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A Mathematical Framework for Cyclic Life Prediction of Directionally Solidified Nickel Superalloys

Roland Mücke

ALSTOM, Brown Boveri Strasse 7, CH-5401 Baden / Switzerland

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Abstract. Modern gas turbines utilize single crystal (SX) and directionally solidified (DS) nickel superalloys which hold a higher cyclic life resistance and an improved creep rupture strength compared to their conventionally cast (CC) version. Both, SX and DS materials feature a significant direction dependence of material properties, which needs to be considered in the constitutive and lifing models. In this context, the paper presents a mathematical framework of cyclic life prediction. Although the method is developed for DS nickel alloys with transverse isotropic material behaviour, a generalisation to common orthotropic materials including SX is straightforward. The proposed procedure is validated by two examples. Moreover, an application to turbine components is shown.

Introduction

The performance of modern gas turbines for electric power generation and aircraft engines is closely related to the material capability of the first turbine stages. In this context, directionally solidified (DS) as well as single crystal (SX) nickel superalloys were developed for achieving an improved fatigue resistance and a significantly increased creep rupture strength by eliminating grain boundaries [1]. However, the advanced material capability of DS and SX alloys can only be fully exploit in the design of turbine parts when both, the constitutive model and the lifetime prediction method appropriately consider the anisotropy of the material.

This short paper focuses on the development of a cyclic life model. It is assumed that the constitutive model (either based on anisotropic viscoplastic constitutive laws [2] or a simple elastic model with anisotropic Neuber correction [3]) appropriately predicts the stabilized load cycle in the investigated part. Moreover, the influence of material damage on the constitutive behaviour is neglected. Therefore, the cyclic life prediction can be applied as post-processing procedure after the constitutive modelling. In the described context, lifing models are often based on empirical approaches. In fact, it has been found and confirmed by several researchers that under the assumption of a stabilized loading cycle a linear relationship on log-log scale is appropriate for describing the failure of a uniaxial, cyclic loaded material. Thereby, the cyclic load is characterized in terms of either the mechanical strain range $\Delta\varepsilon$ or the stress range $\Delta\sigma$. This has led to local stress and local strain approaches based on the Basquin equation and the Coffin Manson equation [4]

$$N_i = C \cdot \Delta\sigma^m \text{ and } N_i = C \cdot \Delta\varepsilon^m . \qquad (1)$$

Thereby, the symbol N_i denotes the number of load cycles related to a suitable end-of-life criterion. The parameters C and m are material constants depending on minimum and maximum temperature, hold time and further characteristics of the load cycle. Of course, the parameters C and m have different numerical values for strain and stress approaches. Nevertheless, the same notation is used here for showing the consistence of the mathematical formulation.

Note that parts of the presented work have recently been published in [5].

Anisotropy of nickel superalloys

The crystal structure of nickel is characterized by a cubic face-centred unit cell, i.e. the lattice vectors are orthogonal and of equal length. Therefore, SX nickel superalloys obey a cubic material symmetry while DS nickel superalloys exhibit a transverse isotropic material symmetry. The polycrystalline microstructure of conventional casts produces an isotropic material behaviour on the macroscopic scale. Note that both DS and SX nickel alloys belong to the more general class of orthotropic materials. Other materials with orthotropic symmetry are some types of composites and textures produced by milling and rolling.

Fig. 1 shows the direction dependence of the Young's modulus for different types of material symmetry. Note that for typical superalloys the ratio between the highest and the lowest stiffness approaches 1.6 to 2.5 dependent on temperature. Note further, that the linear elastic model of DS materials featuring transverse isotropic material symmetry contains five independent elastic constants while for cubic materials only three elastic constants are required [6-9].

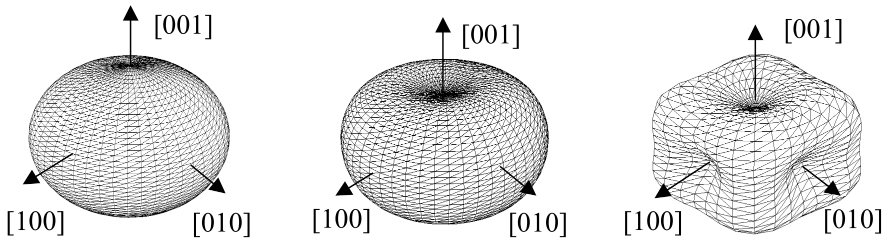

Fig. 1: Direction dependence of Young's modulus.

The type of anisotropy in the elastic model can also be observed for the fatigue strength [10]. Depending on material and temperature, the delta strain vs. cycle curves can have the same slope. In this case, the anisotropy is independent on loading. For fatigue curves with different slopes, the degree of anisotropy additionally depends on the load level, Fig. 2.

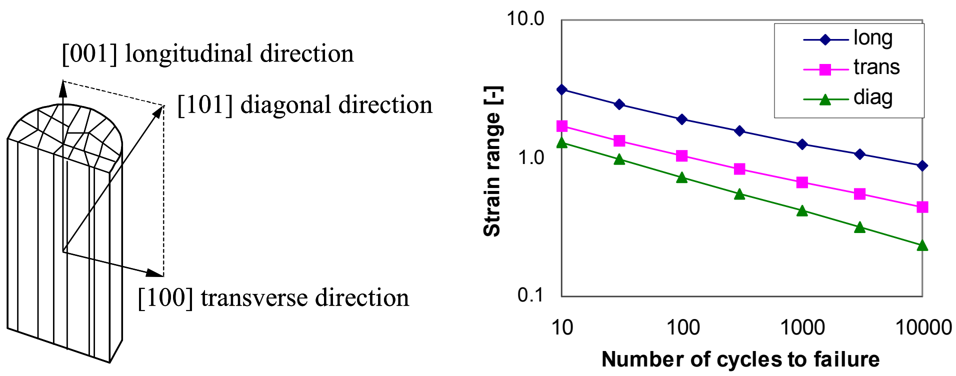

Fig. 2: Example for direction dependence of fatigue strength of a DS superalloy.

Cyclic Life Prediction for DS materials

The local stress approach. The anisotropy of DS materials is fully described by considering the fatigue strength in three independent material directions. In this paper, these directions are referred to as "characteristic directions" and identified by the subscript "d ". Thus, instead of Eq. (1) for isotropic materials, the following three equations characterise the uniaxial cyclic behaviour of DS materials

$$N_{i,d} = C_d \cdot \Delta\sigma_d^{m_d} \quad \text{for } d = 1, 2, 3 \tag{2}$$

where the three pairs of fatigue parameters (C_d, m_d) can be determined by uniaxial cyclic tests in the corresponding material directions.

Above equations characterise the fatigue behaviour for uniaxial loading in either of the characteristic directions of DS materials. Fatigue assessment for engineering structures additionally requires to consider loading in arbitrary directions and to account for multiaxial load cases with all possible combinations of stress tensor components. The relation $\Delta\boldsymbol{\sigma} \Rightarrow N_i$ then forms a surface of equal number of cycles-to-failure. Obviously, for isotropic materials the shape of the surface is a sphere, i.e. the fatigue strength is independent on the material direction. In case of anisotropic materials, the symmetry of the failure surface reflects the symmetry of the fatigue strength.

The three equations (2) are replaced by the general multiaxial formulation

$$N_i = C \cdot f_H^m(\Delta\boldsymbol{\sigma}, \mathbf{F}) \tag{3}$$

where

$$f_H^2(\Delta\boldsymbol{\sigma}, \mathbf{F}) = \mathbf{q}_\sigma^T \mathbf{F} = \begin{bmatrix} (\Delta\sigma_{22} - \Delta\sigma_{33})^2 + (\Delta\sigma_{33} - \Delta\sigma_{11})^2 + 2(\Delta\sigma_{12})^2 \\ (\Delta\sigma_{11} - \Delta\sigma_{22})^2 + 4(\Delta\sigma_{12})^2 \\ 2((\Delta\sigma_{23})^2 + (\Delta\sigma_{31})^2) \end{bmatrix}^T \begin{bmatrix} F \\ H \\ L \end{bmatrix} \tag{4}$$

denotes the square of the Hill equivalent stress function involving the three Hill parameters $\mathbf{F} = [F, H, L]^T$. For details on the Hill function see [7]. Obviously, for the uniaxial load $\Delta\sigma_d$ in either of the characteristic directions both, the general multiaxial approach of Eq (3) and the uniaxial formulation (2) have to predict the same number of cycles-to-failure, i.e. $C_d \cdot \Delta\sigma_d^{m_d} = C \cdot f_H^m(\Delta\boldsymbol{\sigma}_d, \mathbf{F})$, where $\Delta\boldsymbol{\sigma}_d$ denotes the stress tensor components produced by $\Delta\sigma_d$. Further, without loss of generality, the direction "1" can be used as reference direction. Thus, with $C=C_1$ and $m=m_1$ we obtain the three conditions

$$C_d \cdot \Delta\sigma_d^{m_d} = C_1 \cdot f_H^{m_1}(\Delta\boldsymbol{\sigma}_d, \mathbf{F}) \quad \text{for } d = 1, 2, 3 \tag{5}$$

for describing the three Hill parameters $\mathbf{F} = [F, H, L]^T$.

We now consider the right term of Eq (5). A uniaxial stress $\Delta\sigma_d$ in direction d produces the stress tensor

$$\Delta\boldsymbol{\sigma}_d = \mathbf{r}_d \Delta\sigma_d \quad \text{where } \mathbf{r}_d = \Delta\boldsymbol{\sigma}_d / \sqrt{\Delta\boldsymbol{\sigma}_d : \Delta\boldsymbol{\sigma}_d} = \mathbf{D}_d \text{ with } \mathbf{D}_d = \mathbf{d} \otimes \mathbf{d} \tag{6}$$

Thereby, $\Delta\boldsymbol{\sigma}_d : \Delta\boldsymbol{\sigma}_d$ describes the double inner product of the second-rank tensor $\Delta\boldsymbol{\sigma}_d$ and $\mathbf{d} \otimes \mathbf{d}$ denotes the dyadic product of the normalized direction vector $\mathbf{d}$. Thus, the magnitude of the uniaxial stress $\Delta\sigma_d$ can be taken out of the Hill equivalent stress function in order to obtain

$$f_H^2(\Delta\boldsymbol{\sigma}_d, \mathbf{F}) = \boldsymbol{\theta}_d^T \mathbf{F} \cdot (\Delta\sigma_d)^2 = \begin{bmatrix} (r_{22} - r_{33})^2 + (r_{33} - r_{11})^2 + 2r_{12}^2 \\ (r_{11} - r_{22})^2 + 4r_{12}^2 \\ 2(r_{23}^2 + r_{31}^2) \end{bmatrix}^T \begin{bmatrix} F \\ H \\ L \end{bmatrix} (\Delta\sigma_d)^2 \tag{7}$$

Eqs (7) and (5) results after quadration and further rearrangement in

$$(C_d / C_1)^2 \cdot \Delta\sigma_d^{2(m_d - m_1)} = (\boldsymbol{\theta}_d^T \mathbf{F})^{m_1} \quad \text{for } d = 1, 2, 3 \tag{8}$$

The magnitude of loading $\Delta\sigma_d$ can now be eliminated by assuming that the actual multiaxial load $\Delta\boldsymbol{\sigma}$, computed by Finite Element Analysis, produces the same damage as a uniaxial load of the same Hill equivalent stress in either of the characteristic directions. Using Eqs. (4) and (7) immediately gives $\Delta\sigma_d^2 = (\mathbf{q}^T\mathbf{F})/(\boldsymbol{\theta}_d^T\mathbf{F})$. Finally, with Eq. (8) we obtain the governing equations of the multiaxial anisotropic failure surface

$$\phi(\mathbf{F}) := (C_d / C_1)^2 \cdot (\mathbf{q}^T\mathbf{F})^{m_d - m_1} - (\boldsymbol{\theta}_d^T \mathbf{F})^{m_d} = 0 \quad \text{for } d = 1, 2, 3 \tag{9}$$

where C_d and m_d denote the fatigue strength parameters in the directions d = 1, 2, 3, $\mathbf{q}_\sigma$ and $\mathbf{F}$ are defined in Eq (4), and $\boldsymbol{\theta}_d$ follows from Eq (7).

Eq (9) forms a non-linear algebraic system of three equations for the three unknown Hill parameters **F**. The system is solved by the standard Newton method [11], for details see also [5]. After iteration for the Hill parameters, the number of cycles-to-failure is determined by $N_i = C_1 \cdot f_H^{m_1}(\Delta\boldsymbol{\sigma}, \mathbf{F})$ where the Hill function is defined according to Eq (4).

The local strain approach. Due to the Poisson effect, a uniaxial load of material always produces a multiaxial state of strain. Therefore, the strains are the basis for cyclic life prediction and the multiaxial strain state needs to be determined first. Assuming proportional loading the strain $\Delta\varepsilon_d$ in direction d produces the stress

$$\Delta\sigma_d = E_d \Delta\varepsilon_d \quad \text{with } 1/E_d = \mathbf{D}_d : \mathbf{H} : \mathbf{D}_d \tag{10}$$

Note that $\mathbf{D}_d$ is the direction tensor as defined in Eq (6) and **H** denotes the 4th order tensor of elastic compliance. Further, using Eq (6) and the generalized Hooke's law $\Delta\boldsymbol{\varepsilon}_d = \mathbf{H} : \Delta\boldsymbol{\sigma}_d$, the multiaxial state of strain produced by a uniaxial load $\Delta\varepsilon_d$ yields $\Delta\boldsymbol{\varepsilon}_d = \mathbf{H} : \mathbf{D}_d E_d \Delta\varepsilon_d$. Thus the load ratio $\mathbf{r}_d$ is obtained by

$$\mathbf{r}_d = \Delta\boldsymbol{\varepsilon}_d / \Delta\varepsilon_d = \mathbf{H} : \mathbf{D}_d E_d \tag{11}$$

Note that, in contrary to the stress approach, the load ratio now depends on the elastic constants. Therefore, it may slightly change with temperature.

The remaining part of the procedure is the same as for the stress approach when replacing the stress by the strain and the fatigue strength parameters of the stress approach by those of the strain approach.

Generalisation. The above mathematical framework can easily be applied to general orthotropic materials with six Hill parameters [7]. When doing so, the symmetry of transverse isotropic materials, which is described by 3 independent Hill parameters (as for DS nickel superalloys), but also the symmetry of cubic materials with 2 independent Hill parameters (as for SX nickel superalloys) can be interpreted as special cases of general orthotropic symmetry. Moreover, the cyclic life prediction method for isotropic materials directly follows from the general orthotropic case when using the same fatigue strength properties in all six characteristic directions.

Validation for a nickel superalloy

Uniaxial loading in different directions. We first consider uniaxial specimens in different loading directions. To this end, uniaxial thermo-mechanical fatigue (TMF) tests have been carried out in longitudinal (β=0°), diagonal (β=45°) and transverse (β=90°) direction for describing the anisotropic failure surface of the material. Further, additional TMF tests in β=25° and β=70° directions are performed for comparing the fatigue life predictions for these orientations with the additional experimental data. Fig. 3 shows the close prediction of the additional tests by the failure surface. More details are given in [5].

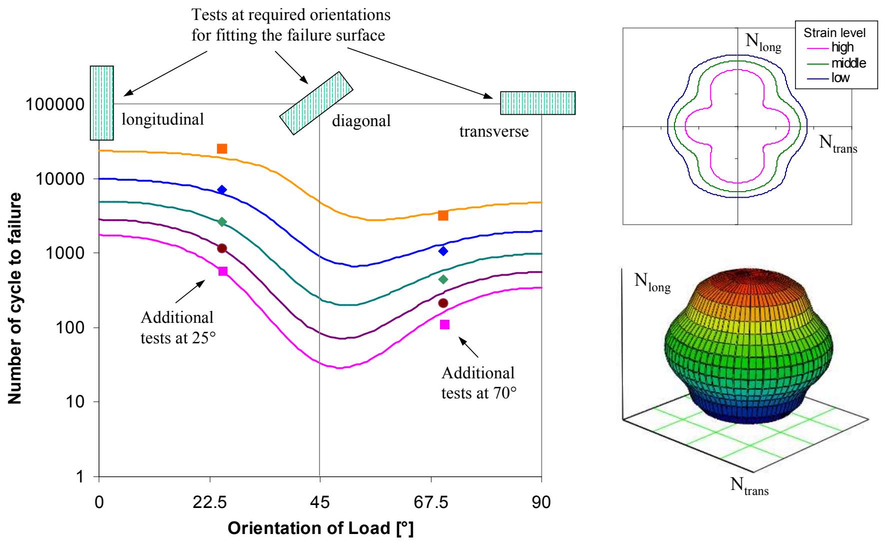

Fig. 3. Failure surface and comparison of predicted and experimental uniaxial test results for the strain approach.

Notch specimen. In a second example the predicted and measured cyclic life of notch specimens is compared. In contrary to the above case, we now address biaxial load conditions produced by the notch effect, Fig. 4.
Table *1* shows the close prediction when using the proposed anisotropic cyclic lifing method [5].

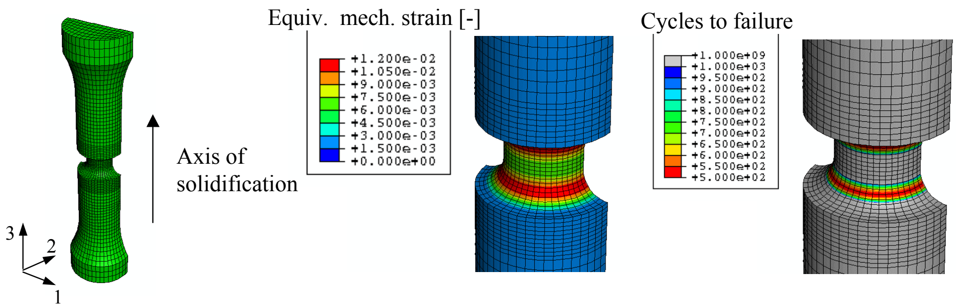

Fig. 4: Results of notch specimen calculation.

Table 1: Test conditions and measured cyclic life of notch specimen testing.

Load conditions	Load levels in axis of solidification	Predicted cyclic life	Measured cyclic life
LCF, R_σ=-1	$\Delta\sigma_{av,notch}$ = 400MPa T = 1000°C	520	616
LCF, R_σ=-1	$\Delta\sigma_{av,notch}$ = 460MPa T = 1000°C	286	313
TMF, R_ε=-1	$\Delta\varepsilon_{av,notch}$ = 0.45%, T = 400-1000°C	454	612
TMF, R_ε=-1	$\Delta\varepsilon_{av,notch}$ = 0.55%, T = 400-1000°C	232	252

Application to turbine components. The anisotropic procedure for cyclic life prediction of DS materials has been implemented in a lifetime analysis tool. The tool is applied as post-processing after a Finite Element Analysis of the turbine components, Fig. 5.

As an example, Fig. 6 shows the metal temperature, the Hill equivalent mechanical strain and the predicted cyclic life of a turbine component after using the proposed anisotropic lifing approach, for details see [5].

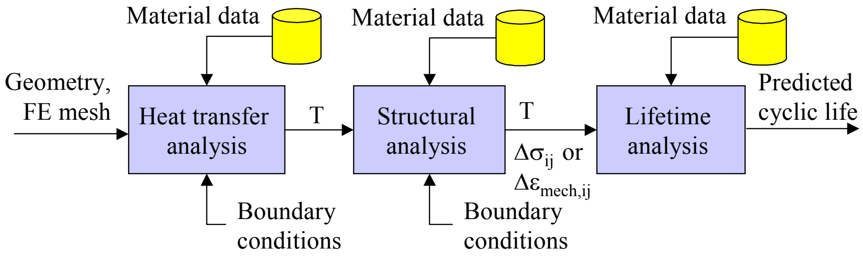

Fig. 5: Sketch of lifetime prediction analysis.

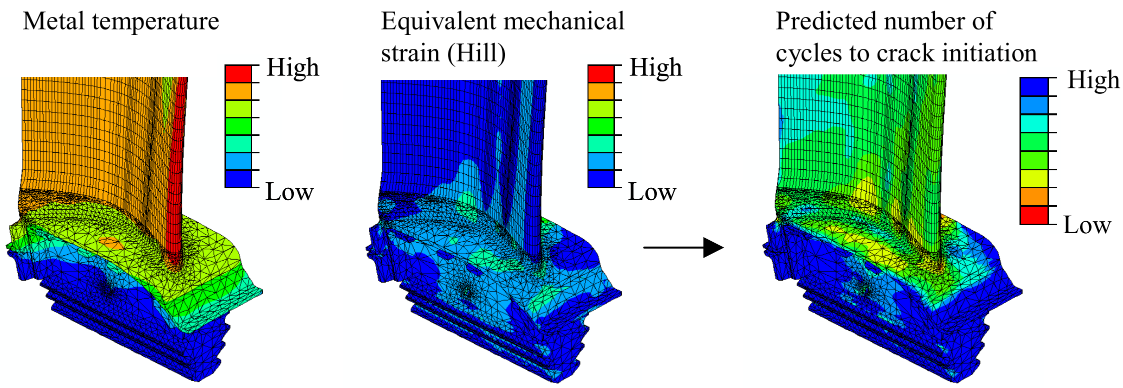

Fig. 6: Example of application to turbine components.

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