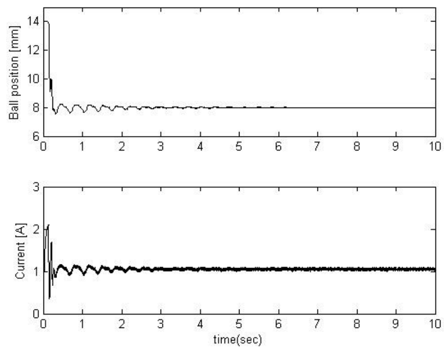

Fig. 4(a) The classical controller : $\Delta M = 0$

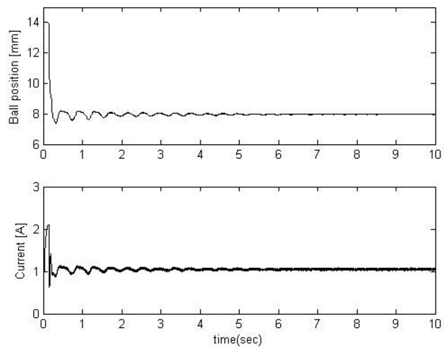

Fig. 4(b) Adaptive controller : $\Delta M = 0$

The effect of mass parameter uncertainty in the EMS is tested and is shown in Fig. 5(a)-(b). In Fig. 5(a), we show that the position of ball is off from the control reference. In Fig. 5(b), we observe that the proposed controller shows stable convergence with uncertain mass of ball. Thus, we find that the proposed controller achieves the control goal whereas the classical controller fails.

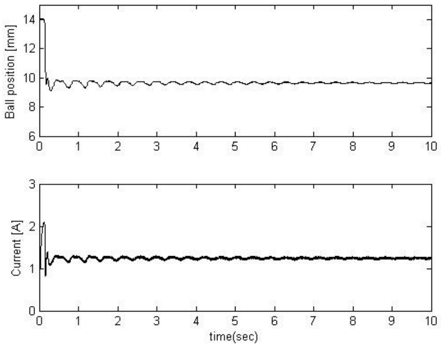

Fig. 5(a) The classical controller : $\Delta M = -0.03$

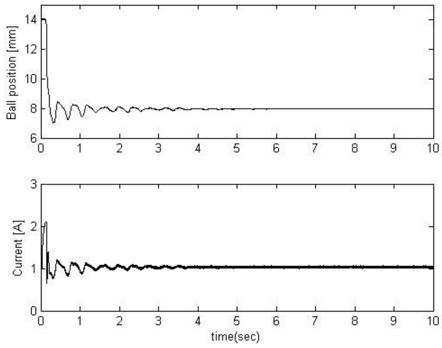

Fig. 5(b) Adaptive controller : $\Delta M = -0.03$

Conclusion

In this paper, we characterize the class of adaptive controller including uncertain nonlinear plants and we show that if a plant is included in this class, then the plant can be linearized and regulated using the adaptive controller. We analytically show that the controlled electromagnetic levitation system is regulated. We verify the validity of the proposed adaptive controller through experiment. In particular, it is shown that the proposed controller adaptively accommodates some unknown ball mass. The experimental results demonstrate the adaptive controller shows better performance than the classical controller.

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