The Analytical Description of Projectile Motion of Cricket Ball in a Linear Resisting Medium the Storm Force

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Abstract. A detailed work based on the second-order ordinary differential equation is presented to solve oscillation in the trajectory projectile motion of cricket ball for damped alternating external force ($f_0$) problems. This paper purpose to compute the distance time depends horizontal and the distance time depends vertical. The parabolic path of trajectories for a projectile motion of cricket ball increase oscillation with the value of parameter $\lambda$ and $f_0$ is the storm force.

Introduction

Peter Coutis used the quadratic drag force to calculate the distance vertical($y(t)$) motion and distance horizontal($x(t)$) in projectile motion of a cricket ball. Sean M. Stewart consider the time of flight, range and the angle which maximizes the range of a projectile motion in a linear resisting medium are expressed in analytic form in terms of the recently defined Lambert W function is to be the inverse of $We^W = z$. Jeffrey Leela et. al, explores the various equation associated with the movement of the projectile motion for ball in flight and they use the velocity depends on $C_D$ and also the drag force couples the equation for the horizontal and vertical component of the velocity see equation (1) and equation (2), where $C_D$ is the drag coefficient

Projectile motion of cricket ball below external force\textbf{\textit{$f_0 e^{-\lambda t} \cos(\omega t)$}}

The parameters governing the projectile of motion of a cricket ball are the launch angle ($\theta$), the speed at which the ball leaves the bat ($v_0$), and the linear drag coefficient per unit mass ($\beta$). If we assume that the only forces acting on the cricket ball in flight are gravity drag force and external forces\textbf{\textit{$f_0 e^{-\lambda t} \cos(\omega t)$}}, Newton’s second law implies

\begin{equation}
mx = -C_D \dot{x} \cos(\theta)
\end{equation}

and

\begin{equation}
my = -C_D \dot{y} \sin(\theta) - mg - f_0 e^{-\lambda t} \cos(\omega t)
\end{equation}

The cricket ball’s motion is best described by separating it into horizontal($x(m)$) and vertical($y(m)$) components as we have already emphasized, the horizontal motion dependent of the vertical motion and then applying the kinematic equation. Here $\dot{x}(t)$ is the velocity time dependent and $x(t)$ is
the acceleration time dependent, \( C_D \) is the drag coefficient, \( \lambda \) is the damped coefficient. Multiplying the above equation by \( 1 / m \) gives

\[
\ddot{x}(t) + \beta \dot{x}(t) \cos(\theta) = 0
\]

and

\[
\ddot{y}(t) + \beta \dot{y} \sin(\theta) = -\left( g + F_0 e^{-\lambda t} \cos(\omega t) \right),
\]

where \( \beta = C_D / m, \ F_0 = f_0 / m. \) We use the auxiliary equation find solution of equation (3)

\[
x(t) = A + Be^{-\beta t \cos(\theta)}.
\]

Here, \( A \) and \( B \) are constants which can be determined from the initial condition \( x(0) = 0, \ \dot{x}(0) = v_0 \cos(\theta) \). A solution to equation (3) is

\[
x(t) = \frac{v_0}{\beta} \left( 1 - e^{-\beta t \cos(\theta)} \right)
\]

From the above result, we gives a time-dependent expression for \( x \), which is the horizontal distance travelled by the cricket ball in time \( t \). Rearrangement of equation (6) provides

\[
t = \ln \left[ \frac{v_0 - \beta x(t)}{v_0} \right]^{-\frac{1}{\beta \cos(\theta)}}.
\]

Find the general solution of the inhomogeneous of equation (4), \( y(t) = y_C(t) + y_P(t) \). Find the fundamental solution to the homogeneous equation the characteristic equation is

\[
y_C(t) = C + De^{-\beta t \sin(\theta)}.
\]

Here, \( y_1(t) = 1, \ y_2(t) = e^{-\beta t \sin(\theta)} \). Compute their Wronskian,

\[
W = -\beta \sin(\theta) e^{-\beta t \sin(\theta)},
\]

\[
W_1 = \left( g + F_0 e^{-\lambda t} \cos(\omega t) \right) e^{-\beta t \sin(\theta)},
\]

\[
W_2 = -\left( g + F_0 e^{-\lambda t} \cos(\omega t) \right).
\]

We compute the function \( u_1(t), \ u_2(t) \), we obtain the particular solution is

\[
y_P(t) = \frac{F_0 e^{-\lambda t} (\lambda \cos(\omega t) - \omega \sin(\omega t))}{\beta \sin(\theta)(\lambda^2 + \omega^2)} - \frac{gt}{\beta \sin(\theta)} + \frac{g}{\beta^2 \sin^2(\theta)}
\]

\[
+ \frac{F_0 e^{-\lambda t} ((\beta \sin(\theta) - \lambda) \cos(\omega t) + \omega \sin(\omega t))}{\beta \sin(\theta)((\beta \sin(\theta) - \lambda)^2 + \omega^2)}
\]

The general solution is

\[
y(t) = C + De^{-\beta t \sin(\theta)} - \frac{gt}{\beta \sin(\theta)} + \frac{F_0 e^{-\lambda t} (\lambda \cos(\omega t) - \omega \sin(\omega t))}{\beta \sin(\theta)(\lambda^2 + \omega^2)}
\]

\[
+ \frac{F_0 e^{-\lambda t} ((\beta \sin(\theta) - \lambda) \cos(\omega t) + \omega \sin(\omega t))}{\beta \sin(\theta)((\beta \sin(\theta) - \lambda)^2 + \omega^2)}
\]

With the initial condition for \( y(t) \), i.e. \( y(0) = 0, \ \dot{y}(0) = v_0 \sin(\theta) \) the expression for \( y(t) \) becomes
\[ y(t) = \left[1 - e^{-\beta \sin(\theta)}\right] \left(\frac{v_0}{\beta} + \frac{g}{\beta^2 \sin^2(\theta)}\right) - \frac{F_0 e^{-\beta t \sin(\theta)}}{\beta \sin(\theta)} \left(\frac{\lambda - \beta \sin(\theta)}{(\beta \sin(\theta) - \lambda)^2 + \omega^2}\right) \\
- \frac{gt}{\beta \sin(\theta)} + \frac{F_0 e^{-\lambda t}}{\beta \sin(\theta)} \left(\frac{\lambda \cos(\omega t) - \omega \sin(\omega t)}{\omega^2 + \lambda^2}\right) - \frac{F_0 \lambda}{\beta \sin(\theta)} \left(\frac{(\beta \sin(\theta) - \lambda) \cos(\omega t) + \omega \sin(\omega t)}{\beta \sin(\theta)}\right) (\beta \sin(\theta) - \lambda)^2 + \omega^2) \right) \]

Results and Discussion

Fig.1 A trajectories projectile of motion for a cricket ball struck at an angle of calculation of 45° in the presence of a linearized drag force by vary the initial velocity.

From figure 1 we illustrate magnitude of the \( x(t) \) horizontal distance time dependent and \( y(t) \) vertical distance time dependent increase from 100 m to 210 m with increasing the velocity of trajectories projectile of motion for a cricket ball.

Fig.2 A trajectories projectile of motion for a cricket ball struck at an angle of calculation of 45° in the presence of a linearized drag force by vary an angle.

Fig.3 A schematic diagram for behavior of a trajectories projectile of motion for a cricket ball in case of \( \lambda = 0.05 \) is the damped coefficient.
From figure 3 and 4, if the value of parameter $\lambda$ is small the parabolic path of trajectories for projectile motion of a cricket ball will increase small oscillation and the value of parameter $\lambda$ is large the parabolic path of trajectories for projectile motion of a cricket ball will decrease small oscillation as figure 4.

**Conclusions**

The parabolic path of trajectories for a projectile motion of a cricket ball increase oscillation with the value of parameter $\lambda$ and $f_0$ is the storm force. When at angle 60° the high parabolic path of cricket ball more than the angle 30° it is influence due to the large storm force affection $x(t)$ horizontal distance time dependent and $y(t)$ vertical distance time dependent little as figure 2.

**References**


