The Analytical Description of Projectile Motion of Cricket Ball in a Linear Resisting Medium the Storm Force

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Abstract. A detailed work based on the second-order ordinary differential equation is presented to solve oscillation in the trajectory projectile motion of cricket ball for damped alternating external force ($f_0$) problems. This paper purpose to compute the distance time depends horizontal and the distance time depends vertical. The parabolic path of trajectories for a projectile motion of cricket ball increase oscillation with the value of parameter $\lambda$ and $f_0$ is the storm force.

Introduction

Peter Coutis used the quadratic drag force to calculate the distance vertical($y(t)$) motion and distance horizontal($x(t)$) in projectile motion of a cricket ball. Sean M. Stewart consider the time of flight, range and the angle which maximizes the range of a projectile motion in a linear resisting medium are expressed in analytic form in terms of the recently defined Lambert W function is to be the inverse of $We^W = z$. Jeffrey Leela et. al, explores the various equation associated with the movement of the projectile motion for ball in flight and they use the velocity depends on $C_D$ and also the drag force couples the equation for the horizontal and vertical component of the velocity see equation (1) and equation (2), where $C_D$ is the drag coefficient

Projectile motion of cricket ball below external force $\left( f_0 e^{-\lambda t} \cos(\omega t) \right)$

The parameters governing the projectile of motion of a cricket ball are the launch angle ($\theta$), the speed at which the ball leaves the bat ($v_0$), and the linear drag coefficient per unit mass ($\beta$). If we assume that the only forces acting on the cricket ball in flight are gravity drag force and external forces $\left( f_0 e^{-\lambda t} \cos(\omega t) \right)$. Newton’s second law implies

$$m \ddot{x} = -C_D \dot{x} \cos(\theta)$$ \hspace{1cm} (1)

and

$$m \ddot{y} = -C_D \dot{y} \sin(\theta) - mg - f_0 e^{-\lambda t} \cos(\omega t)$$ \hspace{1cm} (2)

The cricket ball’s motion is best described by separating it into horizontal($x(m)$) and vertical($y(m)$) components as we have already emphasized, the horizontal motion dependent of the vertical motion and then applying the kinematic equation. Here $\dot{x}(t)$ is the velocity time dependent and $\ddot{x}(t)$ is...
the acceleration time dependent, \( C_D \) is the drag coefficient, \( \lambda \) is the damped coefficient. Multiplying the above equation by \( \frac{1}{m} \) gives

\[
\ddot{x}(t) + \beta \dot{x}(t) \cos(\theta) = 0
\]

(3)

and

\[
\dot{y}(t) + \beta \dot{y} \sin(\theta) = -\left(g + F_0 e^{-\lambda t} \cos(\omega t)\right),
\]

(4)

where \( \beta = C_D / m, \quad F_0 = f_0 / m \). We use the auxiliary equation find solution of equation (3)

\[
x(t) = A + Be^{-\beta t \cos(\theta)}.
\]

(5)

Here, \( A \) and \( B \) are constants which can be determined from the initial condition \( x(0) = 0, \quad \dot{x}(0) = v_0 \cos(\theta) \). A solution to equation (3) is

\[
x(t) = \frac{v_0}{\beta} \left( 1 - e^{-\beta t \cos(\theta)} \right)
\]

(6)

From the above result, we gives a time-dependent expression for \( x \), which is the horizontal distance travelled by the cricket ball in time \( t \). Rearrangement of equation (6) provides

\[
t = \ln \left[ \frac{v_0 - \beta x(t)}{v_0} \right]^{\frac{-1}{\beta \cos(\theta)}}.
\]

(7)

Find the general solution of the inhomogeneous of equation (4), \( y(t) = y_C(t) + y_P(t) \). Find the fundamental solution to the homogeneous equation the characteristic equation is \( y_C(t) = C + De^{-\beta t \sin(\theta)} \). Here, \( y_1(t) = 1, \quad y_2(t) = e^{-\beta t \sin(\theta)} \). Compute their Wronskian,

\[
W = -\beta \sin(\theta) e^{-\beta t \sin(\theta)},
\]

\[
W_1 = \left(g + F_0 e^{-\lambda t} \cos(\omega t)\right)e^{-\beta t \sin(\theta)},
\]

\[
W_2 = -\left(g + F_0 e^{-\lambda t} \cos(\omega t)\right).
\]

We compute the function \( u_1(t), \quad u_2(t) \), we obtain the particular solution is

\[
y_P(t) = \frac{F_0 e^{-\lambda t} \left( \lambda \cos(\omega t) - \omega \sin(\omega t) \right)}{\beta \sin(\theta) \left( \beta \sin(\theta) - \lambda \right)} - \frac{gt}{\beta \sin(\theta) \left( \beta \sin(\theta) - \lambda \right)} + \frac{g}{\beta \sin(\theta) \left( \beta \sin(\theta) - \lambda \right)^2 + \omega^2}
\]

\[
+ \frac{F_0 e^{-\lambda t} \left((\beta \sin(\theta) - \lambda) \cos(\omega t) + \omega \sin(\omega t)\right)}{\beta \sin(\theta) \left((\beta \sin(\theta) - \lambda)^2 + \omega^2\right)}.
\]

(8)

The general solution is

\[
y(t) = C + De^{-\beta t \sin(\theta)} - \frac{gt}{\beta \sin(\theta)} + \frac{F_0 e^{-\lambda t} \left( \lambda \cos(\omega t) - \omega \sin(\omega t) \right)}{\beta \sin(\theta) \left( \beta \sin(\theta) \lambda^2 + \omega^2 \right)}
\]

\[
+ \frac{F_0 e^{-\lambda t} \left((\beta \sin(\theta) - \lambda) \cos(\omega t) + \omega \sin(\omega t)\right)}{\beta \sin(\theta) \left((\beta \sin(\theta) - \lambda)^2 + \omega^2\right)}.
\]

(9)

With the initial condition for \( y(t) \), i.e. \( y(0) = 0, \quad \dot{y}(0) = v_0 \sin(\theta) \) the expression for \( y(t) \) becomes
\[ y(t) = \left[ (1 - e^{-\beta \sin(\theta)}) \left( \frac{v_0}{\beta} + \frac{g}{\beta^2 \sin^2(\theta)} \right) \right] - \frac{F_0 e^{-\beta t \sin(\theta)} (\lambda - \beta \sin(\theta))}{\beta \sin(\theta)((\beta \sin(\theta) - \lambda)^2 + \omega^2)} - \frac{gt}{\beta \sin(\theta)} + \frac{F_0 e^{-\lambda t} (\lambda \cos(\omega t) - \omega \sin(\omega t))}{\beta \sin(\theta)(\lambda^2 + \omega^2)} - \frac{F_0 \lambda}{\beta \sin(\theta)(\lambda^2 + \omega^2)} + \frac{F_0 e^{-\lambda t} ((\beta \sin(\theta) - \lambda) \cos(\omega t) + \omega \sin(\omega t))}{\beta \sin(\theta)((\beta \sin(\theta) - \lambda)^2 + \omega^2)} \]  

Results and Discussion

Fig. 1 A trajectories projectile of motion for a cricket ball struck at an angle of calculation of $45^\circ$ in the presence of a linearized drag force by vary the initial velocity.

From figure 1 we illustrate magnitude of the $x(t)$ horizontal distance time dependent and $y(t)$ vertical distance time dependent increase from 100 m to 210 m with increasing the velocity of trajectories projectile of motion for a cricket ball.

Fig. 2 A trajectories projectile of motion for a cricket ball struck at an angle of calculation of $45^\circ$ in the presence of a linearized drag force by vary an angle.

Fig. 3 A schematic diagram for behavior of a trajectories projectile of motion for a cricket ball in case of $\lambda = 0.05$ is the damped coefficient.
From figure 3 and 4, If the value of parameter $\lambda$ is small the parabolic path of trajectories for projectile motion of a cricket ball will increase small oscillation and the value of parameter $\lambda$ is large the parabolic path of trajectories for projectile motion of a cricket ball will decrease small oscillation as figure 4.

**Conclusions**

The parabolic path of trajectories for a projectile motion of cricket ball increase oscillation with the value of parameter $\lambda$ and $f_0$ is the storm force. When at angle $60^\circ$ the high parabolic path of cricket ball more than the angle $30^\circ$ it is influence due to the large storm force affection $x(t)$ horizontal distance time dependent and $y(t)$ vertical distance time dependent little as figure 2.

**References**


Fig.4 A schematic diagram for behavior of a trajectories projectile of motion for a cricket ball in case of $\lambda = 0.99$ is the damped coefficient.