A Modified Algorithm for Reducing Calculation Errors in Large Strain Measurement with Strain Gauges

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Abstract. There is no simple linear relationship between strain and potential in strain measurement with strain gauges, especially for large strain measurements. In this paper, a modified algorithm was proposed to improve the accuracy of strain obtained from measured voltage. The strain was calculated from a nonlinear relationship between voltage and strain rather than a linear simplification. Moreover, the corrections for   different sensitivity factors of strain gauges and lead wire resistance were considered. The proposed method was suitable for both large and small strain measurements using a quarter bridge, and validated by experimental tests. It is also very easy to be implemented as a software form and used in scientific tests and engineering applications.

Introduction

The strain gauges have widely been used for the small strain measurement in scientific research and engineering applications due to their high precision, low cost and convenience. When large strain measurements are required, optical measurement methods are commonly used. However, on some occasions, for example, for the large strain measurement in a burst test of pressure vessel, strain gauge technology is more suitable because of the high risk and damages.

There are three important things should be considered in using strain gauge technology to measure a large strain[1]. The first is the linearity of strain gauge and adhesion quality. For a small strain measurement, it is a mature technique. Tokyo Measure Institute and other companies have made strain gauge and adhesion suitable for large strain measurement. The second is the linearity of instrument voltage. The linearity within the range of large voltage can be guaranteed by adjusting the amplifier. The third is the relationship between voltage and strain value from the strain gauges. For a small strain measurement, a linear proportional relationship is suitable; for a large strain measurement, using a linear relationship will cause too large an error. In this study, a modified method will be developed to build a nonlinear relationship between the voltage and strain to reduce measurement error.

Basic principle of Strain Gauge Technology

    The relationship between strain and resistance change

The relationship between strain and resistance change can be expressed as

$$k = \frac{\frac{\Delta R}{R}}{\varepsilon} \tag{1}$$

    Where k is the sensitivity coefficient of strain gauge, provided by the manufacturer. From Eq. (1), the strain in a structure can be measured by measuring the resistance change of the strain gauge attached to the structure.
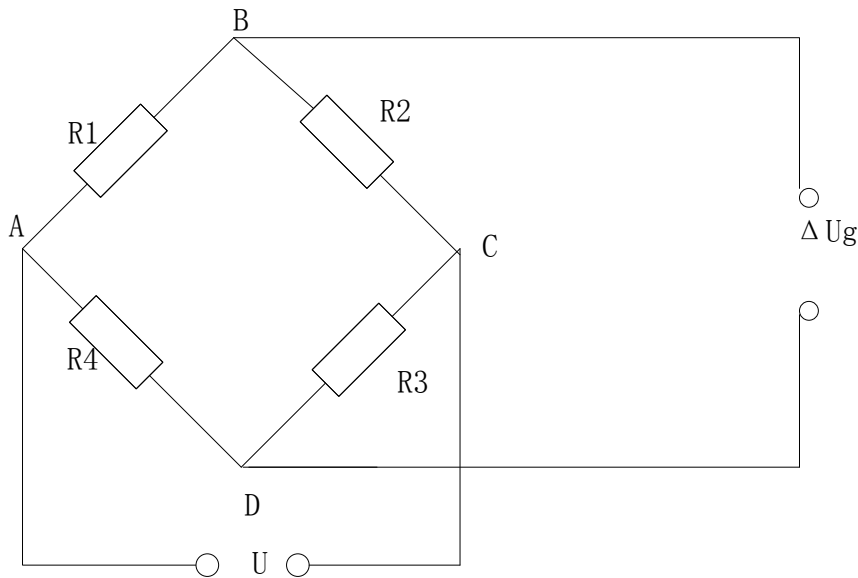
    Bridge Circuit

Figure 1 Wheatstone bridge circuit

Figure 1 shows a Wheatstone bridge circuit. The bridge output voltage between point B and point D is:

$$\Delta Ug = \left(\frac{R_1}{R_1 + R_2} - \frac{R_4}{R_3 + R_4} \right) U \tag{2}$$

where U is the supply voltage.

Standard Strain Gauge Simulator

For a half-bridge circuit, we have

$$R_1 = R + \Delta R$$

$$R_2 = R - \Delta R$$

$$R_3 = R$$

$$R_4 = R$$

Substituting the equations above into Eq.(2) yields

$$\Delta U_g = \frac{U}{2} \cdot \frac{\Delta R}{R} \qquad\qquad (3)$$

Which shows a linear relationship between ΔU_g and ΔR. A standard strain gauge instrument

was made based on this relationship to measure the strain.

Quarter Measuring Bridge

In engineering applications, a quarter bridge is widely used. We have

$$R_1 = R + \Delta R$$

$$R_2 = R_3 = R_4 = R$$

Substituting the equations above into Eq.(2) yields

$$\Delta U_g = \frac{U}{4} \cdot \frac{\Delta R}{R + \frac{1}{2} \cdot \Delta R} \qquad\qquad (4)$$

Strain Calculation Method

Normal Strain Calculation Method

Since $\Delta R / R$ is very small, Equation (4) can be approximately expressed as

$$\Delta U_g = \frac{U}{4} \cdot \frac{\Delta R}{R} \cdot \left[1 - \frac{1}{2} \cdot \frac{\Delta R}{R} + \left(\frac{1}{2} \cdot \frac{\Delta R}{R} \right)^2 - \cdots \cdots \right] \qquad (5)$$

For an approximation, Eq. (5) is rewritten as

$$\Delta U_g = \frac{U}{4} \cdot \frac{\Delta R}{R}$$
$$= \frac{U}{4} \cdot k\varepsilon$$

So

$$\varepsilon = \frac{4}{k} \cdot \frac{\Delta U_g}{U} \qquad\qquad (6)$$

The approximation error introduced by using Eq.(6) to calculate ΔU_g instead of Eq.(5) is

$$e = \frac{1}{2} \cdot \frac{\Delta R}{R} = \frac{k\varepsilon}{2} \qquad (7)$$

When k=2, Eq (7) is expressed as

$$e = \varepsilon \qquad (8)$$

Thus, for a strain of 10%, the error introduced by the approximation is 10%. Since Eq. (6) exhibits a linear relationship between the voltage and strain changes and its approximation error is small for small strain measurements, it is widely used.

Modified algorithm of strain calculation

Eq. (4) shows that the relationship between voltage and strain is not linear. In practical applications of large strain measurements, a relationship curve between voltage and strain is calibrated by experiments in order to reduce the calculation error in [2]. This method is not convenient and not adaptable. Therefore, it is more suitable to directly use Eq. (4) rather than the simplified linear relationship between voltage and strain.

From Equation (4), the resistance change rate can be expressed as

$$\frac{\Delta R}{R} = \frac{1}{\dfrac{1}{4} \cdot \dfrac{U}{\Delta U_g} - \dfrac{1}{2}} \qquad (9)$$

which can be used to calculate the strain.

Correction of measurement Parameters

Temperature Compensation

In a half bridge, two strain gauges are used: one is used to measure strain, the other is for temperature compensation. The resistance can be expressed as

$$R_1 = R + \Delta R + \Delta R_t$$

$$R_2 = R + \Delta R_t$$

$$R_3 = R_4 = R$$

Substituting the resistances into Eq. (3) yields

$$\Delta U_g = \frac{U}{4} \cdot \frac{\Delta R}{R} \cdot \frac{1}{1 + \dfrac{1}{2} \cdot \dfrac{\Delta R}{R} + \dfrac{\Delta R_t}{R}} \qquad (10)$$

In the room temperature $\Delta R_t = 0$, Eqs. (10) and (4) are equivalent.

Sensitivity Correction of Strain Gauge

The sensitivity factor of instrument is different from that of strain gauges, which can be expressed as:

$$\frac{\Delta R}{R} = k_{in} \cdot \varepsilon_{in}$$

Where k_{in} is the sensitivity factor of instrument, ε_{in} is the measurement of instrument.

Thus the strain value from the strain gauge can be written as:

$$\varepsilon = \frac{1}{k} \cdot \frac{\Delta R}{R} = \frac{k_{in}}{k} \cdot \varepsilon_{in} \qquad (11)$$

Equation (11) can be used to correct the sensitivity factor of different strain gauges to be applicable for the same strain measurement instrument.

Lead wire Resistance Attenuation Correction

The error due to lead wire resistance attenuation can be corrected by using a three-wire circuit, as shown in Figure 2, where strain gauges R_1 and R_2 are used for strain measurement and temperature compensation, respectively. The strain with a lead wire resistance correction is written as
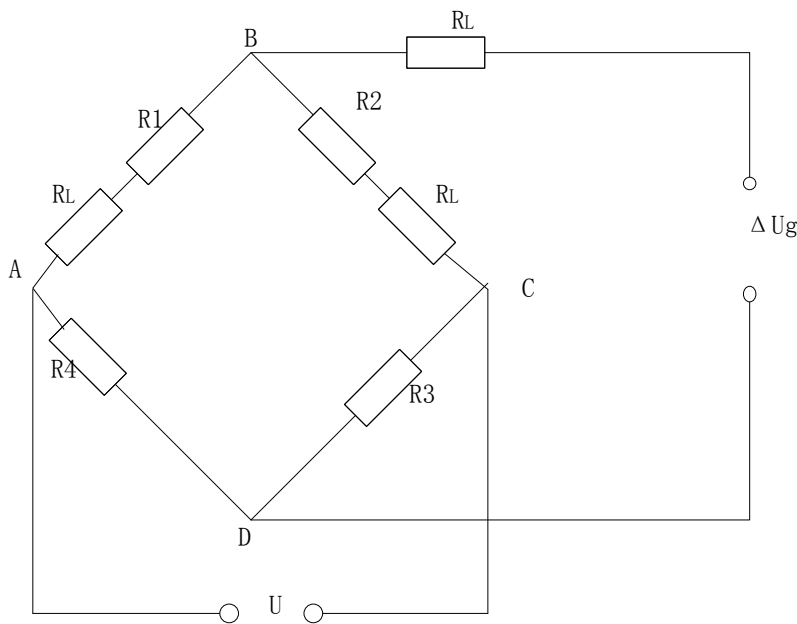

Figure 2   A Three-Wire Circuit diagram

$$R_1 = R + \Delta R + R_L$$

$$R_2 = R + R_L$$

$$R_3 = R_4 = R$$

We can get

$$\varepsilon = \frac{R}{R + R_L} \cdot \varepsilon_{in} \qquad (12)$$

Strain Measurements with corrections

The strain measurement with temperature compensation and lead wire resistance correction can be obtained by combining Eqs.(11) and (12), expressed as:

$$\varepsilon = \frac{k_{in}}{k} \cdot \frac{R + R_L}{R} \cdot \varepsilon_{in} \cdots \cdots \tag{13}$$

Set $C = \frac{k}{k_{in}} \cdot \frac{R + R_L}{R}$ which presents the corrections of temperature compensation and lead wire

resistance corrections, the true strain can be expressed as:

$$\varepsilon = C \cdot \varepsilon_{in} \tag{14}$$

where $\varepsilon_{in} \propto \Delta U_g^{in}$. The voltage change can be expressed as:

$$\Delta U_g = C \cdot \Delta U_g^{in} \tag{15}$$

Substituting Eq. (15) into Eq. (9) yields

$$\frac{\Delta R}{R} = \frac{1}{\frac{1}{4} \cdot \frac{U}{C \cdot \Delta U_g^{in}} - \frac{1}{2}} \tag{16}$$

Then substituting Eq. (16) into Eq.(1) yields

$$\varepsilon = \frac{1}{k} \cdot \frac{\Delta R}{R} = \frac{1}{k} \cdot \frac{1}{\frac{1}{4} \cdot \frac{U}{C \cdot \Delta U_g^{in}} - \frac{1}{2}} \tag{17}$$

Where k is the sensitivity factor of strain gauges, k_{in} is the sensitivity factor of

instrument , $C = \frac{k}{k_{in}} \cdot \frac{R + R_L}{R}$.

Experimental Validation

Experimental Instruments

A dynamic strain gauge instrument, SDY 21202D, developed by Beidaihe Experimental Electric Institution and the author, was used to validate the proposed method. The measure range of strain is 0-320000 $\mu\varepsilon$. The developed instrument was calibrated by using a standard strain gauge instrument ,BYM-6, made by Beidaihe Experimental Electric Institution, a digital multimeter,HP34401A ($6\frac{1}{2}$ byte), made by Hewlett-Packard, and a signal generator,TD1017,made by Tianjin Zhonghuan Electronic Instrument Company. The maximum instrumental linear error of SDY21202D is 0.01%. Other instruments used included a standard strain gauge simulator,SDY2301,made by Beidaihe Experimental Electric Institution, with a sensitivity factor of

2 and measurement ranges are 200000, 1000000,1500000,200000,250000 and 300000 $\mu\varepsilon$, and a standard strain gauge simulator, SDY2306, made by Beidaihe Experimental Electric Institution, with a sensitivity of 2 and a measure range from 0.1-100000 $\mu\varepsilon$ and the maximum error of SDY2301 and SDY2306 is 0.05%.

Experimental Procedure

Strain gauge simulators, SDY 2301 and SDY2306, were used to simulate the behavior of strain. Quarter measuring bridge was used to response the signal from strain gauge simulators, and SDY2102D dynamical strain gauge instrument was used to measure the strain. Quarter measuring bridge could be calculated according to three-wire circuit method as R_L is zero.

Experimental Results

Table 1 shows that the calculation errors of strains obtained from both simplified equation (Eq.(6)) and modified equation (Eq.(17)) are small for a small strain measurement. The error is always small when Eq. (17) is used. However, the error increases with the increasing strain value when Eq. (6) is used: the error increases to more than 1.16% when the strain value reaches 10000 $\mu\varepsilon$, which can not be neglected; the error increases to 9.12% when the strain value reaches 100000 $\mu\varepsilon$, which is not acceptable.

We also did another group experiment by three-wire circuit method and got the same law as Table 1. Lead-wire long is 50 meters and per lead-wire dimension is 0.5mm, we measure lead-wire resistance R_L is 2Ω, R chooses 120Ω.

Conclusion

In this paper, a modified method was proposed to calculate the strain from the resistance change of the strain gauge, which is suitable for both small and large strain measurements. The corrections for temperature compensation and lead wire resistance attenuation were also considered. The developed method can improve the accuracy of strain measurements and can be used in the scientific research and engineering applications.

Table 1 Strain Results Measured from quarter measuring bridge

Strain values ($\mu\varepsilon$)	Measuring Voltage of ΔU_g^{in}		Strain value calculated using Equation(6)		Strain value calculated using Equation(17)	
	ΔU_g^{in} after enlargement by amplifier （V）	Enlargement Factor by amplifier for measuring ΔU_g^{in}	Strain Value ($\mu\varepsilon$)	Error (%)	Strain Value ($\mu\varepsilon$)	Error(%)
500	0.124	250	500	0	500.2501	0.050025
1000	0.249		996	-0.400	996.993	-0.3007
1500	0.374		1496	-0.267	1498.241	-0.11724
2000	0.498		1992	-0.400	1995.976	-0.2012
2500	0.623		2492	-0.320	2498.226	-0.07098
3000	0.747		2988	-0.400	2996.955	-0.1015
3500	0.871		3484	-0.457	3496.181	-0.10912
4000	0.995		3980	-0.500	3995.904	-0.10241
4500	1.119		4476	-0.533	4496.125	-0.08612
5000	1.241		4964	-0.720	4988.764	-0.22472
10000	2.471		9884	-1.160	9982.669	-0.17331
15000	0.369	25	14760	-1.600	14981.12	-0.12586
20000	0.490		19600	-2.000	19991.84	-0.0408
25000	0.609		24360	-2.560	24968.23	-0.1271
30000	0.727		29080	-3.067	29950.97	-0.16342
35000	0.844		33760	-3.543	34939.56	-0.17269
40000	0.960		38400	-4.000	39933.44	-0.16639
45000	1.075		43000	-4.444	44932.08	-0.15093
50000	1.192		47680	-4.640	50067.2	0.134409
75000	1.745		69800	-6.933	75037.63	0.050168
100000	2.272		90880	-9.120	99964.8	-0.0352
125000	2.777		111080	-11.1360	124960.6	-0.0315
150000	3.260		130400	-13.067	149954	-0.03067

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