Plasticity in the contour method of residual stress measurement

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Abstract

The contour method is a powerful measurement technique that can provide two-dimensional maps of residual stress in engineering components. However, like most strain relief techniques, it can lose accuracy owing to plasticity when residual stresses have high magnitude relative to the yield strength of the material being measured. 2D finite element analysis is utilised to provide an insight into how plasticity introduced by material removal can influence the accuracy of the contour method. In addition the effect of component restraint during the cut is investigated and the results discussed with respect to published experimental measurements.

Introduction

The contour method is a relatively new technique for the measurement of residual stress [1]. It involves cutting the component of interest along a flat plane on which residual stresses are desired to be determined, measuring the deformation of the cut surfaces and using this information to back calculate the undisturbed distribution of residual stresses that were present normal to the cut plane.

The main strengths of the contour method are that it provides a two-dimensional (2D) map of stress along the plane of interest in the component and can be implemented in the laboratory with widely available cutting and measurement equipment. Although destructive, the contour method has been recently developed to measure multiple components of the stress tensor [2, 3]. Other non-destructive measurement techniques, such as neutron, synchrotron and X-ray diffraction, can provide similar 2D stress maps but may be affected by variations in the microstructure or limited by the depth to which measurements can be made within the test component [4].

The contour method, like most mechanical strain-relief based measurement techniques, can be affected by plasticity when measuring residual stresses of magnitude approaching the yield strength of the material. The contour method is based on the theory of elasticity and the principle of superposition and therefore assumes that stress relaxation during the cutting is purely elastic. If plasticity occurs during cutting, this can introduce errors in the contour results. This subject has been addressed on a case by case basis using 2D finite element (FE) analysis [5, 6], but further work is desirable to study the influence of plasticity on contour measurements in a systematic way.

In this paper, 2D FE simulations of contour method measurements are conducted for a well-defined residual stress field in a flat plate in order to predict the development of plasticity during a contour cut and quantify how this affects the calculated stress results. In particular the influence of boundary constraint conditions applied to the flat plate is examined. Since plasticity is more likely to occur in plane stress conditions than in plane strain, the plane stress state is assumed for all the FE simulations of the present paper. The results from the FE studies are discussed in the context of published contour method measurements and inferences drawn regarding the influence of plasticity in the actual measurements.
Approach

A well-defined residual stress state is designed in order to evaluate the performance of the contour method in the event of plasticity developing during the cutting step. The initial residual stress state is then mapped onto a new FE model upon which the entire contour method process is simulated. The predicted contour method measured residual stresses are then compared with the initial residual stress state. Several clamping strategies for different levels of plasticity-induced are considered to investigate the effect of different plasticity scenarios during the sample cutting.

Creating a well-defined initial residual stress state

A “top hat” initial residual stress distribution was generated in a flat plate under plane stress conditions (Fig. 1-a) using the ABAQUS [7] finite element code. The plate was 150 mm wide, 300 mm long and consisted of three different sections. Sections A and C were assigned identical material properties while section B was given a different yield stress and thermal expansion coefficient (see Table 1). The materials’ stress-strain behaviours were assumed to be elastic-perfectly-plastic. A 1 mm square mesh of first order elements was used for the entire FE model. Residual stresses were generated by cooling the plate to 20 °C from an initial temperature of 1000 °C. This model was later used to simulate the contour cutting process along a line at mid-length of the plate (see Fig. 1). Throughout this paper this line is referred to as the measurement line.

Fig. 1-a shows the map of generated residual stress acting in the longitudinal direction (300 mm dimension) of the plate. As expected three regions of residual stress corresponding to the three sections of the plate were generated; a central tensile region balanced by compressive residual stresses on either side. The created ‘top hat’ longitudinal residual stress profile along the measurement line is shown in Fig. 1-b.

![Fig. 1](image)

**Table 1** The materials’ properties of the composite plate used for residual stress generation.

<table>
<thead>
<tr>
<th></th>
<th>Young’s Modulus (GPa)</th>
<th>Poisson’s Ratio</th>
<th>Yield Stress (MPa)</th>
<th>Thermal Expansion Coefficient (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sections A &amp; C</strong></td>
<td>200</td>
<td>0.3</td>
<td>200</td>
<td>8.65 \times 10^{-6}</td>
</tr>
<tr>
<td><strong>Section B</strong></td>
<td>200</td>
<td>0.3</td>
<td>400</td>
<td>1.73 \times 10^{-5}</td>
</tr>
</tbody>
</table>
Modelling the contour cut

The generated residual stresses were mapped into a new FE model of the same dimensions but having common material properties across the three sections. This allowed the level of plasticity introduced by simulating a contour cut to be controlled by adjusting the magnitude of the material’s yield stress. Thus for yield stress values significantly greater than the maximum residual stress, no plasticity was expected to occur whereas yield stress values close to the maximum residual stress would probably result in high levels of plasticity. Two extreme yield stress cases, 500 MPa and 1500 MPa, where the maximum tensile residual stress is 80% and 27.5% of yield respectively, are examined here. Mesh elements of $1 \times 1 \text{ mm}^2$ was generated close to the cut plane and up to about 25 mm away from the cut. Beyond this distance, the mesh elements were progressively coarsened up to the free edges parallel to the cut. It is known that the way in which components are clamped can affect the degree of plasticity introduced during a contour cut. This was investigated by examining two boundary conditions idealising symmetric and asymmetric clamping with respect to the cut path. In the former case the plate was rigidly fixed at both extreme (lengthwise) ends of the plate, whereas for the latter case just one end was fixed. Asymmetric clamping is of particular interest because this is a requirement for slitting measurements which can be combined with a contour stress analysis [8]. The contour cut was simulated by incrementally removing mesh elements equal to the width of the cut (assumed to be 0.5 mm) in an elastic-perfectly-plastic analysis under plane stress conditions. The size of the cut elements was $1 \times 0.5 \text{ mm}^2$.

Contour maps showing the distribution of cutting plastic strain (PEEQ) for the two boundary conditions with 500 MPa yield stress are shown in Fig. 2. The grey areas correspond to regions with plastic strains above 0.2 %. It can be noticed that the maps are significantly different for the two restraint conditions. The total area of plastic strains due to the release of compressive stresses in section A is larger for the asymmetric case (see Fig.2-a) whereas the total area of plastic strains induced by the release of tensile stresses (section B) is lower than for the symmetrically restrained model (Fig.2-b). No cutting plastic strain is observed in section C of the asymmetric model while a small region of plastic strains is seen towards the cut end of the symmetric model. Line profiles of plastic strain distribution along the cutting path provide a better comparison (Fig. 2-c). It is evident that asymmetric boundary conditions have introduced higher magnitude of plastic strains. As expected, when a high yield stress value of 1500 MPa is used, no cutting plastic strain is observed.

![Fig. 2](image_url)  
Maps of predicted plastic strain for 500 MPa yield stress for asymmetric (a) and symmetric clamping (b). Only close-up views of the cut vicinity (area of $~ 150 \times 60 \text{ mm}^2$) are shown. Line profiles results of plastic strains along the measurement line for the two cases are also shown in (c).
The contour residual stress measurement procedure was applied using the simulated elastic-plastic analysis displacement results for the cut plate. The displacements normal to the two cut edges were extracted, averaged and applied as boundary conditions to the cut edge of a new FE model made from one half of the cut part.

**Results and discussions**

No plasticity occurred during simulated contour cuts for a yield strength of 1500 MPa, and as expected the predicted contour residual stresses were identical to the original stress distribution for both boundary restraint conditions. This result implies that when peak residual stresses are less than about 30% of the material’s yield strength, plasticity has no influence on measured contour results.

Fig. 3 shows the residual stresses from the simulated contour measurements for both symmetric and asymmetric boundary condition models together with the initial distribution of residual stress and predicted plastic strain profiles (PEEQ) for an assumed material yield stress of 500 MPa. It is evident that the predicted contour method results show different features for the two restraint conditions. For the symmetrically restrained model (Fig. 3-a), development of plastic strains due to the release of compressive stresses, from 10 - 55 mm cut depth, has an increasing effect on the predicted measured compressive stresses (25 - 50 mm). When combined with yielding caused by the release of tensile stresses in the zone from 60 to 105 mm, the predicted measurement of tensile stress is substantially lower than the original field. Furthermore, predicted stresses in the 100-125 mm zone differ significantly from the original profile. Similar features were observed in contour method residual stress measurements in a ferritic weld bead-on-plate sample [9], where the component was symmetrically clamped using four fitted bolts (see Fig. 4-a). The residual stress distribution in the plate was measured by neutron diffraction prior to the contour measurement (Fig. 4-b) [9]. From the comparison of the results from the two measurement techniques (See Fig. 4-b), it can be seen that the magnitude of tensile stress indicated by the contour measurement is significantly lower than that measured by neutron diffraction. It has been shown that the size of the plastic zone ahead of a moving cut can be related to the stress intensity factor for an equivalent static crack [10]. It is noteworthy that the reduced tensile stresses measured by the contour approach coincide with high values of stress intensity factor for such a crack, and therefore the contour measurement may well have been affected by plasticity under symmetric restraint conditions.

![Fig. 3](image_url)

**Fig. 3** Effects of predicted plastic strain accumulated during cutting on the simulated contour method residual stresses for symmetric (a) and asymmetric (b) restraint conditions.

For the asymmetric boundary condition, the effects of plastic strains introduced by the release of compressive stresses on the predicted measured stresses are entirely different (Fig. 3-b). The main feature is an apparent shift in the location of the tensile stress region towards the cut start. The magnitude of the predicted compressive stresses is also increased (up to 30 mm depth). A similar
shift in stress profile measured by the contour method under asymmetric boundary conditions, compared with slitting and neutron diffraction measurements, has been observed previously by Traoré (see Fig. 5) [8]. The two destructive techniques were applied in tandem and only one side of the sample was maintained during the cutting to fulfil the slitting restraining condition. Plasticity assessment was carried out [11] using the stress intensity factor data obtained from the slitting method measurement. It was found that the release of compressive stresses (from 0-10 mm cut depth) resulted in plasticity, which might introduce the small shift of tensile peak stress.

The level of rigidity imposed by “restraining” displacements at a line or along a plane in an FE analysis is rarely reproducible in practice. ‘Finger’ clamp tools are sometimes used to locate the test component during the real cutting process [8], but the function of these is to stop global movement of the test component on the machine bed rather than restricting cut face opening/closure (which can involve large forces). Restraint of cut face opening/closure can be approached perhaps by use of ‘fitted bolts’, but the degree of restraint is uncertain unless quantified by complex FE analysis simulating the interaction of the bolts with the test component. Thus the degree of restraint in most practical contour measurements will lie between the two cases in Figs 3-a and b. A further important consideration is the distance between the restraint boundary condition and the cutting plane; that is the closer the imposed boundary condition the greater the restraint and vice versa.

Fig. 4 Photograph of the 20 mm thick weld bead-on-plate specimen showing the holes for clamping and the location of the contour cut (a). In (b) the measured longitudinal contour results at 17.5 mm from the plate back face are compared with neutron diffraction measurements (made on one side of the plate and mirrored about the weld centre-line).

Fig. 5 Schematic drawing/photograph showing the 16 mm thick compact tension specimen blank studied, with the measurement line marked up. In (b) transverse residual stresses measured by the contour method are compared with neutron diffraction results at mid-thickness and slitting results (averaged over the thickness).
Conclusions
The simulated conditions presented in this paper were chosen to illustrate the potential effect of plasticity in contour method measurement results; that is an extreme idealised, top-hat residual stress field of high magnitude was examined with elastic-perfectly plastic material (no strain hardening) under plane stress conditions.

The finite element study presented, supported by previous experimental work of the authors, has shown that plasticity can occur and affect contour residual stress measurements when the magnitude of stresses in the component approach the material yield stress (80% in the case presented). The effect of plasticity on the measured stress profile determined by the contour method depends on how the test component cut face opening/closure is restrained. The two extreme cases (symmetrically and asymmetrically restrained) have characteristic footprints where measured stresses are locally increased or decreased in magnitude or shifted (in the cutting direction) or show a combination of all three effects. However, when peak residual stresses are less than about 30% of the material’s yield strength, the present analytical results suggest that plasticity has no influence on measured contour results.

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References