The Analytical Description of Projectile Motion of Cricket Ball in a Linear Resisting Medium the Storm Force

Siranee Kuaykaew^{1a}, Supoj Kerdmee^{1b}, Paitoon Banyenugam^{1c},
 Piyaarut Moonsri^{2d}, and Artit Hudem^{1e*}

¹ Physics Division, Faculty of Science and Technology, Phetchabun Rajabhat University,
 Phetchabun, Thailand 67000

² Chemistry Division, Faculty of Science and Technology, Phetchabun Rajabhat University,
 Phetchabun, Thailand 67000

^aP7925@outlook.com, ^bSupojkerdmee@hotmail.com, ^cPaitoonbanyenugam@outlook.com,
^dpiyarutto@hotmail.com, ^emagooohotem@yahoo.com

* Corresponding author. E-mail: magooohotem@yahoo.com,
 Fax: 056-717-123, Mobile: 086-9909536

Keywords: Velocity, Projectile Motion, Storm Force, Damped Coefficient

Abstract. A detailed work based on the second-order ordinary differential equation is presented to solve oscillation in the trajectory projectile motion of cricket ball for damped alternating external force (f_0) problems. This paper purpose to compute the distance time depends horizontal and the distance time depends vertical. The parabolic path of trajectories for a projectile motion of cricket ball increase oscillation with the value of parameter λ and f_0 is the storm force.

Introduction

Peter Coutis used the quadratic drag force to calculate the distance vertical($y(t)$) motion and distance horizontal($x(t)$) in projectile motion of a cricket ball. Sean M. Stewart consider the time of flight, range and the angle which maximizes the range of a projectile motion in a linear resisting medium are expressed in analytic form in terms of the recently defined Lambert W function is to be the inverse of $We^W = z$. Jeffrey Leela et. al, explores the various equation associated with the movement of the projectile motion for ball in flight and they use the velocity depends on C_D and also the drag force couples the equation for the horizontal and vertical component of the velocity see equation (1) and equation (2), where C_D is the drag coefficient

Projectile motion of cricket ball below external force ($f_0 e^{-\lambda t} \cos(\omega t)$)

The parameters governing the projectile of motion of a cricket ball are the launch angle (θ), the speed at which the ball leaves the bat (v_0), and the linear drag coefficient per unit mass (β). If we assume that the only forces acting on the cricket ball in flight are gravity drag force and external forces ($f_0 e^{-\lambda t} \cos(\omega t)$), Newton's second law implies

$$m\ddot{x} = -C_D \dot{x} \cos(\theta) \quad (1)$$

and

$$m\ddot{y} = -C_D \dot{y} \sin(\theta) - mg - f_0 e^{-\lambda t} \cos(\omega t) \quad (2)$$

The cricket ball's motion is best described by separating it into horizontal($x(m)$) and vertical($y(m)$) components as we have already emphasized, the horizontal motion dependent of the vertical motion and then applying the kinematic equation. Here $\dot{x}(t)$ is the velocity time dependent and $\ddot{x}(t)$ is

the acceleration time dependent, C_D is the drag coefficient, λ is the damped coefficient. Multiplying the above equation by $1/m$ gives

$$\ddot{x}(t) + \beta \dot{x}(t) \cos(\theta) = 0 \quad (3)$$

and

$$\ddot{y}(t) + \beta \dot{y} \sin(\theta) = -(g + F_0 e^{-\lambda t} \cos(\omega t)), \quad (4)$$

where $\beta = C_D / m$, $F_0 = f_0 / m$. We use the auxiliary equation find solution of equation (3)

$$x(t) = A + B e^{-\beta t \cos(\theta)}. \quad (5)$$

Here, A and B are constants which can be determined from the initial condition $x(0) = 0$, $\dot{x}(0) = v_0 \cos(\theta)$. A solution to equation (3) is

$$x(t) = \frac{v_0}{\beta} (1 - e^{-\beta t \cos(\theta)}) \quad (6)$$

From the above result, we gives a time-dependent expression for x , which is the horizontal distance travelled by the cricket ball in time t . Rearrangement of equation (6) provides

$$t = \ln \left[\frac{v_0 - \beta x(t)}{v_0} \right]^{\frac{-1}{\beta \cos(\theta)}}. \quad (7)$$

Find the general solution of the inhomogeneous of equation (4), $y(t) = y_c(t) + y_p(t)$. Find the fundamental solution to the homogeneous equation the characteristic equation is $y_c(t) = C + D e^{-\beta t \sin(\theta)}$. Here, $y_1(t) = 1$, $y_2(t) = e^{-\beta t \sin(\theta)}$. Compute their Wronskian,

$$W = -\beta \sin(\theta) e^{-\beta t \sin(\theta)}, \quad W_1 = (g + F_0 e^{-\lambda t} \cos(\omega t)) e^{-\beta t \sin(\theta)},$$

$$W_2 = -(g + F_0 e^{-\lambda t} \cos(\omega t)).$$

We compute the function $u_1(t)$, $u_2(t)$, we obtain the particular solution is

$$y_p(t) = \frac{F_0 e^{-\lambda t} (\lambda \cos(\omega t) - \omega \sin(\omega t))}{\beta \sin(\theta) (\lambda^2 + \omega^2)} - \frac{gt}{\beta \sin(\theta)} + \frac{g}{\beta^2 \sin^2(\theta)} \\ + \frac{F_0 e^{-\lambda t} ((\beta \sin(\theta) - \lambda) \cos(\omega t) + \omega \sin(\omega t))}{\beta \sin(\theta) ((\beta \sin(\theta) - \lambda)^2 + \omega^2)} \quad (8)$$

The general solution is

$$y(t) = C + D e^{-\beta t \sin(\theta)} - \frac{gt}{\beta \sin(\theta)} + \frac{F_0 e^{-\lambda t} (\lambda \cos(\omega t) - \omega \sin(\omega t))}{\beta \sin(\theta) (\lambda^2 + \omega^2)} \\ + \frac{F_0 e^{-\lambda t} ((\beta \sin(\theta) - \lambda) \cos(\omega t) + \omega \sin(\omega t))}{\beta \sin(\theta) ((\beta \sin(\theta) - \lambda)^2 + \omega^2)} \quad (9)$$

With the initial condition for $y(t)$, i.e. $y(0) = 0$, $\dot{y}(0) = v_0 \sin(\theta)$ the expression for $y(t)$ becomes

$$\begin{aligned}
y(t) = & \left[(1 - e^{-\beta t \sin(\theta)}) \left(\frac{v_0}{\beta} + \frac{g}{\beta^2 \sin^2(\theta)} \right) \right] - \frac{F_0 e^{-\beta t \sin(\theta)} (\lambda - \beta \sin(\theta))}{\beta \sin(\theta) ((\beta \sin(\theta) - \lambda)^2 + \omega^2)} \\
& - \frac{gt}{\beta \sin(\theta)} + \frac{F_0 e^{-\lambda t} (\lambda \cos(\omega t) - \omega \sin(\omega t))}{\beta \sin(\theta) (\lambda^2 + \omega^2)} - \frac{F_0 \lambda}{\beta \sin(\theta) (\lambda^2 + \omega^2)} \\
& + \frac{F_0 e^{-\lambda t} ((\beta \sin(\theta) - \lambda) \cos(\omega t) + \omega \sin(\omega t))}{\beta \sin(\theta) ((\beta \sin(\theta) - \lambda)^2 + \omega^2)}
\end{aligned} \quad (10)$$

Results and Discussion

Fig.1 A trajectories projectile of motion for a cricket ball struck at an angle of calculation of 45° in the presence of a linearized drag force by vary the initial velocity.

From figure 1 we illustrate magnitude of the $x(t)$ horizontal distance time dependent and $y(t)$ vertical distance time dependent increase from 100 m to 210 m with increasing the velocity of trajectories projectile of motion for a cricket ball.

Fig.2 A trajectories projectile of motion for a cricket ball struck at an angle of calculation of 45° in the presence of a linearized drag force by vary an angle.

Fig.3 A schematic diagram for behavior of a trajectories projectile of motion for a cricket ball in case of $\lambda = 0.05$ is the damped coefficient.

Fig.4 A schematic diagram for behavior of a trajectories projectile of motion for a cricket ball in case of $\lambda = 0.99$ is the damped coefficient.

From figure 3 and 4, If the value of parameter λ is small the parabolic path of trajectories for projectile motion of a cricket ball will increase small oscillation and the value of parameter λ is large the parabolic path of trajectories for projectile motion of a cricket ball will decrease small oscillation as figure 4.

Conclusions

The parabolic path of trajectories for a projectile motion of cricket ball increase oscillation with the value of parameter λ and f_0 is the storm force. When at angle 60° the high parabolic path of cricket ball more than the angle 30° it is influence due to the large storm force affection $x(t)$ horizontal distance time dependent and $y(t)$ vertical distance time dependent little as figure 2.

References

- [1] R.A. Serway et. al., Physics for Scientists and Engineers with Modern Physics, Fifth Edition, saunders college publishing, 2000, pp.82-90.
- [2] K.F. Riley et. al., Mathematical Method for Physics and Engineering, Third Edition, Cambridge, 2006, pp.490-529.
- [3] S. Thornton and J.B. Marion, Classical Dynamics of Particles and System, Fifth Edition, Thomson publishing, 2002, pp.99-135.
- [4] Peter Coutis, Modeling the projectile motion of a cricket ball, INT. J. MATH. EDUC. SCI. TECHNOL., Vol. 29 No. 6, pp 789-798.(1999)
- [5] Sean M. Stewart, An analytic approach to projectile motion in a linear resisting medium, INT. J. MATH. EDUC. SCI. TECHNOL., Vol. 37 No. 4, 266 pp 411-431.(2006)
- [6] Jeffrey Leela et. al, Modelling the flight characteristics of a soccer ball, Lat. Am. J Phys. Educ. Vol. 8 No.4 pp. 4505-1 – pp.4505-5., (2016)