

## Bipedal Walking Trajectory Generation Using Tchebychev Method

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**Abstract.** It is still important and difficult for a biped robot to optimally generate the stable walking trajectory, because the mechanical limitations of the given biped robot should be also considered carefully. In this paper, we assume that different walking trajectories enable to be generated according to various set of weights of torques loaded in each partial joint (e.g., ankle, knee, and hip joint). We present a method for generating various bipedal walking trajectories corresponding to a set of weighted torques. For this purpose, Tchebychev method and sequential quadratic programming are employed to optimize single cost functions consisting in a set of weight torques. Some notations and constraints introduced in [1] are used and modified in this paper.

### Introduction

A walking trajectory generation for a biped robot has been studied as a biped robot has been highlighted [2]. A biped robot is difficult to keep walking stably because a biped robot tends to fall down in comparison with the other mobile robot platforms such as wheeled types and quadruped types. In order to evaluate a stability of a dynamic gait, a zero moment point (ZMP) has been widely used in the related researches after first introduced in [3]. Unlike the center of gravity (COG) just based on the kinematic information, the ZMP is based on the dynamic information (e.g., mass and inertia of each joint) so that the ZMP is more useful to dynamically verify a stability of a walking trajectory. For this reason, the ZMP is also used as a stability criterion in this paper.

For a bipedal walking trajectory generation, the parametric-based methods are popular due to its intuitiveness and ease. In parametric-based methods, a walking trajectory is formulated into a set of parameters and these parameters should be optimized so that several constraints for a walking trajectory are satisfied and a biped robot can walk stably. Constraints and a biped model introduced in [1] are used in this paper. For parameter optimization, various optimization methods can be employed such as a stochastic global optimization method and a local search method. As in [1], since many constraints should be also considered, a local search method is also utilized for a fast search.

In this paper, we obtained a gait of 3D biped robot with optimization technique using Tchebychev method to optimize a set of single objective functions composed of different weight coefficients with respect to body parts. For this purpose, we first define cost functions with a set of different coefficients. After then, we obtain the multiple walking trajectories according to single objective functions formulated as weighted sum of target torques. At this moment, we employ a sequential quadratic programming (SQP) for solving each single objective optimization and Tchebychev method is utilized to obtain multiple trajectories from each solution obtained by the SQP. The optimization objective function is the time averaged sum of joint torques. In Tchebychev method, only the coefficients of joint torques are different. The organization of subsequent chapters is follows. In chapter 1, the overall biped and gait model is introduced. And the kinematic, dynamic and torque constraints are written in chapter 2. In chapter 3, the optimization parameter and the object function in Tchebychev method is described. In chapter 4, the simulation results are presented. And finally, the conclusion is drawn.

## Overall Biped and Gait Model

In this chapter, the over biped and gait model is described. The biped is 3D robot which has 12 actuated joints and 2 active joints in the feet. Actually, we don't have any robot platform, so we just use the robot in [1]. Please refer to the Figure 1 and Table A I , A II in [1]. Figure 1 in [1] describes the overall robot platform and the coordinates system. Table A I in [1] has the Denavit-Hartenberg parameters in the robot system. And Table A II in [1] describes the mass and geometric parameters of the robot. The notation is Craig's. The overall gait consists of 3 parts. SSP (single support phase), DS1 (double support phase 1), DS2 (double support phase 2). The reason of dividing the double support phase into two parts is that it is much easier to make efficient and realistic gaits. The initial pose of the robot is that the left leg is set behind and the toe of left leg is in contact with the ground. In SSP, the left leg moves and set forward to make contact with ground in the heel. In DS1, the right foot is rotates with the toe contact with ground and the left foot rotates with the heel contact with ground. At the end of DS1, the sole of left foot is completely in contact with the ground. In DS2 phase, the sole of left leg keeps in contact with the ground and only right foot rotates with the toe in contact with the ground. The foot picture in Fig. 1 will demonstrate this situation.

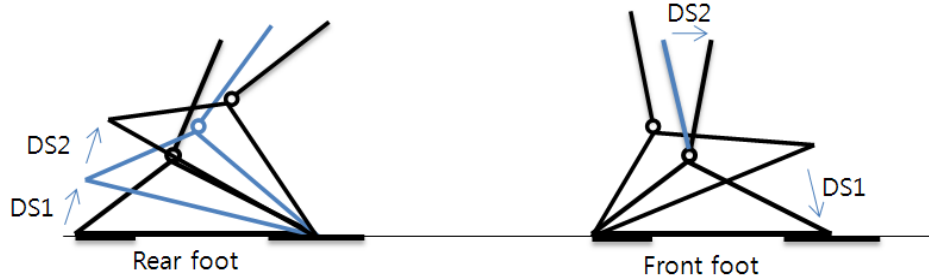


Fig. 1. Movement of Front and Rear foot during DS1 and DS2

The time of overall gait is 2.0 seconds. The SSP time is 0~1.3sec., DS1 is 1.3~1.8sec. DS2 is 1.8~2.0 sec. The SSP is about 65% of overall gait, DS1 is 25%, and DS1 is 10%. And the step length is 0.6m, step width is 0.22m and foot length is 0.27m.

## Gait Constraints

In this chapter, the kinematic, dynamic and torque constraints of gait generation are described. Firstly, the kinematic constraints are described and secondly, the dynamic constraints are described. And finally, the applied torque constraints are described.

There are kinematic constraints in [1]. And in this paper, the kinematic constraints are the same as those of [1]. So Refer to page 566,567 in [1].

And in this paper, the ZMP is calculated separately from left foot to right foot. The ZMP formulas are as follows...

$$O_1 C_1^K = \frac{Z_0 \times M_1^K(O_1)}{Z_0 \bullet F_1^K} \{K = SSP, DS1, DS2\}, A_{13} C_{13}^K = \frac{Z_0 \times M_{13}^K(A_{13})}{Z_0 \bullet F_{13}^K} \{K = DS1, DS2\} \quad (1)$$

The corresponding picture is in Fig. 2

In SSP, the ZMP ( $O_1 C_1^K$ ) must be in Area1. And in DS1, the ZMP of right side ( $O_1 C_1^K$ ) must be in Area2, And the ZMP of left side ( $A_{13} C_{13}^K$ ) must be in the A13 line. In DS2, the ZMP of left foot is in Area3, and the ZMP of right foot must be in the line of O1.

The torque constraints are that the largest part of motor torque in all the time sequence must be below 500Nm. This is for realistic specification of motor torque.

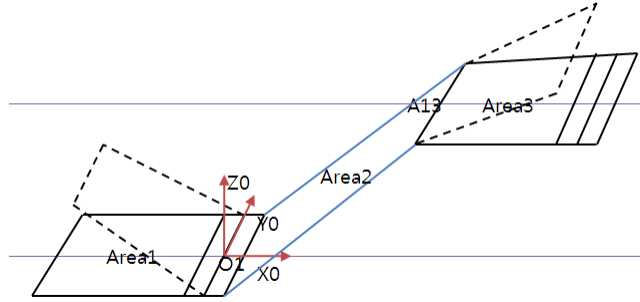


Fig. 2. ZMP coordinates and allowable ZMP area in SSP, DS1, DS2

### The Optimization Parameters and the Object Function in Tchebychev Method

In this chapter, the optimization parameters and object functions are described. The optimization parameters are all 205 parameters. 195 parameters are 13 joint angle curve parameters and the rest 10 parameters are the force parameters of foot sole. Actually each joint angle has 15 parameters and  $15 \times 13$  joint angles = 195 parameters. The each 15 parameters can be used to construct four 4<sup>th</sup> order polynomials in SSP, three 4<sup>th</sup> order polynomials in DS1, three 4<sup>th</sup> order polynomials in DS2. If you need more information on this, refer to page 571 in [1]. The 10 force parameters are divided into 5 DS1 force parameters and 5 DS2 force parameters, and make the linear force curve in each DS1 and DS2 phase. (Z0 direction force only). In program implementation, the moment of sole in DS1, DS2 is set to zero and the X0 direction force is calculated as  $\mu F_z$ . And Y0 direction force is set to zero.

The objective function is as follows

$$Cost = \frac{\int_0^2 \sum_{i=1}^{13} C_i |torque_i| dt}{2.0} \quad (2)$$

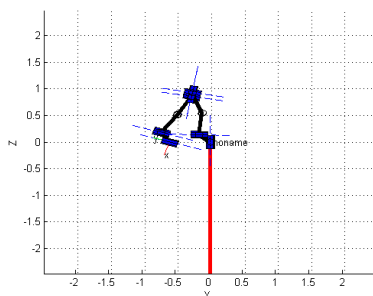
It is time averaged summation of absolute value of each torques. In Tchebychev method, we just changed the  $C_i$  value as follows.

1. All  $C_i$  is 1
2. the knee  $C_i$  is 5 and other  $C_i$  is 3/11
3. the hip  $C_i$  is 5 and other  $C_i$  is 3/11
4. the ankle  $C_i$  is 5 and other  $C_i$  is 3/11

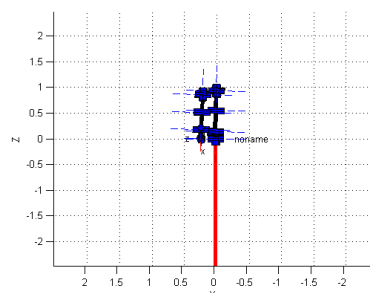
The knee represents the motor 4 and 11, the hip represents motor 5 and 10 and the ankle represents motor 3 and 12.

### The Simulation Results

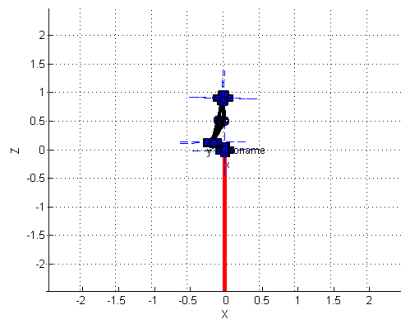
The simulation results are shown in this chapter. Firstly, the overall gait picture is drawn when all the  $C_i$  is 1. The snapshot is taken by using the Matlab Robotics Toolbox. In Fig. 3, left side picture sequence represents the plane view of gait, and right side picture sequence represents the rear view of gait.



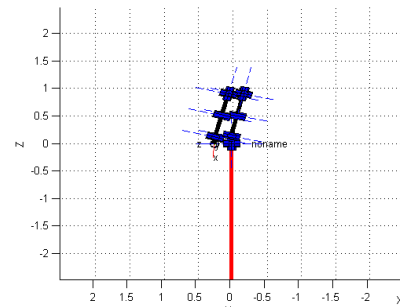
(a) 0 sec



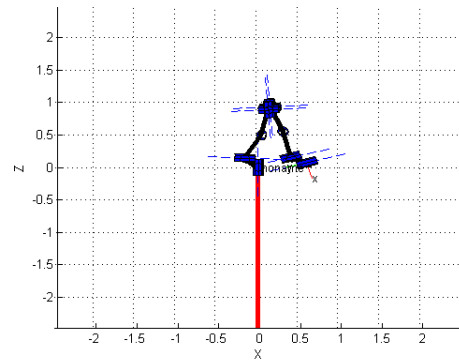
(b) 0 sec



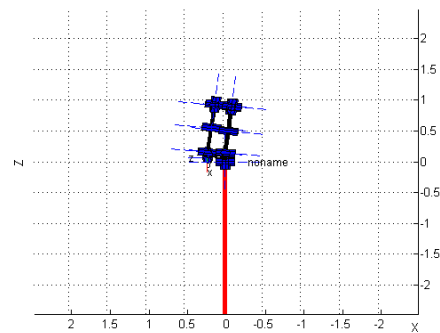
(c) 0.65 sec



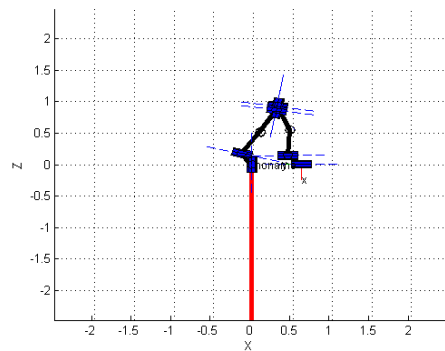
(d) 0.65 sec



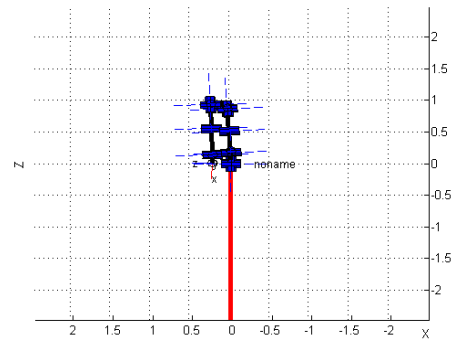
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(f) 1.63 sec



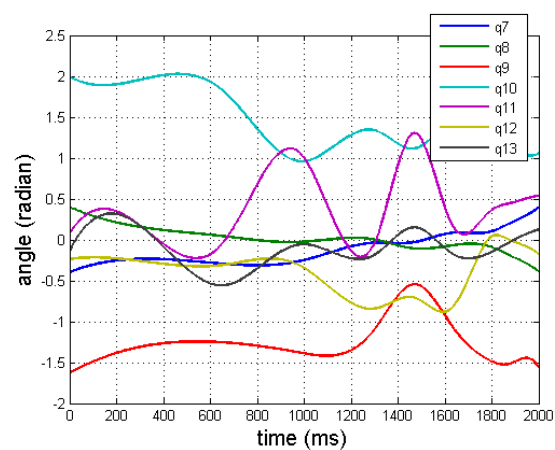
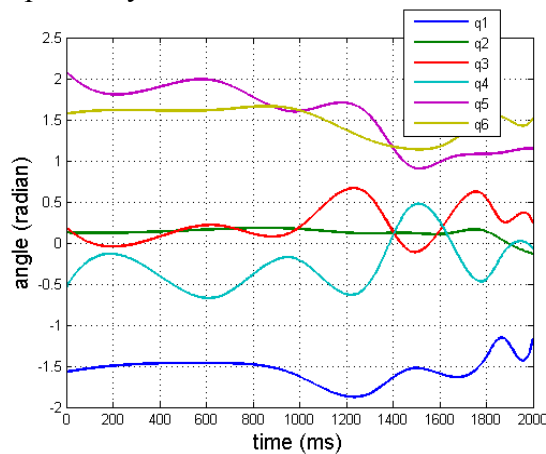
(g) 2sec



(h) 2sec

Fig. 3. Gait sequence when all the  $C_i$  is 1

Secondly, Fig. 4, Fig. 5, Fig. 6 represents the gait curves when the  $C_i$  of ankle, knee, and hip is 5 respectively.

Fig. 4. Gait curve when the  $C_i$  of ankle is 5

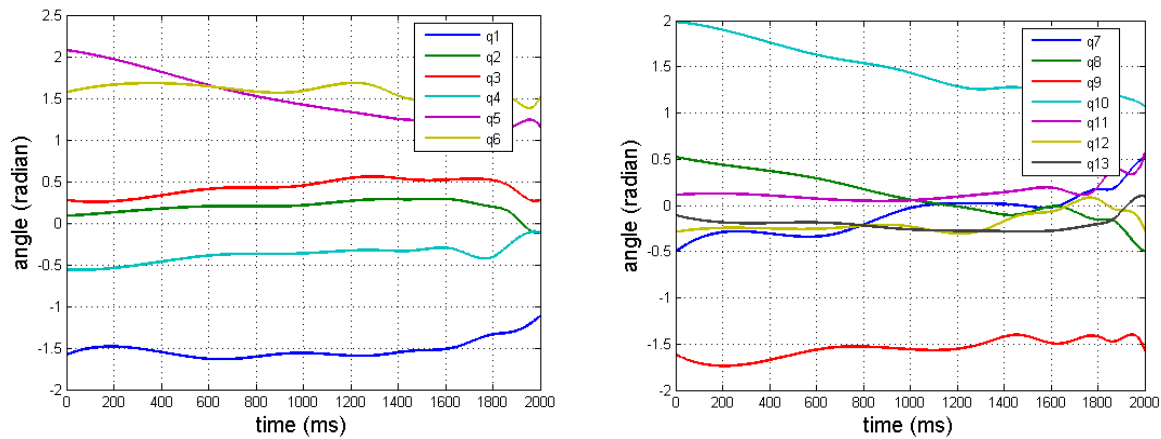


Fig. 5. Gait curve when the Ci of knee is 5

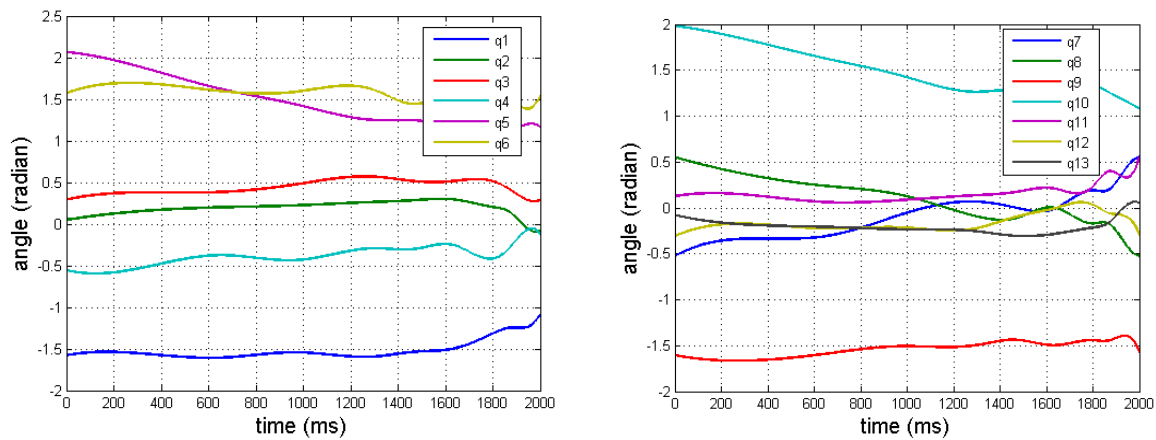


Fig. 6. Gait curve when the Ci of hip is 5

## Conclusion

In this paper, the gait generation by Tchebychev method is investigated. The overall gait synthesis scheme is parametric optimization. And the solver is SQP. The Tchebychev method is applied by augmenting the torque coefficient of ankle, knee, and hip joint. The overall gait picture is represented when all the  $C_i$  is 1. And each gait curve is drawn when  $C_i$  is augmented at ankle, knee, and hip joint respectively. The trajectory of three cases is different. But the hip and knee augmentation case, the trajectory curve is somewhat similar. This is due to the early reach of local minimization point in knee and hip case.

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