

Mathematical Modeling of Grape Organs and Its' Visual

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Abstract. Virtual plant growth is widely applied in many fields such as agriculture, forestry research, green landscape design, education, entertainment and business etc. Plant morphological structure model can simulate the real plant dynamic development, so people may be inclined to use some methods instead of the process; therefore ,multiple produce type or equation are used for modeling of natural phenomena. This paper aims to take grape organs as an example to study 3d plant organ geometric model based on OpenGL software. In this paper, grape's organs (fruit, flower, leaf) geometric structure model and algorithm are presented.

Introduction

Using the computer to study the structure of the plant forms can be traced back to the late sixties century. [1] Plants are the most common living beings in nature, which closely relate to our daily life. Virtual agriculture extends from virtual reality, studies animal, plant growth process, expression, assimilation, alienation process of genetic material changing to virtual reality on computer, and it will be widely applied in the fields of agriculture research, teaching, production, planning, agricultural resources allocation and the circulation of commodities. Virtual agriculture includes virtual crops; agricultural expert system etc. using computer to simulate crops growth process, therefore human can obtain a crop physiology ecosystem and the results of the parallel process of shape structure.[2] The whole process shows the three dimensional animation to simulate crops growth, then the system outputs crop physiology parameters and the relationship. Virtual crops research is of far-reaching significance; hence Virtual technology has widely be applied on the precision of sustainable agriculture. In the future, with little time on field, people may be observing, analyzing the crops' situation on screen. The tectonic model thought is as below:

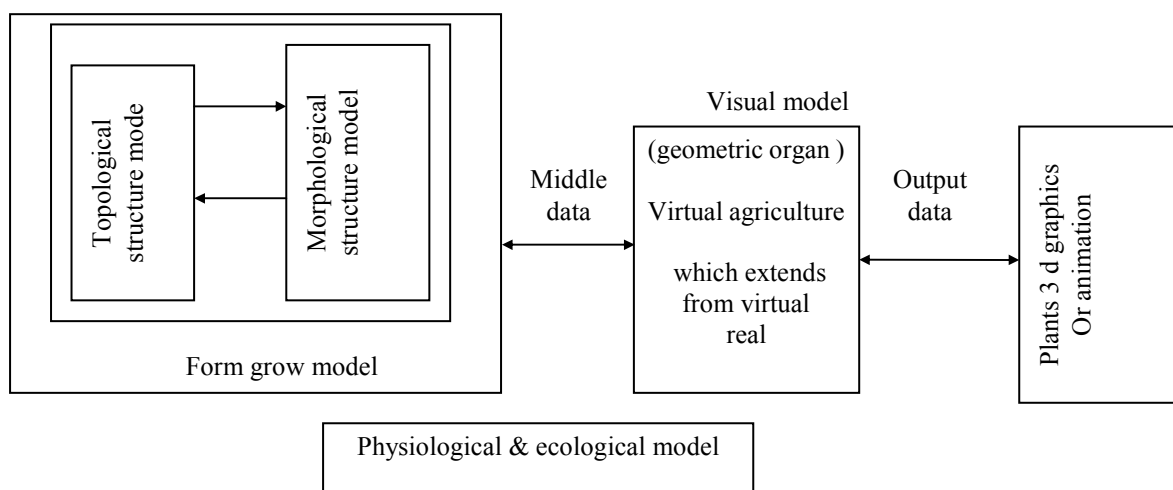


Fig. 1 Virtual crops tectonic model

Grape organ structures of geometric algorithms

3D form characteristics of organs of grape:

Usually the fruit is spherical or ellipsoidal. To reappear with realistic the grapes on computer, you should make sure the axis of the fruit; follow the rule as below: the points of the cross section axis should be the radius that calculate 3d coordinates of the outline.

The grapes form model. The shape of grapes is nearly unchangeable, and their features are prominent, so we can represent their general character in the computers. Hence, people control the results with the least amount of parameters as much as possible. Bezier curve, which has diversity and volatility, can get all kinds of rules of the curve equation, it will lead to the trend namely it is in line with the characteristics of different pattern of fruit. The starting point of the curve, the destination and the corresponding feature conduct as the beginning, the destination coincidence polygon, the first side and polygon last side of the curve in beginning and end the cut in sagittal direction, then the shape of the curve tends to feature polygon. When a given space near a “n + 1” point of the vector PI, the Bezier curve of different points on coordinate difference formula,[3]

$$c(t) = \sum_{i=1}^n p_i B_{in}(t), (0 \leq t \leq 1)$$

$i=1$

P_i Constitutes the curve characteristic polygons, while $B_{in}(t)$ is basis functions, and it is also curve different points on the position vector harmonic function.

$$B_{in}(t) = \frac{n!}{i!(n-i)!} t^i (1-t)^{n-i} = C_n^i t^i (1-t)^{n-i}$$

$$i = 0, 1, \dots, n$$

Considering the uncomplicated elliptic curve of grape: from the four points to control the axis of the fruit shape. Here choose three times Bezier harmonic function

$$B = [B_{0,3}(t), B_{1,3}(t), B_{2,3}(t), B_{3,3}(t)] =$$

$$[t^3 t^2 t^1 1] \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} = TM_2$$

$$C(t) = TM_2 [P_0 P_1 P_2 P_3]^T = \sum_{i=0}^3 P_i B_{i,3}(t) = (1-t)^3 P_0 + 3t(1-t)^2 P_1 + 3t^2(1-t) P_2 + t^3 P_3$$

$$(0 \leq t \leq 1)$$

Here the three-dimensional coordinates, the tangent and the section radius are gained. For convenience of calculation, the Z coordinate of four control points is set to 0. Between the shaft angle of tangent and the XO axis is set as α ; and β act as the angle moving anti-clockwise according to XOY. On above conditions, we can get 3 d coordinate of any points on the contour line via this point. As shown in figure 2.

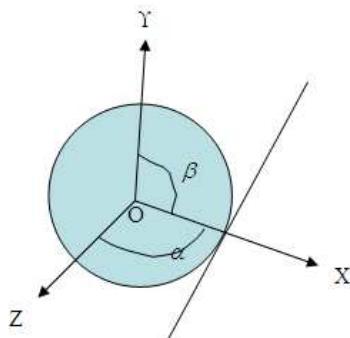


Fig. 2 3 d coordinate calculation chart

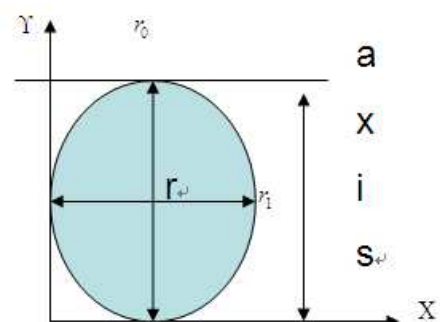


Fig.3 Section radius r calculation schemes

$$\begin{aligned}x &= x_0 + r * \cos(\pi/2 - \alpha) * \cos(\beta); \\y &= y_0 + r * \sin(\pi/2 - \alpha) * \cos(\beta); \\z &= z_0 - r * \sin(\beta);\end{aligned}$$

With the degree of angle of β changing increasingly, the step length is changed from 0 degrees to 360 degrees; in order to calculate the circumvented outline point corresponding around that point on the axis. Through increasing a step length, we can find the next point along axis. The same way is just as the outline points.

Above r is already known. Actually, in the process of computation, r is gained through observation and section function of different forms of fruit. With the example of more normal fruit, r does not have basically mutations. Given a maximum radius r , the position r of other axis is worked out through the section function. For example, the most top part is calculated by quadratic function of three control points. It is shown in figure 3.

X axis represents axis length, the y axis represents its corresponding section radius. Calculation process is as follows:

```
P1x=0; Ply=r0; P2X=1.0/3.0*len; P2y= r0+3.0/4.0*(r1-r0); P3x=len; P3y=r1;
a=((ply-p2y)/(p1x-p2x)-(p2y-p3y)/(p2x-p3x))/(p1x-p3x);
b=(ply-p2y)/(p1x-p2x)-a*(p1x+p2x);
e= P1y-a*P1x*P1x-b*P1x
y=a*step*step+b*step+c;
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The petals of the rendering algorithm

In the section, firstly introduce the petals of the rendering algorithm. It is very necessary for determining a petal of axis line. Due to the relatively simple curve form, using parabolic equation $a * x + xb * x + c = 0$ ($z = 0$) to fit the curve, and z fit coordinate as 0 for convenience. Since this equation, coordinate of any point in the curve can be known.

Secondly, find out the coordinates of any two points in edging lines. As figure 4 shows, a, c and e are the points in one edging line of the petal, b and d are the points in the axis line. Surely, we supposed dividing the two both X coordinates are of the starting point and the ending point into several parts at the same step, therefore 3-dimensional coordinates is built. Knowing the 3-dimensional, we can get the tangent equation and the angle. Assume the angle that between the line determined by these two points and plane xoy as β , and the β should be an angle of a stochastic figure which approximately to 90 degrees, and we can get it by using a stochastic function. The coordinates of the corresponding points in the axis line can be obtained with α and β . Hence, x_1, y_1 are coordinates of a point in the axis line, X_2, Y_2, Z_2 are coordinates of the corresponding point in the edging line of the petal.

$$\begin{cases} x_2 = x_1 + 1 * \cos(\alpha) * \cos(\beta); \\ y_2 = y_1 + 1 * \sin(\alpha) * \cos(\beta); \\ z_2 = -1 * \sin(\beta); \end{cases}$$

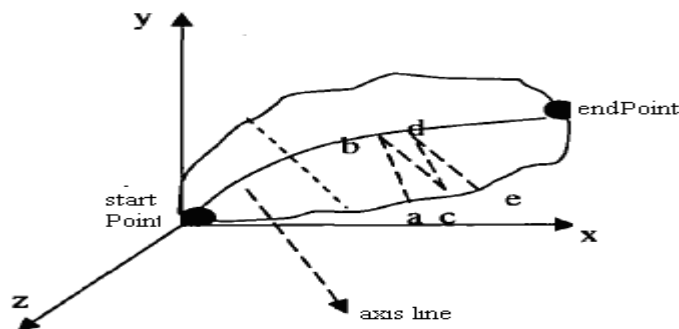


Fig. 4 Diagram of the petal

The method has some advantages in the simulation of the petal: relatively simple control, few parameters, clear mathematical and physical significance. However, its disadvantage is that its biological significance is not clear.

Particle system based on leaf texture algorithm

The irregular surface of the geometric texture should be effectively represented and rendered before using computer to draw a variety of the grape. In 1983, Reeve's presented a simulation method of irregular fuzzy objects, namely, the particle system method, which fully embodies the dynamic and random quality of fuzzy objects. The section demonstrates the construction of leaf veins, which derived from the Reeves's algorithm [3].

Basic principles of particle system Particle system constructional method is based on the idea that many simple shapes of tiny particles as the basic element get together to form an irregular fuzzy objects. To generate a moment of particle system the basic steps are as follows:

- 1) To produce new particles in the system;
- 2) To give each new particle static attributes;
- 3) To delete particles which have been in existence and over the survival time in the system;
- 4) To move and transform particles according to the dynamic properties of the remaining particles;
- 5) To display the image of the live particles.

The basic model structure of leaf texture The blade is the important part of a plant, and it is also the main organ. The texture of leaf is mainly composed of venation of the leaf, or determined by the distribution of veins in the leaves. The following is a universal energy model which used to simulate the texture of leaf by using a particle. The particle represents each small part in the boundary, therefore, one after another the particles in the internal boundary blade disperse; each particle produces the same amount of energy, and the result is that all transmit to petiole. We described this model through the following processes:

- 1) Divide particles in the internal boundary of the blade on average;
- 2) move to the nearest particle;
- 3) Mobile to the target (petiole);
- 4) Repeat 2), until all the particles reach the end.

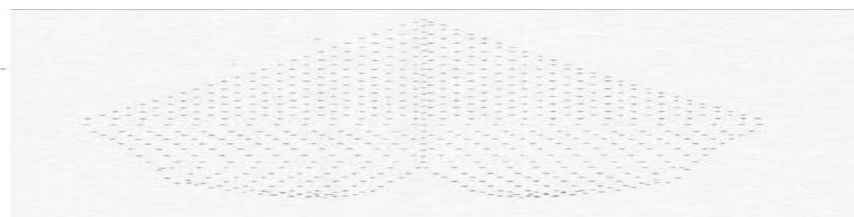
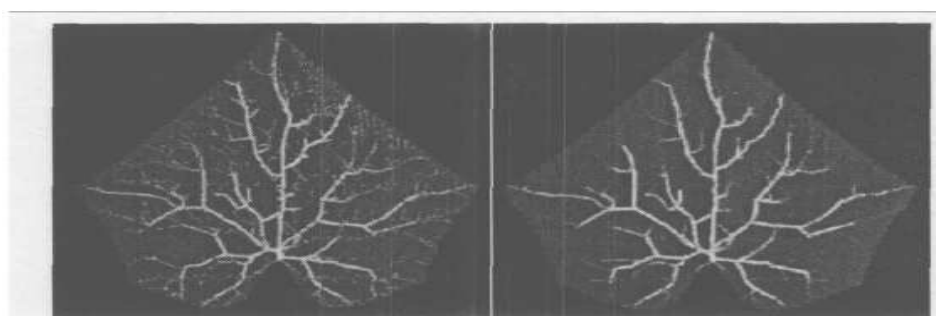


Fig. 5 Diagram of the particles

The sparse of veins can be controlled by changing the distribution density of particle. We can decide the particle motional trajectory which has shown on the screen according to real circumstances (machine speed, approximation degree). Because there is no need to demonstrate the dynamic process of veins formation, we can close timer with OpenGL own in figure 6.



rendering speed slow

rendering speed fast

Fig. 6 After Rendering of the blade

This section presents a method of using particle system to construct the leaf texture,. Particle system can better reflect the real situations of the nature, and sometimes it is very effective to imitate the complex and regular things in the nature. To combine methods of particle system with other method of simulating natural scenery can create more complex and vivid natural scenery image.

Structure algorithm based on leaf of the Nurbs surface

We often used all kinds of spline and spline curve in scientific computing, such as B spline, Bezier curve and surface, which are reflected incisively and vividly in the OpenGL evaluation of command. Based on the above theory, we use NURBS surface to fit the blade [4][5].

The basic idea of the algorithm is: drawing the point of NURBS surface.

First, turn the blade into left and right two parts, each part is drawn by 4 x4 control points of NURBS surface; then join together the curve surface of both sides.

Figure 7 is a divisional map of the control points on the “xoy” plane and the Z coordinate of control points fluctuates around 0.

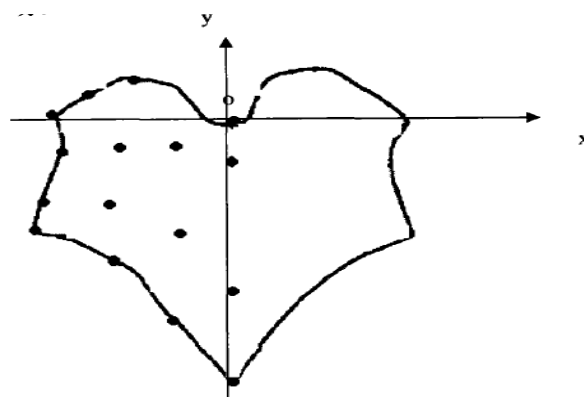


Fig.7 NURBS surface control points

By changing the coordinate of control points, we can easily change the curve surface [6].

Conclusion

Plant 3d visualization and its' mathematical modeling are the key technology in the virtual plant growth. This paper has taken the grapes' organ for example to introduce its drawing algorithm in detail. The process of the 3d plant can be roughly divided into two stages: structuring and rendering, the model then come into being.

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