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Embedding Complete Binary Trees into Locally Twisted Cubes

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Abstract. The locally twisted cube is a newly introduced interconnection network for parallel computing, which possesses many desirable properties. In this paper, the problem of embedding complete binary trees into locally twisted cubes is studied. Let $LTQ_n(V, E)$ denote the n-dimensional locally twisted cube. We find the following result in this paper: for any integer $n \geq 2$, we show that a complete binary tree with $2^n - 1$ nodes can be embedded into the LTQ_n with dilation 2.

1. Introduction

An interconnection network can be represented by a graph $G = (V, E)$, where V represents the node set and E represents the edge set. One of the important properties of interconnection networks is graph embedding ability. Given a host graph $G_2 = (V_2, E_2)$, which represents the network into which other networks are to be embedded, and a guest graph $G_1 = (V_1, E_1)$, which represents the network to be embedded, the problem is to find a mapping from each node of G_1 to a node of G_2, and a mapping from each edge of G_1 to a path in G_2. Two common measures of effectiveness of an embedding are the dilation and expansion. The dilation of embedding ψ is defined as $\text{dil}(G_1, G_2, \psi) = \max\{\text{dist}(G_2, \psi(u), \psi(v))|(u, v) \in E_1\}$, where $\text{dist}(G_2, \psi(u), \psi(v))$ denotes the distance between the two nodes $\psi(u)$ and $\psi(v)$ in G_2. The smaller the dilation of an embedding is, the shorter the communication delay that the graph G_2 simulates the graph G_1 [1]. The expansion of embedding is defined as $\exp(G_1, G_2, \psi) = |V(G_2)|/|V(G_1)|$, which measures the processor utilization. The smaller the expansion of an embedding is, the more efficient the processor utilization that the graph G_2 simulates the graph G_1. Graph embedding has good applications in transplanting parallel algorithms developed for one network to a different one, and allocating concurrent processes to processors in the network. Path, cycle and mesh are three fundamental networks for parallel computing, and much work about path, cycle and mesh embedding [3], [4], [6] appeared in the literature.

Trees are another common interconnection structures used in parallel computing. It is important to study the problem of how to embed different kinds of trees into a host graph. Recently, many tree embedding problems have been studied [9], [7], [10].

The locally twisted cube LTQ_n is a variant of hypercube, proposed by Yang et al. [11]. It has many attractive features superior to those of the hypercube, such as the diameter is only about half of that of Q_n. In particular, Yang et al. [12] showed that LTQ_n is Hamiltonian connected and contains a cycle of every length from 4 to 2^n for $n \geq 3$. Furthermore, LTQ_n was proved to be $(n-2)$-pancyclic [2], for any integer $n \geq 3$. And some other properties of locally twisted cubes were discussed [8],[7],[5].

In this paper, the problem of embedding complete binary trees into locally twisted cubes is studied. We find for any integer $n \geq 2$, a complete binary tree with $2^n - 1$ nodes can be embedded into the LTQ_n with dilation 2.

2. Preliminaries

A binary string x of length n is denoted by $x_1x_2...x_{n-1}x_n$, where x_1 is the most significant bit and x_n is the least significant bit.

Similar to Q_n, LTQ_n is an n-regular graph of 2^n nodes. Every node of LTQ_n is identified by a unique binary string of length n. LTQ_n can be recursively defined as follows.

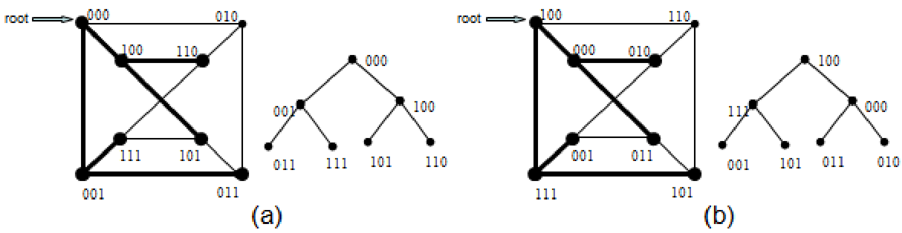

Fig. 1: LTQ_3 and CBT_3.

Definition 1 [11]. For $n \geq 2$, an n-dimensional locally twisted cube, LTQ_n, is defined recursively as follows:

(1) LTQ_2 is a graph consisting of four nodes labeled with 00, 01, 10, and 11, respectively, connected by four edges (00, 01), (00, 10), (01, 11), and (10, 11).

(2) For $n \geq 3$, LTQ_n is built from two disjoint copies of LTQ_{n-1} with the following steps. Let LTQ^0_{n-1} denote the graph obtained by prefixing the label of each node of one copy of LTQ_{n-1} with 0, and LTQ^1_{n-1} denote the graph obtained by prefixing the label of each node of the other copy of LTQ_{n-1} with 1. Connect each node $x = 0x_2x_3...x_n$ of LTQ^0_{n-1} to the node $1(x_2 + x_n)x_3...x_n$ of LTQ^1_{n-1} with an edge, where $'+'$ represents the modulo 2 addition.

We use CBT_n to denote the complete binary tree with $2^n - 1$ nodes which can be embedded into LTQ_n. For any integer $i \in \{0, 1\}$, CBT^i_n denotes to prefix the node labels of CBT_n with i. $|P|$ is the length of path P.

3. Embedding complete binary trees into locally twisted cubes

Lemma 1. A complete binary tree with 3 nodes can be embedded into LTQ_2 with dilation 1 rooted at any node of LTQ_2.

Proof. Obviously, we can embedded a complete binary tree rooted at any node of LTQ_2 into LTQ_2 with dilation 1, the lemma holds. □

Considering the symmetric properties of LTQ_3, we can intuitively find the symmetric properties of LTQ_3 as shown in the following lemma.

Lemma 2. Let f_1, f_2, f_3 be three self-isomorphic mappings from V(LTQ_3) to V(LTQ_3) as follows:

(1) $f_1(0x_2x_3) = f_1(1x_2\bar{x}_3)$ and $f_1(1x_2x_3) = f_1(0\bar{x}_2\bar{x}_3)$;

(2) $f_2(0x_2x_3) = f_2(0x_2\bar{x}_3)$ and $f_2(1x_2x_3) = f_2(1\bar{x}_2\bar{x}_3)$;

(3) $f_3(x_1x_2x_3) = f_3(x_1\bar{x}_2x_3)$;

where $x_1, x_2, x_3 \in \{0, 1\}$.

Lemma 3. A complete binary tree with 7 nodes can be embedded into LTQ_3 with dilation 1 rooted at any node of LTQ_3.

Proof. By Lemma 2, we only need to consider the following two nodes: 000 and 100. Figure 1 demonstrates two complete binary trees which can be embedded into LTQ_3 rooted at 000 and 100, respectively. It is easy to verify that a complete binary tree can be embedded into LTQ_3 rooted at any node of LTQ_3 with dilation 1, see Figure 1, every node of CBT_3 is mapping to a node in LTQ_3 and every edge of CBT_3 is mapping to an edge in LTQ_3, the lemma holds. □

Lemma 4. A complete binary tree with 15 nodes can be embedded into LTQ_4 with dilation 1 rooted at 1000, while another complete binary tree with 15 nodes can be embedded into LTQ_4 with dilation 2 rooted at 1010.

Proof. We can embed a complete binary tree rooted at 1000 into LTQ_4 with dilation 1, see Figure 2 (a), every node of CBT_4 is mapping to a node in LTQ_4 and every edge of CBT_4 is mapping to an edge in LTQ_4. While another complete binary tree rooted at 1010 can be embedded into LTQ_4 with dilation 2, see Figure 2 (b). In the second embedding, edge (1010,0000) in the complete binary tree is mapping to a path P: $1010- > 0010- > 0000$ of LTQ_4. Every node of CBT_4 is mapping to a node in LTQ_4 and every edge of CBT_4 except edge (1010,0000) is mapping to an edge in LTQ_4. By the definition of dilation, the dilation of the second embedding is 2. □

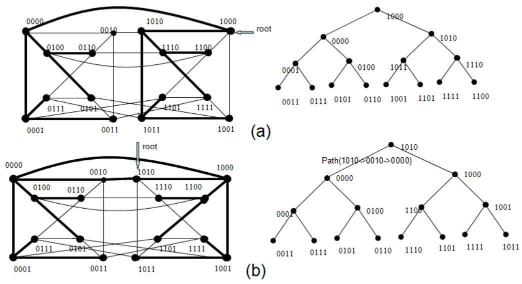

Fig. 2: (a)LTQ_4 and CBT_4 with dilation 1. (b)LTQ_4 and CBT_4 with dilation 2

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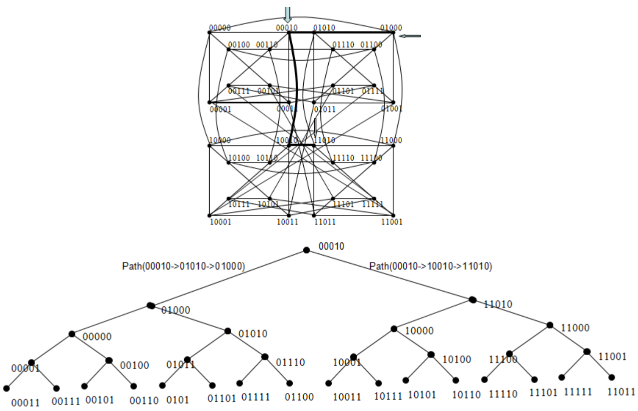

Fig. 3: LTQ_5 and CBT_5.

Lemma 5. A complete binary tree with $2^5 - 1$ nodes can be embedded into LTQ_5 with dilation 2, whose root is 00010.

Proof. By the proof of Lemma 4, we have two complete binary trees which can be embedded into LTQ_4. we can embedded CBT_5 into LTQ_5 with dilation 2, the proof process is as follows. We prefix the label of each node of CBT_4 in the Figure 2 (a) with 0, and prefix the label of each node of CBT_4 in the Figure 2 (b) with 1, and then, we link the roots of this two complete binary trees with two edges (00010,01000) and (00010,11010). It upper steps can construct CBT_5. Edge (00010,01000) of CBT_5 is mapping to the path P_1: $00010- > 01010- > 01000$ of LTQ_5 and edge (00010,11010) of CBT_5 is mapping to the path P_2: $00010- > 10010- > 11010$ of LTQ_5, see Figure 3. By the definition of dilation, the dilation of the embedding is 2.

Every node of CBT_5 is mapping to a node in LTQ_5 and every edge of CBT_5 is mapping to an edge in LTQ_5, except edges (1010,0000),(00010,01000) and (00010,11010), while these three edges are mapping to a path of length 2 in LTQ_5, respectively. □

Lemma 6. A complete binary trees with $2^6 - 1$ nodes can be embedded into LTQ_6 with dilation 2, whose root is 010010.

Proof. By the proof of Lemma 5, we have two complete binary trees which can be embedded into LTQ_5. we can embedded CBT_6 into LTQ_6 with dilation 2, the construction process is as follows. To show the embedded tree clearly, Figure 4 omits some edges in LTQ_6. We prefix the label of each node of CBT_5 in the Figure 3 with 0, and prefix the label of each node of CBT_5 in the Figure 3 with 1, and then, we link the roots of this two complete binary trees with two edges (000010,010010) and (100010,010010). The upper steps can construct CBT_6. Edge (000010,010010) of CBT_6 is mapping to the edge (000010,010010) of LTQ_6 and edge (010010,100010) of CBT_6 is mapping to the path P: $010010- > 110010- > 100010$ of LTQ_6, see Figure 5. By the definition of dilation, the dilation of the embedding is 2. □

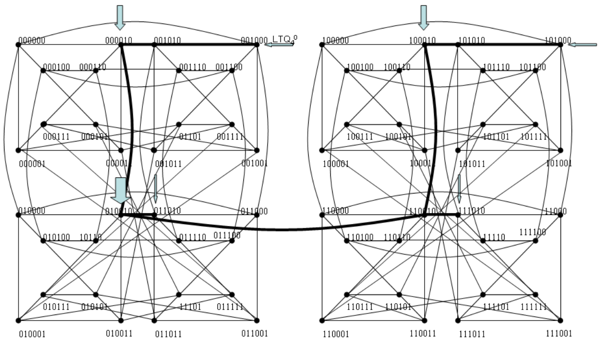

Fig. 4: LTQ_6 with some edges omitted.

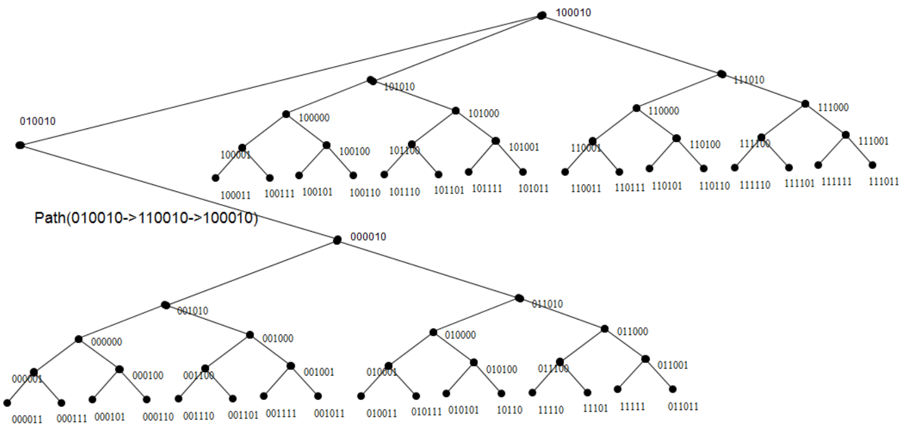

Fig. 5: CBT_6.

Lemma 7. For any integer $n \geq 6$, a complete binary tree with $2^n - 1$ nodes can be embedded into LTQ_n with dilation 2, whose root is $01^{n-5}0010$.

Proof. We prove this lemma by induction on the dimension n of LTQ_n. According to Lemmas 6, this lemma holds when n=6. Supposing that the lemma holds for $n = \tau$ ($\tau \geq 6$), we will prove that the lemma holds for $n = \tau + 1$.

According to the induction of hypothesis, for any integer $\tau \geq 6$, for LTQ_τ, a complete binary tree with $2^\tau - 1$ nodes can be embedded into LTQ_τ with dilation 2, whose root is $01^{\tau-5}0010$.

Now, we will prove that for any integer $\tau \geq 6$, a complete binary tree with $2^{\tau+1} - 1$ nodes can be embedded into $LTQ_{\tau+1}$ with dilation 2, whose root is $01^{\tau-4}0010$.

By the hypothesis, a complete binary tree CBT_τ^0 with $2^\tau - 1$ nodes can be embedded into LTQ_τ^0 with dilation 2, whose root is $001^{\tau-5}0010$. And a complete binary tree CBT_τ^1 with $2^\tau - 1$ nodes can be embedded into LTQ_τ^1 with dilation 2, whose root is $101^{\tau-5}0010$.

The complete binary tree $CBT_{\tau+1}$ with root $01^{\tau-4}0010$ can be gained by CBT_τ^0 and CBT_τ^1 as figure 6. By the Definition 1, we have $(01^{\tau-4}0010,\ 001^{\tau-5}0010), (01^{\tau-4}0010,\ 11^{\tau-4}0010)$,and $(11^{\tau-4}0010,\ 101^{\tau-5}0010)$ are three edges of $LTQ_{\tau+1}$. The edge a of $CBT_{\tau+1}$ is mapping to an edge of $LTQ_{\tau+1}$: $(01^{\tau-4}0010, 001^{\tau-5}0010)$. And the edge b of $CBT_{\tau+1}$ is mapping to a path P of $LTQ_{\tau+1}$: $01^{\tau-4}0010 -> 11^{\tau-4}0010-> 101^{\tau-5}0010$. A complete binary tree $CBT_{\tau+1}$ with root $01^{\tau-4}0010$ can be embedded into $LTQ_{\tau+1}$. The dilations of the embedding CBT_τ^0 and CBT_τ^1 are 2, and $|P| = 2$, therefore, the dilation of embedding of $CBT_{\tau+1}$ into $LTQ_{\tau+1}$ is 2. □

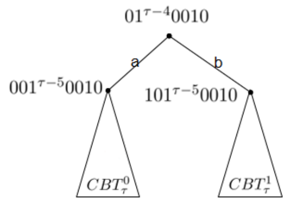

Fig. 6: The complete binary tree $CBT_{\tau+1}$.

Theorem 1. A complete binary tree CBT_n with $2^n - 1$ nodes can be embedded into LTQ_n with dilation 2.

Proof. By Lemma 1 and Lemmas 3 - 7, the theorem holds obviously. □

Since the node number of LTQ_n is 2^n, and CBT_n has 2^n-1 nodes, the expansion of this embedding is $\frac{2^n-1}{2^n}$. When n is big enough, the expansion is almost 1.

Conclusions

In this paper, the problem of embedding complete binary trees into locally twisted cubes is studied. We find the following result in this paper: for any integer $n \geq 2$, a complete binary tree with $2^n - 1$ nodes can be embedded into the LTQ_n with dilation 2.

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