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# The Analytical Description of Projectile Motion of Cricket Ball in a Linear Resisting Medium the Storm Force

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**Abstract.** A detailed work based on the second-order ordinary differential equation is presented to solve oscillation in the trajectory projectile motion of cricket ball for damped alternating external force ( $f_0$ ) problems. This paper purpose to compute the distance time depends horizontal and the distance time depends vertical. The parabolic path of trajectories for a projectile motion of cricket ball increase oscillation with the value of parameter $\lambda$ and $f_0$ is the storm force.

## Introduction

Peter Coutis used the quadratic drag force to calculate the distance vertical(y(t)) motion and distance horizontal(x(t)) in projectile motion of a cricket ball. Sean M. Stewart consider the time of flight, range and the angle which maximizes the range of a projectile motion in a linear resisting medium are expressed in analytic form in terms of the recently defined Lambert W function is to be the inverse of $We^W = z$. Jeffrey Leela et. al, explores the various equation associated with the movement of the projectile motion for ball in flight and they use the velocity depends on $C_D$ and also the drag force couples the equation for the horizontal and vertical component of the velocity see equation (1) and equation (2), where $C_D$ is the drag coefficient

## Projectile motion of cricket ball below external force $\left(f_0 e^{-\lambda t}\cos(\omega t)\right)$

The parameters governing the projectile of motion of a cricket ball are the launch angle ($\theta$), the speed at which the ball leaves the bat ( $v_0$ ), and the linear drag coefficient per unit mass ($\beta$). If we assume that the only forces acting on the cricket ball in flight are gravity drag force and external forces $\left(f_0 e^{-\lambda t}\cos(\omega t)\right)$, Newton's second law implies

$$m\ddot{x} = -C_D \dot{x}\cos(\theta) \tag{1}$$

and

$$m\ddot{y} = -C_D \dot{y}\sin(\theta) - mg - f_0 e^{-\lambda t}\cos(\omega t) \tag{2}$$

The cricket ball's motion is best described by separating it into horizontal(x(m)) and vertical(y(m)) components as we have already emphasized, the horizontal motion dependent of the vertical motion and then applying the kinematic equation. Here $\dot{x}(t)$ is the velocity time dependent and $\ddot{x}(t)$ is the acceleration time dependent, $C_D$ is the drag coefficient, $\lambda$ is the damped coefficient. Multiplying the above equation by $1/m$ gives

$$\ddot{x}(t)+\beta\dot{x}(t)\cos(\theta)=0 \tag{3}$$

and

$$\ddot{y}(t)+\beta\dot{y}\sin(\theta)=-\left(g+F_0e^{-\lambda t}\cos(\omega t)\right), \tag{4}$$

where $\beta=C_D/m,\ F_0=f_0/m$. We use the auxiliary equation find solution of equation (3)

$$x(t)=A+Be^{-\beta t\cos(\theta)}. \tag{5}$$

Here, $A$ and $B$ are constants which can be determined from the initial condition $x(0)=0,\ \dot{x}(0)=v_0\cos(\theta)$. A solution to equation (3) is

$$x(t)=\frac{v_0}{\beta}\left(1-e^{-\beta t\cos(\theta)}\right) \tag{6}$$

From the above result, we gives a time-dependent expression for $x$, which is the horizontal distance travelled by the cricket ball in time $t$. Rearrangement of equation (6) provides

$$t=\ln\left[\frac{v_0-\beta x(\mathrm{t})}{v_0}\right]^{\frac{-1}{\beta\cos(\theta)}}. \tag{7}$$

Find the general solution of the inhomogeneous of equation $(4)$, $y(t)=y_C(t)+y_P(t)$. Find the fundamental solution to the homogeneous equation the characteristic equation is $y_C(t)=C+De^{-\beta t\sin(\theta)}$. Here, $y_1(t)=1$, $y_2(t)=e^{-\beta t\sin(\theta)}$. Compute their Wronskian, $W=-\beta\sin(\theta)e^{-\beta t\sin(\theta)}$, $W_1=\left(g+F_0e^{-\lambda t}\cos(\omega t)\right)e^{-\beta t\sin(\theta)}$, $W_2=-\left(g+F_0e^{-\lambda t}\cos(\omega t)\right)$.

We compute the function $u_1(t)$, $u_2(t)$, we obtain the particular solution is

$$\begin{aligned}y_P(t)=&\frac{F_0e^{-\lambda t}(\lambda\cos(\omega\,\mathrm{t})-\omega\sin(\omega\,\mathrm{t}))}{\beta\sin(\theta)(\lambda^2+\omega^2)}-\frac{gt}{\beta\sin(\theta)}+\frac{g}{\beta^2\sin^2(\theta)}\\&+\frac{F_0e^{-\lambda t}((\beta\sin(\theta)-\lambda)\cos(\omega\,\mathrm{t})+\omega\sin(\omega\,\mathrm{t}))}{\beta\sin(\theta)((\beta\sin(\theta)-\lambda)^2+\omega^2)}\end{aligned} \tag{8}$$

The general solution is

$$\begin{aligned}y(t)=&C+De^{-\beta t\sin(\theta)}-\frac{gt}{\beta\sin(\theta)}+\frac{F_0e^{-\lambda t}(\lambda\cos(\omega\,\mathrm{t})-\omega\sin(\omega\,\mathrm{t}))}{\beta\sin(\theta)(\lambda^2+\omega^2)}\\&+\frac{F_0e^{-\lambda t}((\beta\sin(\theta)-\lambda)\cos(\omega\,\mathrm{t})+\omega\sin(\omega\,\mathrm{t}))}{\beta\sin(\theta)((\beta\sin(\theta)-\lambda)^2+\omega^2)}\end{aligned} \tag{9}$$

With the initial condition for $y(t)$, i.e. $y(0)=0,\ \dot{y}(0)=v_0\sin(\theta)$ the expression for $y(t)$ becomes

$$y(t)=\left[(1-e^{-\beta t\sin(\theta)})\left(\frac{v_0}{\beta}+\frac{g}{\beta^2\sin^2(\theta)}\right)\right]-\frac{F_0 e^{-\beta t\sin(\theta)}(\lambda-\beta\sin(\theta))}{\beta\sin(\theta)((\beta\sin(\theta)-\lambda)^2+\omega^2)}$$
$$-\frac{gt}{\beta\sin(\theta)}+\frac{F_0 e^{-\lambda t}(\lambda\cos(\omega t)-\omega\sin(\omega t))}{\beta\sin(\theta)(\lambda^2+\omega^2)}-\frac{F_0\lambda}{\beta\sin(\theta)(\lambda^2+\omega^2)} \quad (10)$$
$$+\frac{F_0 e^{-\lambda t}((\beta\sin(\theta)-\lambda)\cos(\omega t)+\omega\sin(\omega t))}{\beta\sin(\theta)((\beta\sin(\theta)-\lambda)^2+\omega^2)}$$

**Results and Discussion**

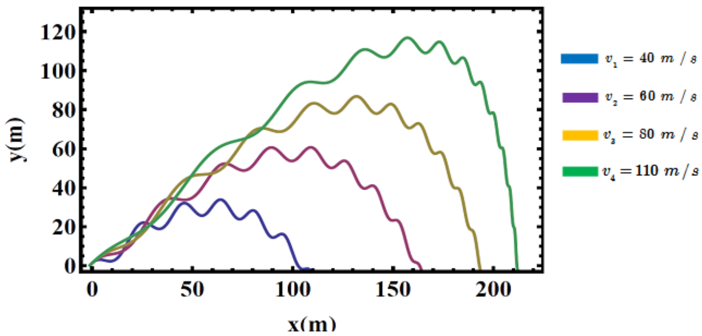


Fig.1 A trajectories projectile of motion for a cricket ball struck at an angle of calculation of 45° in the presence of a linearized drag force by vary the initial velocity.

From figure 1 we illustrate magnitude of the x(t) horizontal distance time dependent and y(t) vertical distance time dependent increase from 100 m to 210 m with increasing the velocity of trajectories projectile of motion for a cricket ball.

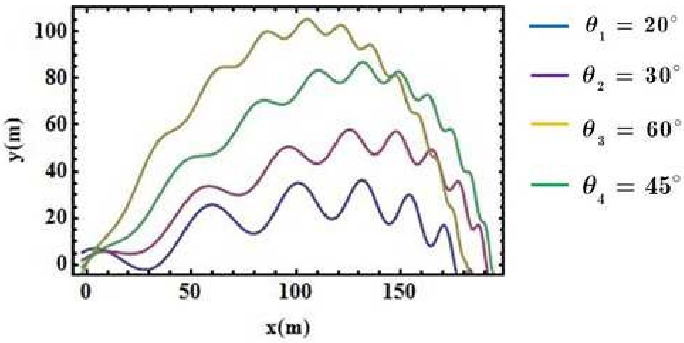


Fig.2 A trajectories projectile of motion for a cricket ball struck at an angle of calculation of 45° in the presence of a linearized drag force by vary an angle.

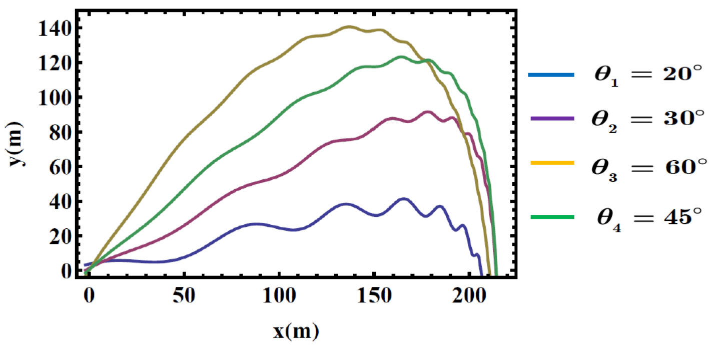


Fig.3 A schematic diagram for behavior of a trajectories projectile of motion for a cricket ball in case of $\lambda = 0.05$ is the damped coefficient.

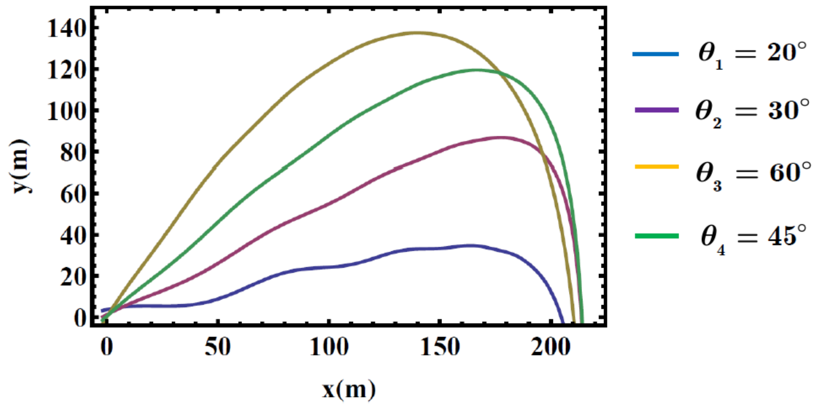


Fig.4 A schematic diagram for behavior of a trajectories projectile of motion for a cricket ball in case of $\lambda = 0.99$ is the damped coefficient.

From figure 3 and 4, If the value of parameter $\lambda$ is small the parabolic path of trajectories for projectile motion of a cricket ball will increase small oscillation and the value of parameter $\lambda$ is large the parabolic path of trajectories for projectile motion of a cricket ball will decrease small oscillation as figure 4.

## Conclusions

The parabolic path of trajectories for a projectile motion of cricket ball increase oscillation with the value of parameter $\lambda$ and $f_0$ is the storm force. When at angle $60^o$ the high parabolic path of cricket ball more than the angle $30^o$ it is influence due to the large storm force affection x(t) horizontal distance time dependent and y(t) vertical distance time dependent little as figure 2.

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