Output Feedback Control of a Ball and Beam System Based on Jacobian Linearization under Sensor Noise

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Abstract. In this paper, we consider a control problem of a ball and beam system with sensor noise on feedback sensor. If sensor noise exists in sensor's signal, it can make feedback signals deformed and then it can lead to control performance degradation or even system failure. Therefore, we need to design a robust controller to deal with the possible sensor noise in the feedback information. We develop an output feedback controller with a gain-scaling factor in order to minimize the effect of AC sensor noise in output feedback information. Our proposed controller is applied to a ball and beam system and verified by analysis and simulation. As a result, our controller reduces the effect of sensor noise to arbitrarily small by increasing a gain-scaling factor.

Introduction

We consider a control problem of a ball and beam system with sensor noise on feedback channel. Control systems operate via measurement data and it is usually assumed that the measured signals are clean. If sensor noise exists in sensor's signal, it can make feedback signals deformed and then it can result in performance degradation or even system failure. Therefore, we need to design a robust controller to accommodate the possible sensor noise in the feedback information [2],[5],[6].

In this paper, we assume that an output sensor of a ball and beam system is coupled with AC noise signal. We apply Jacobian linearization to a ball and beam system and propose an output feedback controller compensating sensor noise of feedback sensor for a ball and beam system. Our controller is equipped with a gain-scaling factor to minimize the effect of sensor noise in output feedback information. We give a theoretical analysis of the controlled system using Lyapunov stability theorems and Laplace transform [2],[4]. We illustrate the improved control performance via simulation for a ball and beam system.

A ball and beam system model based on Jacobian linearization and sensor noise

The ball and beam system can be modeled as [7]

$$\ddot{r} = K_{bb} \sin \theta$$
$$\ddot{\theta} = -\frac{1}{\mu} \dot{\theta} + \frac{K_{srv}}{\mu} u \tag{1}$$

where r is the position of ball, θ is the angle of motor, K_{bb} is the model gain of the ball and beam system, K_{srv} is a steady-state gain, μ is a time constant and u is an input motor voltage.
We define the states of the system as follows

$$r = x_1, \quad \dot{r} = \dot{x}_1 = x_2, \quad \ddot{r} = \dot{x}_2$$
$$\theta = x_3, \quad \dot{\theta} = \dot{x}_3 = x_4, \quad \ddot{\theta} = \dot{x}_4 \tag{2}$$

Then, we obtain the following state equation

$$\dot{x}_1 = x_2, \quad \dot{x}_2 = K_{bb} \sin x_3, \quad \dot{x}_3 = x_4, \quad \dot{x}_4 = -\frac{1}{\mu} x_4 + \frac{K_{srv}}{\mu} u, \quad y = x_1 \tag{3}$$

The Jacobian linearized system is as follows

$$\dot{x} = Ax + Bv + \delta(x) \tag{4}$$

$$y = Cx \tag{5}$$

where $x = [x_1, x_2, x_3, x_4]^T$ is the state, $v = \frac{K_{srv}}{\mu} u$ is an internal input, $u \in R$ is an input, $y \in R$ is an output, $\delta(x) = [0, 0, 0, \delta_4(x)]^T$ and $\delta_4(x) = -\frac{1}{\mu} x_4$. The system matrices are

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & K_{bb} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$$

We suppose that sensor noise enters at the output sensor of a ball and beam system and the measured output becomes as follows

$$\overline{y} = y + s(t) \tag{6}$$

where $s(t)$ is sensor noise. Sensor noise is often modeled as a sinusoidal function [1]. Therefore, we introduce the following condition on $s(t)$.

Assumption 1. There exist constants $\alpha \geq 0$ and $\omega \geq 0$ such that

$$s(t) = \alpha \sin \omega t \tag{7}$$

Design and analysis of proposed controller

In the presence of sensor noise, the proposed output feedback controller equipped with a gain-scaling factor is given by

$$\dot{\hat{x}} = A\hat{x} + Bv - L(\varepsilon)(\overline{y} - C\hat{x}) \tag{8}$$

$$v = K(\varepsilon)\hat{x} - \delta_4(\hat{x}) \tag{9}$$

where $\hat{x} = [\hat{x}_1, \hat{x}_2, \hat{x}_3, \hat{x}_4]^T$ is the observed state, $\delta(\hat{x}) = [0, 0, 0, \delta_4(\hat{x})]^T$, $\delta_4(\hat{x}) = -\frac{1}{\mu}\hat{x}_4$, $L(\varepsilon) = [\frac{l_1}{\varepsilon}, \frac{l_2}{\varepsilon^2}, \frac{l_3}{\varepsilon^3}, \frac{l_4}{\varepsilon^4}]^T$ and $K(\varepsilon) = [\frac{k_1}{\varepsilon^4}, \frac{k_2}{\varepsilon^3}, \frac{k_3}{\varepsilon^2}, \frac{k_4}{\varepsilon}]$ with $\varepsilon > 0$. We define $e = [e_1, e_2, e_3, e_4]^T$ where $e_i = x_i - \hat{x}_i, i = 1, \cdots, 4$. By subtracting (8) from (4) and with the controller (9), the augmented closed-loop dynamics is

$$\dot{e} = A_L(\varepsilon)e + L(\varepsilon)s(t) + \delta(x) - \delta(\hat{x}) \tag{10}$$

$$\dot{x} = A_K(\varepsilon)x - BK(\varepsilon)e + \delta(x) - \delta(\hat{x}) \tag{11}$$

where $A_L(\varepsilon) = A + L(\varepsilon)C$ and $A_K(\varepsilon) = A + BK(\varepsilon)$. We must select $L = [l_1, l_2, l_3, l_4]^T$ and $K = [k_1, k_2, k_3, k_4]$ such that $A_L = A_L(1)$ and $A_K = A_K(1)$ are Hurwitz. The Lyapunov equation for (10) is $A_L^T P_L + P_L A_L = -I$. With $E(\varepsilon) = \text{diag}[1, \varepsilon, \varepsilon^2, \varepsilon^3]$, we have the following equalities [3]:

$$\varepsilon A_L(\varepsilon) = E(\varepsilon)^{-1} A_L E(\varepsilon)$$
$$A_L^T(\varepsilon) P_L(\varepsilon) + P_L(\varepsilon) A_L(\varepsilon) = -\varepsilon^{-1} E(\varepsilon)^2 \tag{12}$$
$$P_L(\varepsilon) = E(\varepsilon) P_L E(\varepsilon)$$

We set $V_o(e) = e^T P_L(\varepsilon)e$ for (10). Then, along the trajectory of (10), we have

$$\dot{V}_o(e) = -\varepsilon^{-1} e^T E(\varepsilon)^2 e + 2e^T E(\varepsilon) P_L E(\varepsilon) L(\varepsilon)s(t) + 2e^T E(\varepsilon) P_L E(\varepsilon)\left(\delta(x) - \delta(\hat{x})\right)$$
$$\leq -\varepsilon^{-1} \|E(\varepsilon)e\|^2 + 2\varepsilon^{-1}\left\{E(\varepsilon)e\right\}^T P_L L s(t) + 2\|P_L\|\|E(\varepsilon)e\|\|E(\varepsilon)\left(\delta(x) - \delta(\hat{x})\right)\|_1 \tag{13}$$

We have $\left\| E(\varepsilon)\left(\delta(x) - \delta(\hat{x})\right) \right\|_1 \le \frac{1}{\mu} \left\| E(\varepsilon)e \right\|$ for (13), and then we can obtain

$$\dot{V}_o(e) \le -\varepsilon^{-1} \left\| E(\varepsilon)e \right\|^2 + 2\varepsilon^{-1} \left\{ E(\varepsilon)e \right\}^T P_L Ls(t) + 2\frac{2}{\mu} \left\| P_L \right\| \left\| E(\varepsilon)e \right\|^2$$

$$\le -N_0 \left\| E(\varepsilon)e \right\|^2 - \frac{1}{2}\varepsilon^{-1} \left\{ E(\varepsilon)|e| \right\}^T E(\varepsilon) \left(\begin{bmatrix} |e_1| \\ |e_2| \\ |e_3| \\ |e_4| \end{bmatrix} - 4\alpha \left\| P_L \right\| \left\| L \right\| \begin{bmatrix} 1 \\ \varepsilon^{-1} \\ \varepsilon^{-2} \\ \varepsilon^{-3} \end{bmatrix} \right) \tag{14}$$

where $N_o = \frac{1}{2}\varepsilon^{-1} - \frac{2}{\mu}\left\| P_L \right\|$ and $|s(t)| = |\alpha \sin \omega t| \le \alpha$.

Satisfying $N_o > 0$, the ultimate bound of e is as follows

$$UB\left(\begin{bmatrix} |e_1| \\ |e_2| \\ |e_3| \\ |e_4| \end{bmatrix} \right) \le 4\alpha \left\| P_L \right\| \left\| L \right\| \begin{bmatrix} 1 \\ \varepsilon^{-1} \\ \varepsilon^{-2} \\ \varepsilon^{-3} \end{bmatrix} \tag{15}$$

If ε is increased, the ultimate bounds of e_2, e_3 and e_4 except for e_1 can be made arbitrarily small. Because the Lyapunov stability theorem is a sufficient condition, we analyze the ultimate bound of e_1 via Laplace transform. Applying Laplace transform for the system (10), $E_1(s)$ is as follows

$$E_1(s) = \frac{\left(s + \frac{1}{\mu}\right)\left(s^2 e_1(0) + s e_2(0) + K_{bb} e_3(0)\right) + K_{bb} e_4(0)}{\sigma(s)} + \frac{\phi(s)}{\sigma(s)}\frac{\alpha\omega}{s^2 + \omega^2} \tag{16}$$

where $\sigma(s) = s^4 + \frac{1}{\mu}s^3 - \phi(s)$, $\phi(s) = \frac{l_1}{\varepsilon}s^3 + \left(\frac{1}{\mu}\frac{l_1}{\varepsilon} + \frac{l_2}{\varepsilon^2}\right)s^2 + \left(\frac{1}{\mu}\frac{l_2}{\varepsilon^2} + K_{bb}\frac{l_3}{\varepsilon^3}\right)s + \left(\frac{K_{bb}}{\mu}\frac{l_3}{\varepsilon^3} + K_{bb}\frac{l_4}{\varepsilon^4}\right)$.

When $\sigma(s)$ is a Hurwitz polynomial, $E_1(s)$ applied by the final-value theorem remains as follows

$$\frac{D_1}{s + j\omega} + \frac{D_2}{s - j\omega} = \frac{D_1(s - j\omega) + D_2(s + j\omega)}{s^2 + \omega^2} \tag{17}$$

where D_1 and D_2 is

$$D_1 = \frac{\alpha}{2}\frac{\gamma(\omega) + j\beta(\omega)}{\left(\frac{1}{\mu}\omega^3 - \beta(\omega)\right) - j\left(\omega^4 - \gamma(\omega)\right)}, \quad D_2 = \frac{\alpha}{2}\frac{\gamma(\omega) - j\beta(\omega)}{\left(\frac{1}{\mu}\omega^3 - \beta(\omega)\right) + j\left(\omega^4 - \gamma(\omega)\right)}$$

where $\gamma(\omega) = \text{Re}\left(\phi(s)\big|_{s=jw}\right)$ and $\beta(\omega) = \text{Im}\left(\phi(s)\big|_{s=jw}\right)$. Now that $\frac{D_1}{s+j\omega}$ and $\frac{D_2}{s-j\omega}$ are vibrating terms in steady state, the ultimate bound of e_1 is minimized if $D_1(s - j\omega) + D_2(s + j\omega)$, that is the size of (17), is smaller.

$$D_1(s - j\omega) + D_2(s + j\omega)$$

$$= \frac{\alpha}{2}\left[\frac{\gamma(\omega) + j\beta(\omega)}{\left(\frac{1}{\mu}\omega^3 - \beta(\omega)\right) - j\left(\omega^4 - \gamma(\omega)\right)}(s - j\omega) + \frac{\gamma(\omega) - j\beta(\omega)}{\left(\frac{1}{\mu}\omega^3 - \beta(\omega)\right) + j\left(\omega^4 - \gamma(\omega)\right)}(s + j\omega) \right]$$

$$= \alpha\left[\frac{\varepsilon^8\frac{1}{\mu}\gamma(\omega)\omega^3 - \varepsilon^8\beta(\omega)\omega^4}{\varepsilon^8\left(\frac{1}{\mu}\omega^3 - \beta(\omega)\right)^2 + \varepsilon^8\left(\omega^4 - \gamma(\omega)\right)^2}s + \frac{\varepsilon^8\gamma(\omega)\omega^4 + \varepsilon^8\frac{1}{\mu}\beta(\omega)\omega^3 - \varepsilon^8\gamma(\omega)^2 - \varepsilon^8\beta(\omega)^2}{\varepsilon^8\left(\frac{1}{\mu}\omega^3 - \beta(\omega)\right)^2 + \varepsilon^8\left(\omega^4 - \gamma(\omega)\right)^2}\omega \right] \tag{18}$$

Substituting $\gamma(\omega) = \text{Re}\left(\phi(s)\big|_{s=jw}\right)$ and $\beta(\omega) = \text{Im}\left(\phi(s)\big|_{s=jw}\right)$ into (18), we can rewrite (18) for ε as follows

$$D_1(s-j\omega)+D_2(s+j\omega) = \alpha\left[\frac{l_1\left(\omega^7 - \frac{1}{\mu}\omega^5\right)\varepsilon^7 + O(\varepsilon^6)}{\left(\omega^8 + \frac{1}{\mu}\omega^6\right)\varepsilon^8 + O(\varepsilon^7)}s + \frac{\left(l_1^2\omega^6 - l_2\omega^6 + \frac{1}{\mu^2}l_1^2\omega^4\right)\varepsilon^6 + O(\varepsilon^5)}{\left(\omega^8 + \frac{1}{\mu}\omega^6\right)\varepsilon^8 + O(\varepsilon^7)}\omega\right]$$

(19)

If ε is increased, the size of $D_1(s-j\omega)+D_2(s+j\omega)$ is decreased. Thus, the ultimate bound of e_1 can be reduced by increasing ε. The Lyapunov equation for (11) is $A_K^T P_K + P_K A_K = -I$, and we have the following equalities:

$$\varepsilon A_K(\varepsilon) = E(\varepsilon)^{-1} A_K E(\varepsilon)$$
$$A_K^T(\varepsilon)P_K(\varepsilon) + P_K(\varepsilon)A_K(\varepsilon) = -\varepsilon^{-1}E(\varepsilon)^2$$
$$P_K(\varepsilon) = E(\varepsilon)P_K E(\varepsilon)$$

(20)

We set $V_c(x) = x^T P_K(\varepsilon)x$ for (11). Then, along the trajectory of (11), we have

$$\dot{V}_c(x) = -\varepsilon^{-1}\|E(\varepsilon)x\|^2 - 2\varepsilon^{-1}x^T E(\varepsilon)P_K BKE(\varepsilon)e + 2x^T E(\varepsilon)P_K E(\varepsilon)\{\delta(x)-\delta(\hat{x})\}$$
$$\leq -\varepsilon^{-1}\|E(\varepsilon)x\|^2 - 2\varepsilon^{-1}\{E(\varepsilon)x\}^T P_K BKE(\varepsilon)e + 2q\|E(\varepsilon)x\|\|E(\varepsilon)e\|$$

(21)

where $q = \frac{1}{\mu}\|P_K\|$.

We set a Lyapunov function for entire system (10)-(11) to $V_c(e,x) = V_o(e) + dV_c(x), d > 0$. Then, along the trajectories of (10) and (11), we obtain the following from (14) and (21).

$$\dot{V}(e,x) \leq -\Sigma^T M \Sigma - \frac{1}{2}\varepsilon^{-1}\{E(\varepsilon)|e|\}^T E(\varepsilon)\left(\begin{bmatrix}|e_1| \\ |e_2| \\ |e_3| \\ |e_4|\end{bmatrix} - 4\alpha\|P_L\|\|L\|\begin{bmatrix}1 \\ \varepsilon^{-1} \\ \varepsilon^{-2} \\ \varepsilon^{-3}\end{bmatrix}\right)$$
$$-\frac{1}{2}\varepsilon^{-1}d\{E(\varepsilon)|x|\}^T E(\varepsilon)\left(\begin{bmatrix}|x_1| \\ |x_2| \\ |x_3| \\ |x_4|\end{bmatrix} - 4\|P_K\|\|K\|\begin{bmatrix}1 \\ \varepsilon^{-1} \\ \varepsilon^{-2} \\ \varepsilon^{-3}\end{bmatrix}\left(|e_1| + \varepsilon|e_2| + \varepsilon^2|e_3| + \varepsilon^3|e_4|\right)\right)$$

(22)

where

$$\Sigma = \begin{bmatrix}\|E(\varepsilon)e\| \\ \|E(\varepsilon)x\|\end{bmatrix}, \quad M = \begin{bmatrix}N_o & -dq \\ -dq & \frac{d}{2}\varepsilon^{-1}\end{bmatrix}$$

If $d < \frac{N_o}{2\varepsilon q^2}$ is satisfied, M becomes positive definite matrix. Thus, the ultimate bound of (x_1, x_2, x_3, x_4) is as follows

$$UB\left(\begin{bmatrix}|x_1| \\ |x_2| \\ |x_3| \\ |x_4|\end{bmatrix}\right) \leq 4\|P_K\|\|K\|\begin{bmatrix}1 \\ \varepsilon^{-1} \\ \varepsilon^{-2} \\ \varepsilon^{-3}\end{bmatrix}\left(|e_1| + \varepsilon|e_2| + \varepsilon^2|e_3| + \varepsilon^3|e_4|\right)$$

(23)

Substituting the ultimate bound of (e_1, e_2, e_3, e_4) into (23), we can obtain

$$UB\left(\begin{bmatrix} |x_1| \\ |x_2| \\ |x_3| \\ |x_4| \end{bmatrix}\right) \leq 64\alpha \left\|P_K\right\| \left\|K\right\| \left\|P_L\right\| \left\|L\right\| \begin{bmatrix} 1 \\ \varepsilon^{-1} \\ \varepsilon^{-2} \\ \varepsilon^{-3} \end{bmatrix} \tag{24}$$

If ε is increased in (24), the ultimate bounds of x_2, x_3 and x_4 is decreased except for x_1. Thus, we analyze the ultimate bound of x_1 via Laplace transform.

Applying Laplace transform for the system (11), $X_1(s)$ is as follows

$$X_1(s) = \frac{s^3 - \left(\frac{k_4}{\varepsilon} - \frac{1}{\mu}\right)s^2 - \frac{k_3}{\varepsilon^2}s - \frac{K_{bb}k_2}{\varepsilon^3}}{\eta(s)} x_1(0) + \frac{s^2 - \left(\frac{k_4}{\varepsilon} - \frac{1}{\mu}\right)s - \frac{k_3}{\varepsilon^2}}{\eta(s)} x_2(0) + \frac{\left(s - \frac{k_4}{\varepsilon} + \frac{1}{\mu}\right)K_{bb}}{\eta(s)} x_3(0)$$

$$+ \frac{K_{bb}}{\eta(s)} x_4(0) + \frac{1}{\eta(s)}\frac{K_{srv}}{\mu}\left(\frac{k_1}{\varepsilon^4}E_1(s) + \frac{k_2}{\varepsilon^3}E_2(s) + \frac{k_3}{\varepsilon^2}E_3(s) + \frac{k_4}{\varepsilon}E_4(s)\right) \tag{25}$$

where $\eta(s) = s^4 - \left(\frac{k_4}{\varepsilon} - \frac{1}{\mu}\right)s^3 - \frac{k_3}{\varepsilon^2}s^2 - \frac{k_2 K_{bb}}{\varepsilon^3}s - \frac{k_1 K_{bb}}{\varepsilon^4}$.

When $\eta(s)$ is a Hurwitz polynomial, the terms with initial value for (25) become 0 by the final-value theorem and the remaining terms($X_{r1}(s)$) is as follows

$$X_{r1}(s) = \frac{1}{\eta(s)}\frac{K_{srv}}{\mu}\left(\frac{k_1}{\varepsilon^4}E_1(s) + \frac{k_2}{\varepsilon^3}E_2(s) + \frac{k_3}{\varepsilon^2}E_3(s) + \frac{k_4}{\varepsilon}E_4(s)\right) \tag{26}$$

We assume that $\eta(s)$ has separate roots for analysis and substitute $E_{bi}(s), i = 1, \cdots, 4$, that is the ultimate bound of $E_i(s), i = 1, \cdots, 4$, into (26) as follows.

$$\frac{\frac{K_{srv}}{\mu}}{(s-\lambda_1)(s-\lambda_2)(s-\lambda_3)(s-\lambda_4)}\left(\frac{k_1}{\varepsilon^4}E_{b1}(s) + \frac{k_2}{\varepsilon^3}E_{b2}(s) + \frac{k_3}{\varepsilon^2}E_{b3}(s) + \frac{k_4}{\varepsilon}E_{b4}(s)\right) = \left(\sum_{i=1}^{4}\frac{a_i}{s+\lambda_i}\right)\left(\sum_{i=1}^{4}\frac{k_i}{\varepsilon^{5-i}}E_{bi}(s)\right) \tag{27}$$

where $\lambda_i, i = 1, \cdots, 4$ are roots of $\eta(s)$, $a_i, i = 1, \cdots, 4$ are constants by partial fraction expansion for $\frac{K_{srv}/\mu}{(s-\lambda_1)(s-\lambda_2)(s-\lambda_3)(s-\lambda_4)}$. Applying the inversion of Laplace transform for (27)

$$\left(\sum_{i=1}^{4}a_i \exp(-\lambda_i t)\right) * \left(\sum_{i=1}^{4}\frac{k_i}{\varepsilon^{5-i}}e_{bi}(t)\right) = \int_0^t \left(\sum_{i=1}^{4}a_i \exp\left(-\lambda_i(t-\tau)\right)\right) * \left(\sum_{i=1}^{4}\frac{k_i}{\varepsilon^{5-i}}e_{bi}(\tau)\right)d\tau \tag{28}$$

Because the ultimate bounds of (x_1, x_2, x_3, x_4) are reduced by increasing ε as shown in (15) and (19), we set maximum value of $e_{bi}(t)$ as $\frac{b_i}{\varepsilon}, i = 1, \cdots, 4$.

$$\int_0^t \left(\sum_{i=1}^{4}a_i \exp\left(-\lambda_i(t-\tau)\right)\right) * \left(\sum_{i=1}^{4}\frac{k_i}{\varepsilon^{5-i}}e_{bi}(\tau)\right)d\tau \leq \left(\sum_{i=1}^{4}\frac{a_i}{\lambda_i}\left[1-\exp\left(-\lambda_i t\right)\right]\right)\left(\sum_{i=1}^{4}\frac{k_i}{\varepsilon^{5-i}}\frac{b_i}{\varepsilon}\right) \tag{29}$$

The exponential terms $\exp(-\lambda_i t), i = 1, \cdots, 4$ become 0 in steady state. Thus, the ultimate bound of x_1 in steady state is as follows

$$UB(x_1) \leq \frac{1}{\varepsilon}\left(\frac{a_1}{\lambda_1} + \frac{a_2}{\lambda_2} + \frac{a_3}{\lambda_3} + \frac{a_4}{\lambda_4}\right)\left(\frac{k_1 b_1}{\varepsilon^4} + \frac{k_2 b_2}{\varepsilon^3} + \frac{k_3 b_3}{\varepsilon^2} + \frac{k_4 b_4}{\varepsilon}\right) \tag{30}$$

If ε is increased in (30), the ultimate bounds of x_1, x_2, x_3 and x_4 can be reduced.

Simulation

In simulation for a ball and beam system, we set the initial conditions as $x_1(0) = 10cm$, $x_2(0) = 0$, $x_3(0) = 0°$ and $x_4(0) = 0$ and sensor noise as $s(t) = 3\sin 3t$. The matrices A_L and A_K are Hurwitz if choosing $L = [-100, -3080, -71235, 1567]^T$ and $K = [-0.3490, -0.8725, -0.3242, 0.5320]$. When the value of ε changed, the position of ball and the angle of motor are shown as Fig. 1 and Fig. 2. As shown in Fig. 1 and Fig. 2, the ultimate bounds of x_1 and x_3 is reduced when ε is increased. Thus, our proposed controller can reduce the effect of sensor noise by increasing ε.
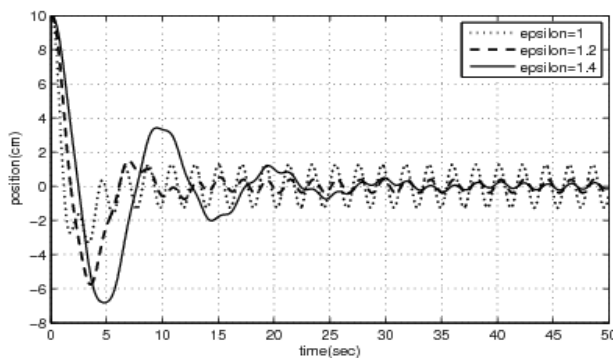
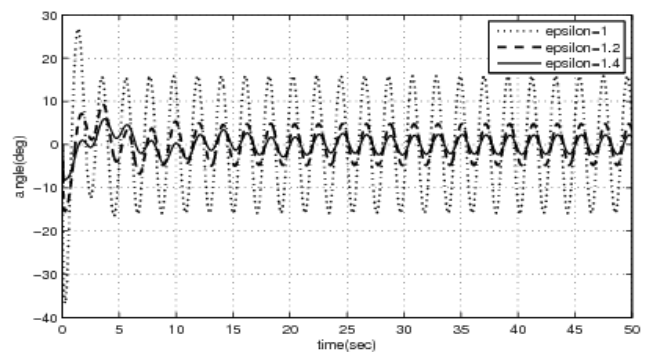Fig. 1. Position of ball(x_1)   Fig. 2. Angle of motor(x_3)

Conclusions

In this paper, we assume that AC sensor noise is included in the output sensor of a ball and beam system. We design an output feedback controller with a gain-scaling factor to minimize the effect of sensor noise. We analyze the ultimate bounds of e and x by the Lyapunov stability theorem and Laplace transform for controlled system. As a result, we show that the effect of sensor noise is arbitrarily reduced by increasing ε via analysis and simulation.

Acknowledgement

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