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# Supply Chain Network Equilibrium with Risk-Averse Members

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**Abstract.** To obtain equilibrium patterns of a competitive supply chain network with stochastic demand, risk-averse channel members, production capacity constraints and price ceilings, we derived the optimization conditions of manufacturers, retailers and demand markets via variational inequality respectively, and then established the whole supply chain network equilibrium problem. Projection and contraction method was utilized to solve the model. Numerical examples were given to illustrate the impact of risk-averse degree of manufacturers and retailers on network equilibrium patterns in case of capacity constraints and price ceilings. The results show that under the same condition, the system profits decrease to a lower level when the manufacturers' risk degree increase compared with retailers. The retailers bear more loss due to their positions in the supply chain.

## Introduction

With the development of modern economics, the competition fashions among enterprises are mainly expressed as the competition among supply chains. The issue how to obtain the equilibrium conditions of supply chain network has already been the critical topic over the years.

Several researches have contributed to literature body of supply chain network equilibrium (SCNE) problems. The seminal paper by Nagurney [1] whose research focused on SCNE with deterministic demand has triggered a variety of related studies. Nagurney [2] developed a framework for the formulation and computation of solutions to supply chain network equilibrium problems in the presence of electronic commerce. Dong [3] studied SCNE with stochastic demand. The multi-tier supply chain network model with B2B e-commerce, supply and demand side risk and uncertainty were presented in Nagurney [4, 5]. More recently, a few authors extended traditional SCNE models into multi-period horizons [6, 7].

However, all of the above literatures assumed the production capacity of manufacturers were infinite, which didn't coincide with the fact. Meng [8] developed a SCNE model with production capacity constraints under deterministic environment. Besides, the government often intervenes in the market to curb the price fluctuation so that the commodity price should be within certain ranges. Nagurney [9] studied spatial price equilibrium problem with price-rigidities on oligopolistic markets, but without considering distribution channels.

Similar to Nagurney [4, 5], we have full realization that the widespread risk existed in supply chain operations may result in the decision-makers being risk-averse. They not only seek to maximize the profits but also minimize their risks. Different form Nagurney [4-5], we make efforts to investigate the impact of channel members' risk-averse degree on supply SCNE with capacity constraints and price rigidities, in which three cases are included: only the manufacturer is risk-averse, only the retailer is risk-averse and both of them are risk-averse.

The paper is arranged as follows. In section 2, we developed the network equilibrium model and derived the optimality conditions of the various decision-makers, and then established the equilibrium conditions as a finite-dimensional variational inequality problem. In section 3 we carried out numerical examples to illustrate the effects of risk on the equilibrium shipments and system profits. Conclusions and suggestions for future research were presented in section 4.

## Supply chain network equilibrium models with risk-averse channel members

In this section, as is illustrated by a two-tire network described in Fig. 1, we consider $M$ manufacturers are involved in the production of homogeneous products which are then shipped to $N$ retailers who are faced with random and price-sensitive market demand.

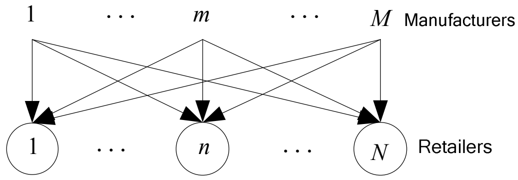


Fig.1 The structure of supply chain network

**The equilibrium of the manufacturer market.** Let $q_m$ denote the non-negative production output of manufacturer $m$, group the production output of all manufacturers into the column vector $q \in R_+^M$. The product shipment between manufacturer $m$ and retailer $n$ is denoted by $q_{mn}$, group the product shipments between all pairs of manufacturers and retailers into a column vector $Q^1 \in R_+^{MN}$. Let $\rho_{mn}$ denote the wholesale price charged by manufacturer $m$ to retailer $n$. Let $C_m$ represent the upper bound of the production capacity for manufacturer $m$.

In order to express competition among manufacturers, here we assume that each manufacturer $m$ is faced with production cost function $f_m(q)$, which may depend not only on the production output of its own but also on those of the other manufacturers.

The transaction cost associated with manufacturer $m$ transacting with retailer $n$ is denoted by $c_{mn}(q_{mn})$. We can express the criterion of profit maximization for manufacturer $m$ as:

$$\max_{(q_m,(q_{mn})_{N\times 1})\in\Omega^M} \pi_m(q,Q^1) = \sum_{n=1}^{N}\rho_{mn}q_{mn} - f_m(q) - \sum_{n=1}^{N}c_{mn}(q_{mn}) \tag{1}$$

Where $\Omega^M = \left\{(q_m,(q_{mn})_{N\times 1}) \in R_+^{1+N} \mid \sum_{n=1}^{N} q_{mn} \le q_m, q_m \le C_m\right\}$.

Note that the first term in (1) corresponds to the revenue whereas the subsequent two terms represent the production cost and the transaction costs, respectively.

In addition to the criterion of profit maximization we also assume that each manufacturer is risk-averse and concerned with risk minimization. The risks perceived by manufacturer $m$ are dependent not only on the flows that he controls but also upon those by other manufacturers. Hence, the second criterion of manufacturer $m$ can be expressed as [4-5]:

$$\min_{q_{mn}\ge 0} r_m = r_m(Q^1)\ , \forall m. \tag{2}$$

The manufacturer $m$ associates a non-negative weight $\eta_m$ which we term as "risk-averse degree". So the multi-criterion decision-making problem for manufacturer $m$ is transformed into:

$$\max_{(q_m,(q_{mn})_{N\times 1})\in\Omega^M} \pi_m(q,Q^1) = \sum_{n=1}^{N}\rho_{mn}q_{mn} - f_m(q) - \sum_{n=1}^{N}c_{mn}(q_{mn}) - \eta_m r_m(Q^1) \tag{3}$$

Where $\Omega^M = \left\{(q_m,(q_{mn})_{N\times 1}) \in R_+^{1+N} \mid \sum_{n=1}^{N} q_{mn} \le q_m, q_m \le C_m\right\}$.

We presume that the production cost functions, the transaction cost functions and the risk functions for each manufacturer are continuous and convex. The optimal conditions for all manufacturers can be described as the following inequality: determine the optimal $(q^*,Q^{1*}) \in \Omega^M$, satisfying:

$$\sum_{m=1}^{M}\frac{\partial f_m(q^*)}{\partial q_m}[q_m - q_m^*] + \sum_{m=1}^{M}\sum_{n=1}^{N}[\frac{\partial c_{mn}(q_{mn}^*)}{\partial q_{mn}} - \rho_{mn}^* + \eta_m\frac{r_m(Q^{1*})}{\partial q_{mn}}]\times[q_{mn} - q_{mn}^*] \ge 0\ , \forall (q,Q^1) \in \Omega^M \tag{4}$$

Where $\Omega^M = \left\{(q,Q^1) \in R_+^{M+MN} \mid \sum_{n=1}^{N} q_{mn} \le q_m, q_m \le C_m\right\}$.

**The equilibrium of the retailer market.** The retailers transact with the manufacturers as well as the consumers. We suppose that $\hat{d}_n(\rho_n)$ is the market demand at the price of $\rho_n$ at retail $n$, where $\hat{d}_n(\rho_n)$ is a random variable with a density function of $\phi_n(x;\rho_n)$, with $\rho_n$ serving as a parameter. Let $\Phi_n(x;\rho_k)=\int_0^x \phi_n(x;\rho_n)\,\mathrm{d}x$ be the probability distribution function of $\hat{d}_n(\rho_n)$. Let $s_n=\sum_{m=1}^{M} q_{mn}$, in turn, denote the total supply at retailer $n$ that he obtains from all the manufacturers, and group these amounts into the column vector $s\in R_+^N$. The retailer $n$ can sells no more than the minimum of his supply or consumers' demand; that is, the actual sale cannot exceed $\min\{s_n,\hat{d}_n\}$. The retailer $n$ is faced with a handling cost $c_n(s)$, which may include the display and storage cost associated with the product. We assume that $c_n(s)$ is continuous and convex for $s$. $\lambda_n^+(>0)$ and $\lambda_n^-(>0)$ are unit penalty of having excess supply and excess demand at retailer outlet, respectively.

Given $s_n$, for retailer $n$, expected sales, inventory and shortage amounts are scalar functions of $s_n$ and $\rho_n$. Particular, let $S_n(s_n,\rho_n)$, $H_n(s_n,\rho_n)$ and $Q_n(s_n,\rho_n)$ denote these values respectively, that are:

$$S_n(s_n,\rho_n)=\mathrm{E}[\min\{\hat{d}_n(\rho_n),s_n\}]=s_n-\int_0^{s_n}(s_n-x)\mathrm{d}\Phi_n(x,\rho_n) \tag{5}$$

$$H_n(s_n,\rho_n)=\mathrm{E}[\max\{0,s_n-\hat{d}_n(\rho_n)\}]=\int_0^{s_n}(s_n-x)\mathrm{d}\Phi_n(x,\rho_n) \tag{6}$$

$$Q_n(s_n,\rho_n)=\mathrm{E}[\max\{0,\hat{d}_n(\rho_n)-s_n\}]=\int_{s_n}^{+\infty}(x-s_n)\mathrm{d}\Phi_n(x,\rho_n) \tag{7}$$

Furthermore, the retailers are also assumed to be risk-averse like manufacturers. For retailer n, the corresponding risk function can be expressed as:

$$\min_{q_{mn}\ge 0} r_n = r_n(Q^1),\ \forall n. \tag{8}$$

It is assumed that risk-averse degree of retailer $n$ is $\eta_n$. Then by simplification of (5) - (7) and combined with (8), the optimization problem of retailer $n$ can be described as

$$\pi_n(s,Q^1)=\rho_n s_n-(\rho_n+\lambda_n^+)\int_0^{s_n}(s_n-x)\mathrm{d}\Phi_n(x,\rho_n)-\lambda_n^-\int_{s_n}^{+\infty}(x-s_n)\mathrm{d}\Phi_n(x,\rho_n)-c_n(s)-\sum_{m=1}^{M}\rho_{mn}q_{mn}-\eta_n r_n(Q^1) \tag{9}$$

Where $\Omega^R=\left\{(s_n,(q_{mn})_{M\times 1})\in R_+^{1+M}\mid s_n=\sum_{m=1}^{M}q_{mn}\right\}$.

Objective function (9) shows that the expected profit is the expected revenues minus the sum of the expected penalty cosr, the handling cost, and the payout to the manufacturers. The optimality conditions for all the retailers can be expressed as the variational inequality problem: determine the optimal $(s^*,Q^{1*})\in\Omega^R$, satisfying

$$\sum_{n=1}^{N}[(\rho_n^*+\lambda_n^++\lambda_n^-)P_n(s_n^*,\rho_n^*)-\rho_n^*-\lambda_n^-+\frac{\partial c_n(s^*)}{\partial s_n}]\times\left[s_n-s_n^*\right]+\sum_{n=1}^{N}\sum_{m=1}^{M}(\rho_{mn}^*+\eta_n\frac{r_n(Q^{1*})}{\partial q_{mn}})\times\left[q_{mn}-q_{mn}^*\right]\ge 0,$$
$$\forall(s,Q^1)\in\Omega^R,\ \text{Where } \Omega^R=\left\{\left(s,Q^1\right)\in R_+^{N+MN}\mid s_n=q_{mn}\right\} \tag{10}$$

In the process of derivation inequality (10), we have not had the prices charged be variables. These variables become endogenous in the complete supply chain network equilibrium model.

**The equilibrium of the demand market by price-ceilings.** In order to protect the benefit of consumers, the government often intervenes in the market by setting price ceilings. Let $\bar{\rho}_n$ denote the price ceiling in demand market $n$ by government. $d_n(\rho_n^*)$ is the expected value of random demand. Therefore, the equilibrium condition of consumers purchased from retailer $n$ is equivalent to the variational inequality problem as follows, after taking the expected value: determine $\rho_n^*\in[0,\bar{\rho}_n]$, satisfying (Nagurney [9])

$$\left[s_n^*-d_n(\rho_n^*)\right]\times[\rho_n-\rho_n^*]\ge 0,\ \ \forall\rho_n\in[0,\bar{\rho}_n] \tag{11}$$

Where $\Omega^C=\left\{\rho\in R_+^N\mid \rho_n\le\bar{\rho}_n\right\}$

**The equilibrium condition of the supply chain network.** In equilibrium state, the optimality conditions for all manufacturers, all retailers and the market equilibrium conditions must be satisfied simultaneously. Hence, we get the equilibrium condition of the supply chain network.

**Theorem**. A product shipment and price pattern $(q^*, Q^{1*}, s^*, \rho^*) \in \Omega$ is an equilibrium pattern of the supply chain network model with risk-averse channel members, production capacity constraints, price rigidities if and only if it satisfies the variational inequality problem given by: determine $(q^*, Q^{1*}, s^*, \rho^*) \in \Omega$, satisfying

$$\begin{aligned}&\sum_{i=m}^{M}\frac{\partial f_m(q^*)}{\partial q_m}\times[q_m-q_m^*]+\sum_{m=1}^{M}\sum_{n=1}^{N}[\frac{\partial c_{mn}(q_{mn}^*)}{\partial q_{mn}}+\eta_m\frac{r_m(Q^{1*})}{\partial q_{mn}}+\eta_n\frac{r_n(Q^{1*})}{\partial q_{mn}}]\times[q_{mn}-q_{mn}^*]+\sum_{n=1}^{N}\left[s_n^*-d_n(\rho_n^*)\right]\times[\rho_n-\rho_n^*]\\&+\sum_{n=1}^{N}[(\rho_n^*+\lambda_n^++\lambda_n^-)P_n(s_n^*,\rho_n^*)-\rho_n^*-\lambda_n^-+\partial c_n(s^*)/\partial s_n]\times\left[s_n-s_n^*\right],\forall(q,Q^1,s,\rho)\in\Omega\end{aligned}\tag{12}$$

Where $\Omega = \Omega^M \times \Omega^R \times \Omega^C$, i.e., $\Omega = \left\{ \begin{array}{l|l} (q,Q^1) \in R_+^{M+MN} & q_m \le C_m, \sum_{n=1}^{N} q_{mn} \le q_m \\ (s,\rho) \in R_+^{N+N} & s_n = \sum_{m=1}^{M} q_{mn},\ \rho_n \le \bar{\rho}_n, \end{array} \right\}$.

Let $\lambda \in R_+^M$ and $\delta \in R_+^M$ denote the Lagranian multipliers column vector with respect to the constraints $q_m \le C_m$ and $\sum_{n=1}^{N} q_{mn} \le q_m$, respectively. $\gamma \in R_+^N$ and $\upsilon \in R_+^N$ denote Lagranian multipliers column vector with respect to the constraints $s_n = \sum_{m=1}^{M} q_{mn}$ and $\rho_n \le \bar{\rho}_n$, respectively.

## Numerical examples and Sensitivity Analysis

Now consider a supply chain network includes 2 manufacturers, 2 retailers and 2 demand markets. It is assumed that the random market demands follow uniform distribution: $\hat{d}_n(\rho_n) \sim [0, b_n/\rho_n], b_n = 20$, for $n = 1,2$. The maximal production capacity of each manufacturer is $C_1 = C_2 = 1$; The price rigidity of each demand market is $\bar{\rho}_1 = \bar{\rho}_2 = 18$. The production cost functions for the manufacturers are $f_m(q) = 2q_m^2 + q_m q_{3-m} + 2.5 q_m$, $m$=1,2; the transactions cost functions of the manufacturers and associated with the retailers are given by $c_{mn}(q_{mn}) = q_{mn}^2 + 3q_{mn}$, for $m = 1,2; n = 1,2$. The handling costs of the retailers are given by $c_n(s) = 0.6 s_n^2$, for $n = 1,2$. Set the unit costs of excess supply and excess demand $\lambda_n^+ = \lambda_n^- = 1$, for $n = 1,2$. The risk functions of manufacturers are given by $r_m = (\sum_{n=1}^{N} q_{mn} - 1)^2, m = 1,2$. The risk functions of retailers are given by $r_n = (\sum_{m=1}^{M} q_{mn} - 0.8)^2, n = 1,2$.

We utilize the Project and contraction method (Korpelevich [10]) to solve the variational inequality problem and yield the equilibrium patterns of the SCNE with and without production capacity constraints, price ceilings. The results are shown in Table 1 to Table 7.

Table 1 Solutions of the SCNE with risk-averse manufacturers but without production capacity constraints and price ceilings

| $C_1 = C_2 = \infty, \bar{\rho}_1 = \bar{\rho}_2 = \infty$ | | | | |
|---|---|---|---|---|
| | $\eta_1^m = \eta_2^m = 0$ | $\eta_1^m = \eta_2^m = 0.2$ | $\eta_1^m = \eta_2^m = 0.5$ | $\eta_1^m = \eta_2^m = 0.8$ |
| $q_m$ | 0.5347 | 0.5422 | 0.5533 | 0.5641 |
| $q_{mn}$ | 0.2674 | 0.2711 | 0.2766 | 0.282 |
| $\rho_{mn}$ | 8.4411 | 8.4823 | 8.5431 | 8.6025 |
| $\rho_n$ | 18.7004 | 18.442 | 18.0739 | 17.7279 |
| $\pi_m$ | 0.5719 | 0.5461 | 0.5125 | 0.4844 |
| $\pi_n$ | 2.5472 | 2.453 | 2.3129 | 2.1745 |
| $\Sigma \pi_m + \pi_n$ | 6.2382 | 5.9983 | 5.6508 | 5.3178 |

Table 2 Solutions of the SCNE with risk-averse retailers but without production capacity constraints and price ceilings

| $C_1 = C_2 = \infty, \bar{\rho}_1 = \bar{\rho}_2 = \infty$ | | | | |
|---|---|---|---|---|
| | $\eta_1^n = \eta_2^n = 0$ | $\eta_1^n = \eta_2^n = 0.2$ | $\eta_1^n = \eta_2^n = 0.5$ | $\eta_1^n = \eta_2^n = 0.8$ |
| $q_m$ | 0.5347 | 0.539 | 0.5452 | 0.5512 |
| $q_{mn}$ | 0.2674 | 0.2695 | 0.2726 | 0.2756 |
| $\rho_{mn}$ | 8.4411 | 8.4645 | 8.4987 | 8.5317 |
| $\rho_n$ | 18.7004 | 18.5528 | 18.3415 | 18.1415 |
| $\pi_m$ | 0.5719 | 0.581 | 0.5945 | 0.6077 |
| $\pi_n$ | 2.5472 | 2.4802 | 2.383 | 2.2897 |
| $\Sigma\pi_m+\pi_n$ | 6.2382 | 6.1224 | 5.955 | 5.7948 |

Table 3 Solutions of the SCNE with risk-averse manufacturers and retailers but without production capacity constraints and price ceilings

| $C_1 = C_2 = \infty, \bar{\rho}_1 = \bar{\rho}_2 = \infty$ | | | | |
|---|---|---|---|---|
| | $\eta_1^m = \eta_2^m = 0.2,$ $\eta_1^n = \eta_2^n = 0.2$ | $\eta_1^m = \eta_2^m = 0.5,$ $\eta_1^n = \eta_2^n = 0.5$ | $\eta_1^m = \eta_2^m = 0.8,$ $\eta_1^n = \eta_2^n = 0.2$ | $\eta_1^m = \eta_2^m = 0.2,$ $\eta_1^n = \eta_2^n = 0.8$ |
| $q_m$ | 0.5464 | 0.5631 | 0.5679 | 0.5583 |
| $q_{mn}$ | 0.2732 | 0.2815 | 0.2839 | 0.2791 |
| $\rho_{mn}$ | 8.5051 | 8.5968 | 8.6234 | 8.5705 |
| $\rho_n$ | 18.3021 | 17.7602 | 17.6091 | 17.9123 |
| $\pi_m$ | 0.5559 | 0.5386 | 0.4956 | 0.5843 |
| $\pi_n$ | 2.3878 | 2.1597 | 2.1147 | 2.2024 |
| $\Sigma\pi_m+\pi_n$ | 5.8874 | 5.3966 | 5.2206 | 5.5734 |

According to Table 1 and Table 2, it can be observed that in the absent of production capacity constraints and price rigidities, no matter the manufacturers' or the retailers' risk-averse degrees increase, the transaction amounts between all pairs increase, meanwhile the market prices decrease. However, which is difference, higher risk-averse degrees of manufacturers lead to fewer profits of all channel members, whereas higher risk-averse degrees of retailers result in fewer profits of retailers and total system, but more profits of manufacturers. Furthermore, from Table 3 we can reasonably draw the conclusions that when both of manufacturers and retailers are risk-averse, the impacts of the manufacturers' risk attitudes on system profits are larger than retailers.

Table 4 Solutions of the SCNE with risk-averse manufacturers, production capacity constraints and price ceilings

| $C_1 = C_2 = 0.5, \bar{\rho}_1 = \bar{\rho}_2 = 18$ | | | | |
|---|---|---|---|---|
| | $\eta_1^m = \eta_2^m = 0$ | $\eta_1^m = \eta_2^m = 0.2$ | $\eta_1^m = \eta_2^m = 0.5$ | $\eta_1^m = \eta_2^m = 0.8$ |
| $q_m$ | 0.5 | 0.5 | 0.5 | 0.5 |
| $q_{mn}$ | 0.25 | 0.25 | 0.25 | 0.25 |
| $\rho_{mn}$ | 9.15 | 9.35 | 9.65 | 9.95 |
| $\rho_n$ | 18 | 18 | 18 | 18 |
| $\pi_m$ | 0.95 | 1 | 1.075 | 1.15 |
| $\pi_n$ | 1.9694 | 1.8694 | 1.7194 | 1.5694 |
| $\Sigma\pi_m+\pi_n$ | 5.8389 | 5.7389 | 5.5889 | 5.4389 |

Table 5 Solutions of the SCNE with risk-averse manufacturers, production capacity constraints and price ceilings

| $C_1 = C_2 = 0.5, \bar{\rho}_1 = \bar{\rho}_2 = 18$ | | | | |
|---|---|---|---|---|
| | $\eta_1^m = \eta_2^m = 0$ | $\eta_1^m = \eta_2^m = 0.2$ | $\eta_1^m = \eta_2^m = 0.5$ | $\eta_1^m = \eta_2^m = 0.8$ |
| $q_m$ | 0.5 | 0.5 | 0.5 | 0.5 |
| $q_{mn}$ | 0.25 | 0.25 | 0.25 | 0.25 |
| $\rho_{mn}$ | 9.15 | 9.27 | 9.45 | 9.63 |
| $\rho_n$ | 18 | 18 | 18 | 18 |
| $\pi_m$ | 0.95 | 1.01 | 1.1 | 1.19 |
| $\pi_n$ | 1.9694 | 1.8914 | 1.7744 | 1.6574 |
| $\Sigma\pi_m+\pi_n$ | 5.8389 | 5.8029 | 5.7489 | 5.6949 |

Table 6 Solutions of the SCNE with risk-averse manufacturers and retailers, production capacity constraints and price ceilings

| $C_1 = C_2 = 0.5, \bar{\rho}_1 = \bar{\rho}_2 = 18$ | | | | |
|---|---|---|---|---|
| | $\eta_1^m = \eta_2^m = 0.2$, $\eta_1^n = \eta_2^n = 0.2$ | $\eta_1^m = \eta_2^m = 0.5$, $\eta_1^n = \eta_2^n = 0.5$ | $\eta_1^m = \eta_2^m = 0.8$, $\eta_1^n = \eta_2^n = 0.2$ | $\eta_1^m = \eta_2^m = 0.2$, $\eta_1^n = \eta_2^n = 0.8$ |
| $q_m$ | 0.5 | 0.5 | 0.5 | 0.5 |
| $q_{mn}$ | 0.25 | 0.25 | 0.25 | 0.25 |
| $\rho_{mn}$ | 9.47 | 9.95 | 10.07 | 9.83 |
| $\rho_n$ | 18 | 18 | 18 | 18 |
| $\pi_m$ | 1.06 | 1.225 | 1.21 | 1.24 |
| $\pi_n$ | 1.7914 | 1.5244 | 1.4914 | 1.5574 |
| $\Sigma\pi_m+\pi_n$ | 5.7029 | 5.4989 | 5.4029 | 5.5949 |

According to Table 4 to Table 6, when setting the production capacity constraints and price rigidities, as the increase of risk-averse degrees of manufacturers or retailers, the transaction amounts between manufacturers and retailers and the prices in the demand markets are invariable, which indicates that the relative constraints take effect so that the manufacturers couldn't reach the optimal output quantities in the absent of capacity constraints. The transaction prices between all pairs increase, which means that the manufacturers can deal with the risk by raising the wholesale prices. As a result, the manufacturers' profits get more as the increase of risk-averse degree. In contrast, retailers' profits become fewer due to the inability to make a corresponding higher commodity price because of price ceilings by government. Meanwhile, the total system profits also decease. Through comparative analysis of Table 4 and Table 5, along with the third column and the fourth column of Table 6, we can draw the conclusions that under the same condition, the system profits will decrease to a much lower level when the manufacturers' risk degrees increase compared with retailers.

The numerical examples and sensitivity analysis show that risk attitude of the manufacturers and retailers have significant impacts on the equilibrium solutions and system profits. In addition, if the manufacturers have production capacity constraints and the government intervenes in the market by setting price ceilings, the supply chain network can't reach the optimal natural conditions. The retailers will bear more loss because of their positions in the supply chain.

## Conclusions

This paper develops a general supply chain network equilibrium model with risk-averse channel members in case of production capacity constraints and the price rigidities, which is described by the variational inequalities and solved by Project and contraction algorithm. This research may be useful to risk-averse decision-makers included in the supply chain network and the policy makers from the governments who face with the long-term strategic decisions of supply chain development. We will investigate the multi-period supply chain and closed-loop supply chain network equilibrium models with risk-averse channel members in future research.

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## References

[1] A. Nagurney, J. Dong, D. Zhang, A supply chain network equilibrium model, Transportation Research Part E: Logistics and Transportation Review, 38(2002) 281-303.

[2] A. Nagurney, J. Loo, J. Dong, Supply chain networks and electronic commerce: a theoretical perspective, Netnomics, 4(2002) 187-220.

[3] J. Dong, D. Zhang, A. Nagurney. A supply chain network equilibrium model with random demands, European Journal of Operational Research, 156(2004) 194-212.

[4] A. Nagurney, J. Cruz, J. Dong, D. Zhang. Supply chain networks, electronic commerce, and supply side and demand side risk, European Journal of Operational Research,164(2005) 120-142.

[5] A. Nagurney, D. Matsypura, Global supply chain network dynamics with multicriteria decision-making under risk and uncertainty, Transportation Research Part E: Logistics and Transportation Review, 41(2005) 585-612.

[6] Y. Hamdouch. Multi-period supply chain network equilibrium with capacity constraints and purchasing strategies, Transportation Research Part C, 19(2011) 803-820.

[7] J. M. Cruz, Z. G. Liu, Modeling and analysis of the multiperiod effects of social relationship on supply chain networks, European Journal of Operational Research, 214(2011) 39-52.

[8] Q. Meng, Y. Huang, R. L. Cheu. Competitive facility location on decentralized supply chains, European Journal of Operational Research, 196(2009) 487-499.

[9] A. Nagurney, L. Zhao. Disequilibrium and variational inequalities, Journal of Computational and Applied Mathematics, 33(1990) 181-198.

[10] G. M. Korpelevich. The extra gradient method for saddle points and other problems, Matckon, 13(1997) 35-49.