*Advanced Engineering Forum* *Online: 2011-12-22*
*ISSN: 2234-991X, Vols. 2-3, pp 822-827*
*doi:10.4028/www.scientific.net/AEF.2-3.822*


# Fluid-Structure Interaction Vibration of Hydraulic Pipe System

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**Keywords:** nonlinear equation, hydraulic pipe, fluid-structure interaction vibration, complex modal method

**Abstract.** Based on Newton method, the nonlinear differential equation of FSI vibration of hydraulic pipe on aero-engine has been established. The equation include visco-elastic coefficient, and the dimensionless equation was got. The influence of mass ratio, velocity of fluid and axial force on natural frequency of the pipe was researched by analyzing the FSI vibration equation of the pipe. The influence of fluid pressure on natural frequency was verified by experiment. And vibration response of the pipe was obtained by experiment at different driving frequency. The conclusion of the experiment was consistent with the result of the theory.

## Introduction

The hydraulic pipe of aero-engine is mainly used to transport fuel, lubricating oil and other media such as air. It is an important part of accessories system on aero-engine. At the same time, the working environment of hydraulic pipe is quite bad. Once the pipe fracturing, the consequence is extremely serious. After previous investigation and study, Wang guopeng (2001) found that the breakdown of aero-engine is mostly due to hydraulic pipe rupture, and fluid-structure interaction (FSI) vibration of the pipe is one of the main reasons for the cause [1]. Therefore, the reliability of hydraulic pipe directly affects the engine's performance and security. There is important significance to research the FSI vibration mechanism and inherent characteristics of hydraulic pipe on the aero-engine.

In recent years, with the development and improvement of nonlinear dynamic theory, many researchers began to research the vibration of pipe conveying fluid [2,3]. The aero-engine hydraulic pipe can be considered as pipe conveying fluid, thus its' mathematical model may consult to the model of pipe conveying fluid. In 1994, Semler et al. have derived the nonlinear equations of motion of the cantilevered pipe used energy method and Newton method [4]. Y. Modarres-Sadeghi (2009) analyzed post-divergence behavior of extensible fluid-conveying pipe supported at both end using the weakly nonlinear equations derived by Semler and Paidousiss [5].

## The equations of hydraulic pipe of Aero-engine

Because there are unbalance force induced by aircraft engine rotor, all accessory system of aero-engine casing vibrate with the casing. Hydraulic pipe system are fixed on the aero-engine casing by tube hoop, So the tube hoop and hydraulic pipe will vibrate with aero-engine casing. The motion form of tube hoop can be seen as the harmonic movement. The mechanical model as shown in Fig. 1 below can describe the motion of the hydraulic pipe on aero-engine. The pipe of length $L$, with fluid of velocity $u$ and tube hoop simulation $D\sin(\omega t)$. $D$ is the amplitude simulation of the tube hoop. Thus, the pipe will do micro vibration under harmonic oscillation of the aero-engine casing.

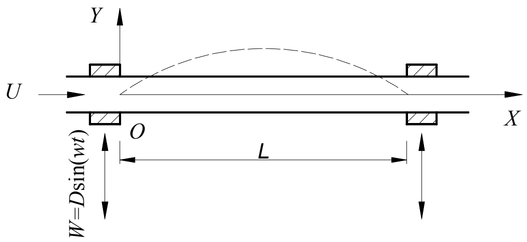



Fig. 1. The model of pipe element with support ends moving

Infinitesimal unit system, as shown in Fig. 2, consists of a fluid element and pipe element. The transverse displacement of pipe is $y(x, t)$. and $p(x, t)$ represents fluid pressure. The fluid of mass per unit length $m_1$, and $m_2$ represents mass per unit length of the pipe. When the pipe vibrate, there produce bending moment $M(x, t)$ and shear force $Q(x, t)$. The pipe is considered to be extensible, so both ends of the pipe element are acted by axial force $T(x, t)$. $q(x, t)$ represents the tangential force between the pipe and fluid, and $F(x, t)$ the normal force. $A$ is the cross-sectional area of the fluid element. The circulation perimeter of fluid is $S$.

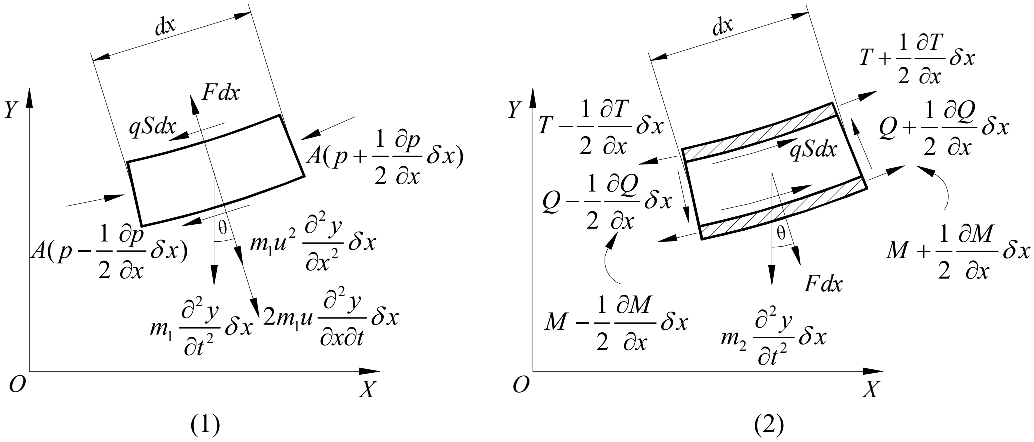



Fig. 2. Forces acting on fluid element and pipe element

After derivation, the resultant equations of motion are

$$aEI\frac{\partial^5 y}{\partial x^4 \partial t}+EI\frac{\partial^4 y}{\partial x^4}+(m_2+m_1)\frac{\partial^2 y}{\partial t^2}+2m_1 u\frac{\partial^2 y}{\partial x \partial t}+$$
$$[m_1 u^2 - T + AP(1-2\nu)+m_1\frac{\partial u}{\partial t}(L-x)-\frac{E\tilde{A}}{2L}\int_0^L(\frac{\partial y}{\partial x})^2 dx-\frac{aE\tilde{A}}{L}\int_0^L\frac{\partial y}{\partial x}\frac{\partial^2 y}{\partial x \partial t}dx]\frac{\partial^2 y}{\partial x^2} \tag{1}$$
$$=(m_1+m_2)D\omega^2\sin\omega t.$$

Boundary conditions are

$$\begin{aligned} &y(0,t)=y(L,t)=0\\ &y(0,t)'=y(L,t)'=0. \end{aligned} \tag{2}$$

In equation (1), $EI$ is the flexural rigidity of the pipe. And a is viscoelastic coefficient. $\tilde{A}$ represents effective cross-sectional area of pipe.

**Dimensionless equations.** The problem may be expressed in dimensionless terms by defining the following dimensionless parameters:

$$\eta=\frac{y}{L},\ \xi=\frac{x}{L},\ \beta=\frac{m_1}{m_1+m_2},\ \tau=\frac{t}{L^2}\left(\frac{EI}{m_1+m_2}\right)^{1/2},\ \upsilon=uL\left(\frac{m_1}{EI}\right)^{1/2},\ Z=\frac{TL^2}{EI}$$

$$p=\frac{PAL^2}{EI},\ \gamma=\frac{\tilde{A}L^2}{2I},\ \alpha=\left(\frac{EI}{m_1+m_2}\right)^{1/2}\frac{a}{L^2},\ \bar{\omega}=\left(\frac{m_1+m_2}{EI}\right)^{1/2}\omega L^2,\ d=\frac{D}{L}. \tag{3}$$

Substituting these dimensionless parameters of equation (3) to equation (1), with $(\ )'$ means $\frac{\partial(\ )}{\partial\xi}$ and $(\dot{\ })$ means $\frac{\partial(\ )}{\partial\tau}$. Let dimensionless axial force parameter $\Gamma=Z-p(1-2\nu)$, one obtains the dimensionless equation of motion

$$\begin{aligned}&\eta^{(4)}+\alpha\dot{\eta}^{(4)}+2\upsilon\beta^{\frac{1}{2}}\dot{\eta}'+\upsilon^2\eta''+\ddot{\eta}+[\beta^{\frac{1}{2}}\dot{\upsilon}(1-\xi)-\Gamma\\&-\gamma\int_0^1(\eta')^2d\xi-2\alpha\gamma\int_0^1\eta'\dot{\eta}'d\xi]\eta''=\bar{\omega}^2d\sin\bar{\omega}\tau.\end{aligned} \tag{4}$$

And the dimensionless boundary conditions

$$\begin{aligned}&\eta(0,\tau)=\eta(1,\tau)=0\\&\eta(0,\tau)'=\eta(1,\tau)'=0.\end{aligned} \tag{5}$$

## The inherent characteristics of the pipe vibration

**The complex modal method.** By using the complex modal method to solve the linear partial differential equation of motion, thus the influence of different parameters on stability of the pipe can be obtained. When $\beta=0.6$, axial force parameter $\Gamma=0$ .The relation curve between natural frequency and dimensionless velocity can be obtained. The figure shows that when the velocity is more than 6.2, the first mode natural frequency is almost zero, namely the critical velocity of flow is 6.2. The former three mode natural frequency all decrease with the velocity increase. In fact, the dimensionless velocity can't be so large because $\upsilon=uL(m_1/EI)^{1/2}$. The flexural rigidity of the pipe is larger than other parameters like $m_1$ and $uL$. So the velocity can't induce instability of the pipe at most time.

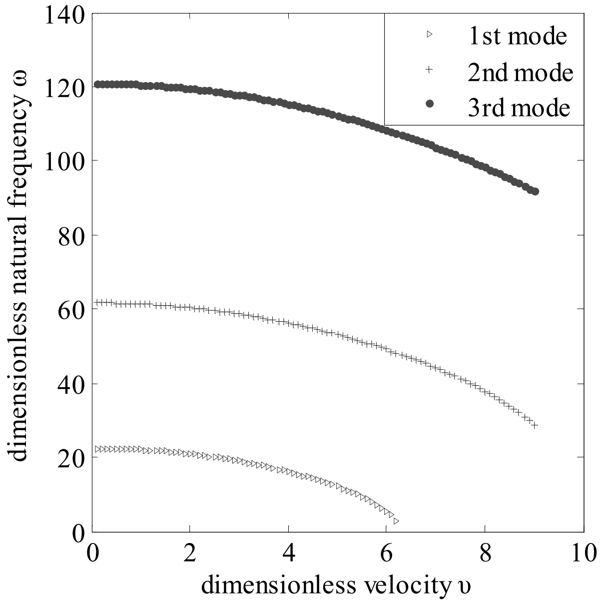
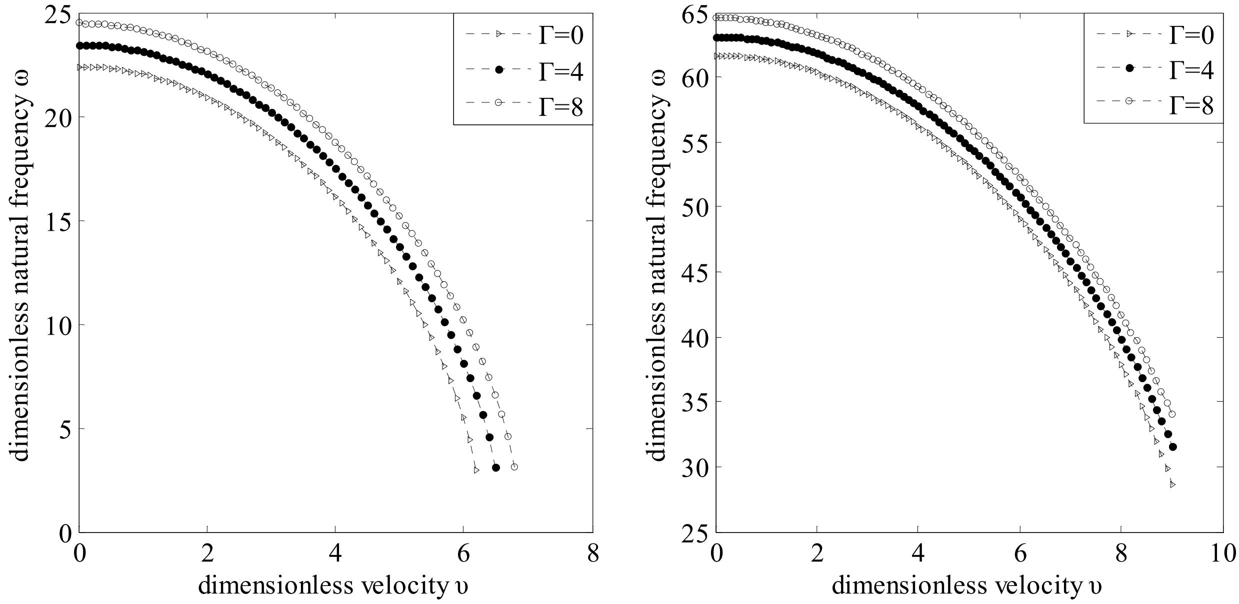



Fig. 3. Influence of fluid velocity on former three mode natural frequency

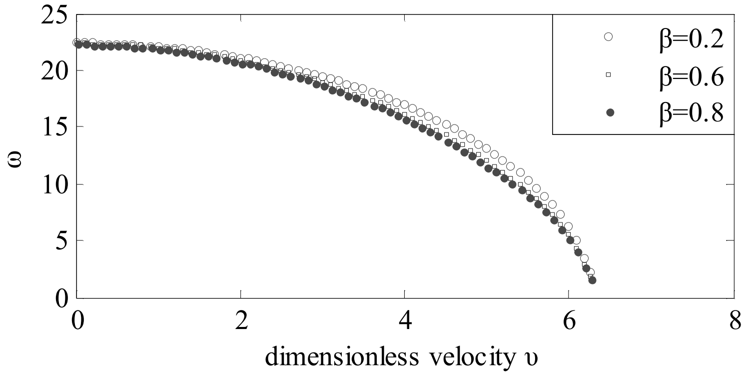
The influence of axial force parameters on characteristics of pipe is showed in Fig. 4. The curves indicate that, the larger the axial force parameter, the larger the natural frequency. Pressure of fluid on the natural frequency is opposite to the axial force parameter. That is to say, when the pressure increase the natural frequency of the pipe would decrease.

When $\Gamma=0$, let $\beta=0.2$, $\beta=0.6$ and $\beta=0.8$. The relation between mass ratio and natural frequency can be described in Fig. 4. The increase of mass ratio would minish the first mode natural frequency but let up the second mode natural frequency. When velocity is zero, the mass ratio do not affect the natural frequency of the pipe. The difference of three curves in Fig. 5(a) and Fig. 5(b) is not large, so the influence of mass ratio on the natural frequency is less.

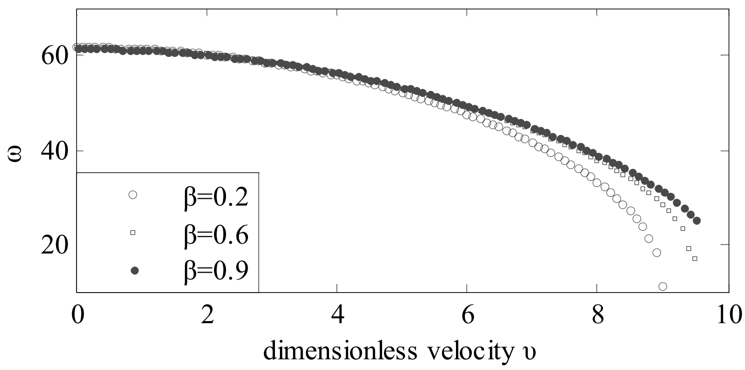



(a) (b)

Fig. 4. The influence of dimensionless axial force parameter on former two mode natural frequency




(a)




(b)

Fig. 5. The influence of mass ratio on former two mode natural frequency

## Experiment of pipe vibration

**The experiment system.** The experiment system (Fig. 6) consists of hydraulic pressure station, fixed bed for pipe, vibrator control system and signal acquisition systems. Hydraulic station can realize the adjustment of fluid pressure. vibrator system consists of industrial computer, power amplifier and vibrator can simulate rotor imbalance force of aircraft engine. Signal acquisition system including sensors, data collection system and software system, may realize measurement of natural frequencies and real-time dynamic response.


Fig. 6. Simulate experiment units of aero-engine hydraulic pipe
1—hydraulic unit, 2—Industrial Personal Computer, 3—power amplifier, 4—vibrator, 5—fixed bed for pipe

**Test procedure of the experiment.** In this experiment, first step is use hammer hit method to obtain the frequency spectrum of pipe at different pressure and flow rate so as to determine the natural frequency at different conditions. And then use vibrator vibrate the pipe at frequency has been tested at same flow and pressure. At last, use data collection system test the acceleration of the pipe. After discrete data process, the natural frequency and displacement response can be obtained.

Table 1 Statistics of the experiment data

| Pressure | $p$=0 Mpa | $p$=2 Mpa | $p$=4 Mpa |
|---|---|---|---|
| 1st mode | 324 Hz | 322 Hz | 295 Hz |
| 2nd mode | 467 Hz | 462 Hz | 449 Hz |
| 3rd mode | 608 Hz | 596 Hz | 576 Hz |

Table 1. shows the former three mode natural frequencies at different fluid pressure. When the pressure is 0 Mpa meaning the pipe include fluid with no flow and no pressure. At this time, the natural frequency is the largest. With the increase of fluid pressure, the former three mode natural frequencies all descend. This result is corresponding to the theory in this paper.

In Fig. 7(a) and (c) are obtained at 240 Hz under 2 Mpa and 4 Mpa respectively. Fig. 7(b) and Fig. 7(d) are obtained at 1st mode natural frequency under 2 Mpa and 4 Mpa respectively. The curves indicate that, displacement response is larger at driving frequency with natural frequency. And the displacement increase with the up of pressure.

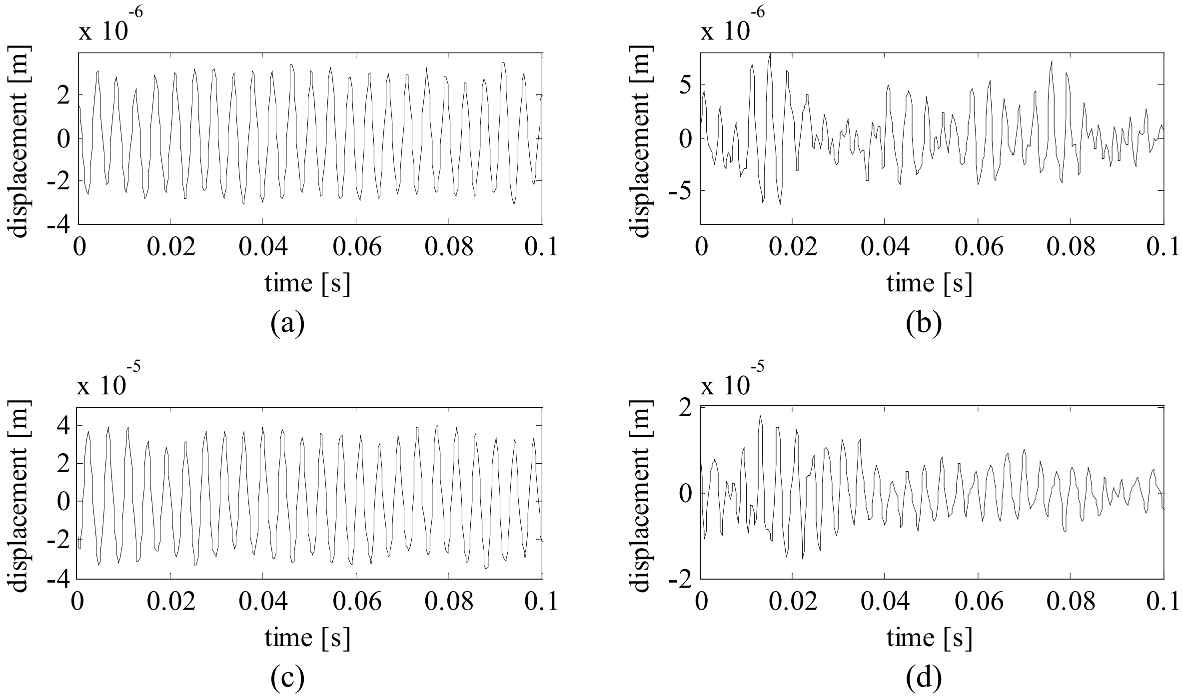



Fig. 7. The displacement response at driving frequency 240 Hz (a) and 322 Hz (b) under pressure of 2 Mpa, (c) and (d) under pressure of 4 Mpa

## Conclusions

The inherent characteristics of the pipe vibration on aero-engine were obtained by using complex modal method based on nonlinear differential equation of FSI vibration of the pipe. The influence of parameters like velocity, axial force and mass ratio on natural frequencies was discussed. The experiment of response of the hydraulic pipe showed that the theory results are right, and the nonlinear differential equation may describe the characteristics of the FSI vibration of the pipe on aero-engine.

## Acknowledgement

The National Natural Science Foundation of PR China (Project No. 51075062) supported this study.

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