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Numerical Analysis of Structure Acoustic Radiation Based on Wave Superposition Method

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Abstract. To overcome the non-uniqueness of solution at eigenfrequencies in the boundary integral equation method for structural acoustic radiation, wave superposition method is introduced to study the acoustics characteristics including acoustic field reconstruction and sound power calculation. The numerical method is implemented by using the acoustic field from a series of virtual sources which are collocated near the boundary surface to replace the acoustic field of the radiator, namely the principle of equivalent. How to collocate these equivalent sources is not indicated definitely. Once wave superposition method is applied to sound power calculation, it is necessary to evaluate its accuracy and impact factors. In the paper, the basic principle of wave superposition method is described, and then the integral equation is discretized. Also, the impact factors including element numbers, frequency limitation, and distance between virtual source and integral surface are analyzed in the process of calculate the acoustic radiation from the simply supported thin plate under concentrated force. The extensive measures of acoustic field at the thin plate are compared with results obtain using different numerical methods. The results show that: (a) The agreement between the results from the above numerical methods is excellent. The wave superposition method requires fewer elements and hence is faster. But the extensive numerical modeling suggests that as long as $ka \leq 1$ the volume velocity matching yields more than adequate accuracy. (b) The equivalent sources should be collocated inside the radiator. And the accuracy of a given Gauss integration formula will decrease as the source approaches the boundary surface. (c) The numerical method is applicable to the acoustic radiation of structure with complicated shape. (d) The method described in this paper can be used to perform effectively sound power calculation, and its application range can be extended on the basis of these conclusions.

Introduction

As a basic problem of acoustics computing, research on structural acoustic radiation is sum up to solve the wave equation satisfying odd-order boundary conditions for an arbitrary closed surface exterior region. For complicated shape and boundary quantities existing in engineering structure, analytical solution can not be generally found and numerical analysis of acoustic radiation has been the subject of a great deal of interest. Due to the advantages of reducing the computational dimension of the problem, automatically satisfying the Helmholtz equation and the Somerfield condition, the boundary integral methods are more efficient than FEM for solving the problem. But there are two difficulties of numerical computation with BEM, such as, singular integral and the unique solutions at the eigenfrequencies. Also calculation speed, storage space and result precision trivial are effected by the trivial integral method. Hence, it is urgent and vital to seek an effective method substituting BEM.

Koopmann and Fahnline [1] et al. proposed an indirect boundary integral method namely wave superposition method to study the acoustic characteristic including acoustic field reconstruction and sound power calculation in 1989. The numerical method is implemented by using the acoustic field from a series of virtual sources which are collected near the boundary surface to replace the actual acoustic field of radiator, called the principle of equivalent. Jeans, Mathews [2] and Xiang Yu [3] successively presented the Burton-Miller's superposition integral and the wave superposition method with complex radius vector. Chen Xinzhao, Yu Fei [4,5] and Li Jiaqing [6] et al. developed the mechanism of near field acoustic holography based on wave superposition method and related experiment, but the acoustic field prediction and how to collocate these equivalent sources are not researched. Xiang Yang [7] utilized the method to calculate sound power of pulsating sphere source, and complicated surface vibration characteristic is not taken into account. Once wave superposition method is applied to acoustic radiation, it is necessary to evaluate its accuracy and impact factors.

The basic principle of wave superposition method is described, and then the integral equation is discretized. Also, the impact factors including element numbers, frequency limitation, and distance between virtual source and integral surface are analyzed in the process of calculate the acoustic radiation from the simply supported thin plate under concentrated force. The extensive measures of acoustic field at the thin plate are compared with results obtain using different numerical methods: an analytic method, a traditional direct boundary element method and a wave superposition method. Thus, the engineering application range can be broadened on the basis of these conclusions.

Basic Equation of Acoustic Radiation

Considering vibrating structure surrounded by an homogeneous and ideal fluid with density ρ and sound speed c, the three-dimensional space is compartmentalized into interior region D and exterior region E. P is an arbitrary point in the exterior region and n is a outward normal of radiator surface, as shown in Fig.1.

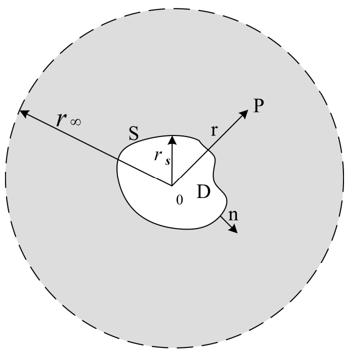

Fig. 1. Vibrating structure with boundary surface S in infinite domain

Acoustic radiation problem for harmonic vibrations in exterior infinite region is expressed by the wave equation, Neumann boundary condition and Sommerfeld radiation condition.

$$\nabla^2 p + k^2 p = 0 \qquad r \in E \tag{1}$$

$$\frac{\partial p}{\partial n} = ik\rho c r_s \qquad r \in S \tag{2}$$

$$\lim_{r\to\infty}\left[r\left(\frac{\partial p}{\partial r} + ikp\right)\right] = 0 \qquad r \in E \tag{3}$$

where $k = \omega / c$ denotes the acoustic wave number of fluid, ω and v_n are the excited angular frequency and velocity vibrating structure. Because analytical solutions for partial differential equation can not be found generally, it is possible to derive the solution of the problem in the form of an integral equation, where the integration extends only over the boundary surface of the radiator. According the acoustic pressure and normal velocity over the boundary surface S, the acoustic field of vibrating structure with arbitrary shape can now be written as

$$p(r) = -\iint_S [G(r, r_s)\nabla_s p(r_s) - p(r_s)\nabla_s G(r, r_s)] \cdot n_s dS \tag{4}$$

Noting that $\nabla_s G(r, r_s) = 0$, when the Green's function of the second kind satisfies the Neumann boundary condition, Eq. 4 is simplified as

$$p(r) = -ik\rho c \iint_S G(r, r_s) v(r_s) \cdot n_s dS \tag{5}$$

where r is placed in exterior region E, ρc is characteristic impedance. Eq. 5 shows that the radiation is only dependent on the normal surface velocity over S, not the pressure over the entire boundary surface. Thus, an indirect boundary integral method based on the volume velocity matching, namely wave superposition method, is proposed to solve Kichhoff-Helmholtz integral equation.

For acoustic radiation problem, the approximate solution for the pressure at the field point P takes the form

$$p(r) = \sum_{v=1}^{N} s_v p_v(r) \tag{6}$$

where the are functions satisfying both the Helmholtz equation in the solution domain E and Sommerfeld radiation condition at infinite distance, the undetermined coefficients s_v donates the elemental source amplitude. Using Euler's equation to rewrite Eq. 6 in terms of the particle velocity at the field point P as

$$v_n(r) = \frac{1}{ik\rho c} \sum_{v=1}^{N} s_v \nabla p_v(r) \tag{7}$$

Numerical Implement of Wave Superposition Method

To avoid the difficulty thar the normal the traditional direct boundary element method adopts surface velocity $v_n(r)$ to mimic boundary condition, the wave superposition method allows the boundary surface to be discretized into smaller elements. Then the Kichhoff-Helmholtz integral equation is convered to a lumped parameter model, and the actual boundary condition is replaced by a volume velocity boundary condition [8], as shown in Eq. 8.

$$u_v = \iint_{S_v} v_n dS(r) = \frac{1}{ik\rho c} \sum_{\mu=1}^{N} s_\mu \iint_{S_v} \nabla p_\mu(r) \cdot n dS(r) \tag{8}$$

Choosing simple and dipole source field as a set of basis function, as follows

$$p_v(r) = \alpha_v G + \beta_v [\nabla_S G \cdot n_S] \tag{9}$$

The approximate solution for acoustic pressure and surface normal velocity are taken as

$$\begin{cases} p(r) = \sum_{v=1}^{N} s_v \left\{ \alpha_v G + \beta_v \left[\nabla_S G \cdot n_S \right] \right\} \\ v_n(r) = \frac{1}{ik\rho c} \sum_{v=1}^{N} s_v \nabla \left\{ \alpha_v G + \beta_v \left[\nabla_S G \cdot n_S \right] \right\} \end{cases} \tag{10}$$

where α_v and β_v are known constants, chosen 1 and 0 to represent the surface element in the plane of an infinite baffle, 0 and $\frac{i}{k}$ to represent element of a surface enclosing no volume, 1 and $\frac{i}{k}$ to represent element of a surface enclosing volume. The volume velocity over element can be written in matrix form as

$$u = U\ s \tag{11}$$

where the individual terms of the matrix U become

$$\begin{cases} U_{\mu v} = \frac{1}{ik\rho c} \iint_{S_v} \nabla \left\{ \alpha_v G + \beta_v \left[\nabla_S G \cdot n_S \right] \right\} \cdot ndS(r) \\ s = \left[s_1\ s_2\ \ \cdots\ \ s_N \right]^T \end{cases} \tag{12}$$

Once the element discretization is defined, the matrix U can be explicitly calculate and the sources amplitudes producing the specified volume velocity distribution can be written as

$$s = U^{-1} u \tag{13}$$

Consider the partial differential equation governing the acoustic field of simple and dipole source, the sound power can be derived by substituting Eq. 9 into Helmholtz equation

$$\Pi_{av} = \Pi_{av,s} + \Pi_{av,d} + \Pi_{av,c} = \frac{1}{2} s^H S\ s \tag{14}$$

where,
$$\begin{cases} \Pi_{av,s} = \frac{2\pi}{\rho c} \sum_{\mu=1}^{N} \sum_{v=1}^{N} \alpha_\mu^* \alpha_v \hat{s}_\mu^* \hat{s}_v j_0\left(kR_{\mu v}\right) \\ \Pi_{av,c} = \frac{2\pi ik}{\rho c} \sum_{\mu=1}^{N} \sum_{v=1}^{N} \alpha_\mu^* \beta_v \hat{s}_\mu^* \hat{s}_v \frac{\left(x_\mu - x_v\right)\left(n_\mu + n_v\right)}{R_{\mu v}} j_1\left(kR_{\mu v}\right) \\ \Pi_{av,d} = \frac{2\pi ik}{\rho c} \sum_{\mu=1}^{N} \sum_{v=1}^{N} \beta_\mu^* \beta_v \hat{s}_\mu^* \hat{s}_v \left\{ \frac{n_\mu \cdot n_v}{kR_{\mu v}} j_1\left(kR_{\mu v}\right) - \frac{\left(x_\mu - x_v\right) \cdot n_\mu}{R_{\mu v}} \frac{\left(x_\mu - x_v\right) \cdot n_v}{R_{\mu v}} j_2\left(kR_{\mu v}\right) \right\} \end{cases} \tag{15}$$

Aiming at validity and accuracy, the acoustic field of simply supported thin plate under concentrated force is researched in the paper and it is convenient to compare and validate the results.

Numerical Example Problems

The length a and width b of simply supported thin plate are 1m and 0.7m, respectively. A concentrated harmonic force works at the place with $(0.5, 0.35)$. And the thin plate is composed of aluminum that thickness h is $0.003m$, density ρ is $7800\, kg/m^3$, the Poisson ratio μ is 0.3, and elastic modulus E $2.1 \times e^{11}\, N/m^2$. The structure model and the element meshes used in the numerical analysis are shown Fig. 2.

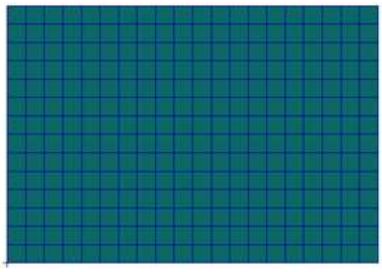

Fig. 2. Element meshes for the plate

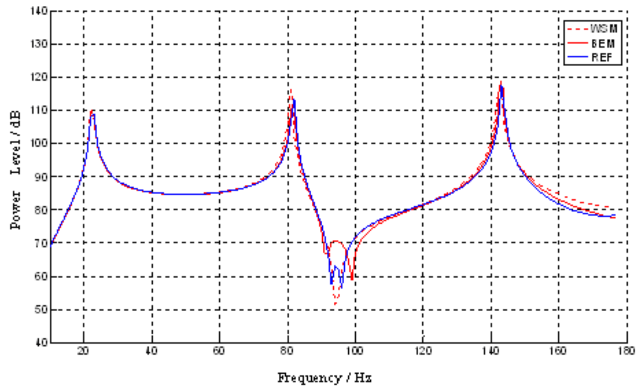

Fig. 3. Power level of the plate using different methods

The plate structure is discretized into 20×14 quadrilateral elements, and first the acoustic meshes should be identical to the structural meshes. As to analyze the impact factor as element numbers, then there are three different acoustic element meshes corresponded to the structural element meshes.

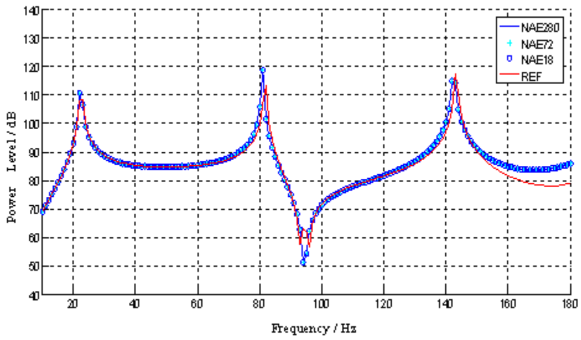

Fig.4. Calculation precision of different acoustic element meshes

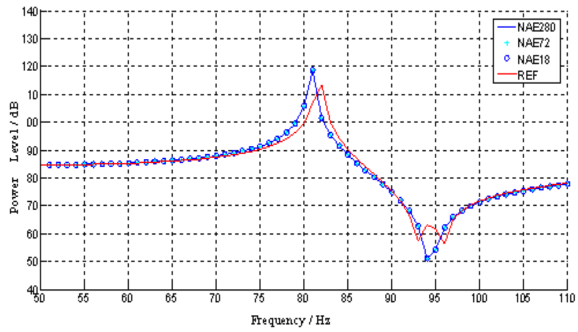

Fig. 5. Calculation precision of different acoustic element meshes (sectional enlargine)

Fig. 3 gives the predicted power level of the simply supported thin plate under concentrated force using different methods, such as, analytical method, traditional direct boundary element method and wave superposition method. The agreement between the results is excellent. However, the source superposition technique requires less elements and hence is faster. But the extensive numerical modeling suggests that as long as $ka \le 1$ the volume velocity matching yields more than adequate accuracy.

Both FEM and BEM are valid in the low to mid frequency range and standard implementations require six elements per wavelength. In many cases of practical interest, the structural wavelengths are much smaller than the acoustic. The same mesh for both methods results in an overly refined acoustic mesh. But the time required to solve the fully populated, complex matrix system depends on the number of unknowns cubed. Thus, the extra refinement results in considerable computational expense in the acoustic analysis. Fig. 4 and Fig. 5 show that the efficiency of the acoustic analysis can be significantly increased by constructing the acoustic element mesh from conglomerations of structural elements and the frequency rang can be broadened employing the wave superposition.

In the process of debugging procedure for calculate acoustic field, there is a phenomena that as the source approaches the element surface, the velocity field of source becomes more concentrated near the location of the source, and more Gauss points are needed to evaluate the volume velocity. So, the accuracy of a given Gauss integration formula will decrease as the source approaches the boundary surface. In general, the method described in this paper can be used to perform effectively acoustic field calculation, and its application range can be extended on the basis of these conclusions.

Summary

The basic principle of wave superposition method is described, and the impact factors including element numbers, frequency limitation, and distance between virtual source and integral surface are analyzed in the process of calculate the acoustic radiation from the simply supported thin plate under concentrated force. The results show that: (a) The agreement between the results from the above numerical methods is excellent. The wave superposition method requires fewer elements and hence is faster. But the extensive numerical modeling suggests that as long as $ka \leq 1$ the volume velocity matching yields more than adequate accuracy. (b) The equivalent sources should be collocated inside the radiator. And the accuracy of a given Gauss integration formula will decrease as the source approaches the boundary surface. (c) The numerical method is applicable to the acoustic radiation of structure with complicated shape. (d) The method described in this paper can be used to perform effectively sound power calculation, and its application range is extended on the basis of these conclusions.

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