Engineering Innovations
ISSN: 2813-1002, Vol. 11, pp 83-98
doi:10.4028/p-i6uNLi

Submitted: 2024-01-14
Revised: 2024-03-24
Accepted: 2024-03-25
Online: 2024-06-26

Minimizing Conditional Mean Tardiness and Mean Earliness on a Single Machine with the Due Windows Approach

Saheed Akande[1,a], Ganiyu, O Ajisegiri[2,b], Temitayo, M. Azeez[3,c],

[1,3]Department of Mechanical and Mechatronics Engineering, Afe Babalola University, Ado-Ekiti, Nigeria

[2]Department of Industrial and Production Engineering, University of Ibadan, Nigeria

[a]sakande1@abuad.edu.ng, [b]ajisegiriganiyu@gmail.com, [c]Azeeztemitayo221@abuad.edu.ng

Keywords: Conditional Mean Tardiness, Conditional Mean Earliness, Ideal Just-In-Time, Schedule, Due Window, Benchmarking

Abstract. A Multicriteria scheduling problem that involves four due date-based performance measures; the total tardiness, proportion of tardy jobs, total earliness as well as the proportion of early jobs was reduced to an equivalent bicriteria problem of minimizing conditional mean Tardiness and conditional mean earliness. A schedule that optimizes these measures tend towards an ideal Just-In-Time (JIT) schedule. Two solution methods named GOA 1 and GOA 2 were proposed to solve the equivalent reduced problem. A single machine and due window approach was explored. The ideal JIT schedule with zero tardiness and zero earliness was set as the optimal and used for benchmarking the proposed solution as well as other methods found in the literature. The results show that the proposed heuristics yielded results that are not significantly different from the optimal in some instances.

Introduction

The Just-In-Time (JIT) scheduling problems have two major application areas: Production and computer science**.** The production system is considered in this work. The problem application is related to the due date-based types of scheduling problems [1]. Just-in-time is a Production Planning and Control (PPC) philosophy that seeks zero waste and zero inventories [2]. Completing a task before or after the due date incurs additional cost, which is a waste of resources in JIT philosophy. Therefore, in JIT system, it is desirable to complete a task within its due date windows. The advantages of achieving JIT schedule include minimization of complexity associated with detailed material planning, ease of control of shop-floor activities, and minimization of inventory cost among others [3]. However, JIT schedule is an ideal schedule [4], and involves a balance between two opposing due date performance measures; tardiness and the earliness based. To achieve JIT, both the earliness and the tardiness parameters must be zero. This is almost impossible or requires a prohibitive computation time and there is always an associated deviation. To minimize this deviation, total tardiness, the proportion of tardy jobs, total earliness as well as the proportion of early jobs must be minimized simultaneously. However, aggregation of these four performance measures will yield a multicriteria scheduling problem which mostly is NP-hard [5]. However, Akande *et al.,* [6] stated that the complexity of scheduling problems grows as the number of objectives increases. Thus, to minimize the complexity associated with solving the four performance measures of interest, the problem is reduced to a Bicriteria scheduling problem **using methodology of (Akande *et al.*, [7]) .**

Notation Used

Table 1 is the list of notations used in this work

Table 1. Notations used for solving the problem

n	Number of jobs.
J	The set of jobs to be scheduled, J = {J1, J2, . . . ,Jn}.
Pi	Processing time of job Ji, i= 1, 2, . . . ,n.
di	Due date of job Ji, i= 1, 2, . . . ,n.
Ci	Completion time of job Ji, i= 1, 2, . . . ,n.
Li	The lateness of jobJi, i= 1, 2, . . . ,n.
Ei	The earliness of job Ji, i= 1, 2, . . . ,n.
C_{tot}	The total completion time of jobs
T_{tot}	The total tardiness of jobs
E_{tot}	The total Earliness of jobs
D_j^e	The earliest due date for job j.
D_i	The original due date
D_j^L	The latest due date for job j.
A_j	flow allowance assigned to job j at time zero.
R_i	The release dates of the job

Reducibility of the Multicriteria Problem to Bicriteria Problem

The scheduling problem of interest involves multiple criteria in which the objective function is the minimization of the total tardiness, the proportion of tardy jobs, total earliness as well as the proportion of early jobs.
This problem can be expressed mathematically as Eq. 1

$$\text{Min } \alpha \sum_{i=1}^{n} \boldsymbol{T_i}\ (\boldsymbol{x}) + \boldsymbol{\beta} \sum_{i=1}^{n} \boldsymbol{U_i}\ (\boldsymbol{x}) + \gamma \sum_{i=1}^{n} \boldsymbol{E_i}\ (\text{x}) + \delta \sum_{i=1}^{n} \boldsymbol{N_i}\ (\boldsymbol{x}) \quad (1)$$

Subject to $x \in \mathcal{S}$ *W*here:
S is the set of feasible solutions, ***x is*** the decision vector, $\boldsymbol{x} = (x_1, x_2, x_3 - - - - x_n)$
Where:
$\alpha, \beta . \gamma$ *and* δ are the are relative weights of tardiness ($\boldsymbol{T_i}$), earliness (E_i), number of tardy jobs, (U_i) and proportion of early jobs ($\boldsymbol{N_i}$) to be optimized respectively
$\alpha + \beta + \gamma + \delta = 1$
These criteria tardiness ($\boldsymbol{T_i}$), earliness (E_i), number of tardy jobs, (U_i) and proportion of early jobs ($\boldsymbol{N_i}$) are all defined as follows in Eq. 2 - Eq. 6.

$$T_i = max\ \{0, (C_i - d_i)\} \quad (2)$$

$$T_i = max\ \{0, (L_i)\} \quad (3)$$

The total tardiness is defined

$$T_{tot} = \sum_{i=1}^{n} T_i \quad (4)$$

$for\ i = 1, 2, \ldots, n$
The total number of tardy jobs is given by

$$N_T = \sum_{i=1}^{n} U_i \quad (5)$$

Where

$$U_i = \begin{cases} 1\ ; T_i > 0 \\ 0\ Otherwise \end{cases} \quad (6)$$

The Conditional Mean Tardiness (CMT) is defined as a measure of the average amount of tardiness for the completed jobs which are found to be tardy. Mathematically, CMT is defined by Eq. 7

$$\text{CMT} = \frac{\sum_{i=1}^{n} Tj}{Uj} \tag{7}$$

The definition of earliness that applies to JIT problem according to Akande et al, [8] is given as:

$$\text{Thus, } E_i = \max\{-L_i, 0\} \tag{8}$$

$for\ i = 1, 2, \ldots, n$

The total earliness is given as

$$E_{tot} = \sum_{i=1}^{n} \boldsymbol{E_i} \tag{9}$$

$for\ \ i = 1, 2, \ldots, n$

The Proportion of early jobs (Z_i) is given as

$$N_E = \sum_{i=1}^{n} Z_i \tag{10}$$

Where

$$Z_i = \begin{cases} 1\,; E_i > 0 \\ 0\ Otherwise \end{cases} \tag{11}$$

The Conditional Mean Earliness (CME) is a measure of the average amount of earliness for the completed jobs which are found to be early. CME is defined mathematically by Eq. 12

$$\text{CME} = \frac{\sum_{i=1}^{n} Ej}{\boldsymbol{Z_i}} \tag{12}$$

Substitute Eqn (7) and Eqn (12) into Eqn (1) using the Akande et al., [7] reducibility of multicriteria to bicriteria problem procedure. The multicriteria scheduling problem can then be reduced to Eq. 13

$$\text{Min } \left(\boldsymbol{A_1} \frac{\sum_{i=1}^{n} Tj}{Uj} + \boldsymbol{A_2} \frac{\sum_{i=1}^{n} Ej}{\boldsymbol{Z_i}}\right) \tag{13}$$

Where A_1 and A_2 are constant and measure the priority or importance attached to conditional mean tardiness and conditional mean earliness respectively.

$$A_1 = f\ (\alpha\ ) \tag{14}$$

$$A_2 = f\ (\gamma\ ) \tag{15}$$

Assuming equal importance to the two performance measures. Then

$$A_1 = A_2 = 0.5 \tag{16}$$

Therefore, the problem is reduced to a bicriteria scheduling problem. [7] stated that multicriteria scheduling problems can be solved effectively using a polynomial-time algorithm that solves bicriteria or single criteria equivalent problems. In this regard, two non-regular performance measures; Conditional mean tardiness (CMT) and conditional mean earliness (CME) are minimized with a due **window subject to static constraint.** Minimizing these performance measures will ensure that the inevitable deviation from the perfect JIT schedule is minimized. Thus, the cost of inventory, wastage as well as other penalty costs associated with early and tardy jobs can be minimized.

Literature Review

Job tardiness is an important scheduling objective and can be measured through several performance measures like total tardiness, maximum tardiness, and Conditional Mean Tardiness (CMT) among others. While other measures have been exhausted by different researchers, the use of CMT is sparse in the literature. The conditional mean tardiness measures the average amount of tardiness for the completed jobs which are found to be tardy [9]. For instance, consider two scheduling solution methods that yield the same value of the total tardiness, the solution method that has a greater number of tardy jobs will exhibit less conditional mean tardiness than the other. CMT is not a regular performance measure, It means that it can decrease while the completion times are not decreasing [10]. [11] explored modified operation due date (MOD) rule for mean job tardiness and the proportion of tardy jobs, and the two measures together imply the conditional mean tardiness. The MOD rule was compared to several well-known tardiness-oriented priority rules, such as minimum slack-per-operation (S/OPN), smallest critical ratio (SCR), and COVERT. The MOD rule tends to achieve lower levels of mean tardiness than the other rules and lower CMT for a given proportion of the number of tardy jobs, except under conditions of high differences in due dates. **[9]** proposed an effective procedure to estimate the first two central moments (i.e., the mean and the variance) of the conditional mean tardiness and from this to compute a probabilistic bound for the maximum tardiness. These estimates are computed from the evaluation of the total tardiness, the number of tardy jobs, and the root mean square tardiness obtained through a stochastic simulation. Different evaluations done by simulation show the effectiveness of the bound obtained. It was concluded the bound is effective from the results obtained. [12] reported that that the use of critical ratio priorities is effective for minimizing the conditional mean tardiness (*CMT*).

Different criteria which are derivatives of job earliness have also been considered by different researchers. However, most researchers have considered earliness as a bicriteria performance measure by optimizing hierarchically, simultaneously or in Pareto form with tardiness, number of tardy jobs, or some measures as criteria. For instance, Rosa et al., [13] solved the single machine scheduling problem with distinct time windows and sequence-dependent setup times. The objective was to minimize the total weighted earliness and tardiness. The problem involves determining the job execution sequence and the starting time for each job in the sequence. An implicit Enumeration algorithm (an exact algorithm), and a general variable neighborhood search algorithm (a heuristic algorithm) were proposed to determine the job scheduling. The Enumeration algorithm yielded optimal solutions of all instances up to 15 jobs within a feasible computational time while the proposed general variable neighborhood search algorithm produces better-quality solutions for larger problem instances with less computational time compared with the other algorithm from the literature. [14] proposed a Pareto-optimal solution algorithm that efficiently generated Pareto-optimal solutions for any possible number of tardy jobs for a single-machine scheduling problem to minimize the summation of the weighted earliness and tardiness, subject to the number of tardy jobs. The work was benchmarked against scheduling on a single machine against restrictive and unrestrictive common due dates(see [15]) to show the superior accuracy and run time of the proposed Pareto solution. The computational analysis also shows that approximately 47.15% of the nodes in the branching tree can be eliminated. [16] developed an approximation optimization approach, which is based on the imperialist competitive algorithm hybridized with an efficient neighborhood search for solving a job shop problem for minimizing the sum of the maximum earliness and tardiness criteria. A mixed integer linear programming (MIP) formulation of the job shop scheduling problem with the objective function was developed. The MIP model was validated on different problem sizes through a set of experiments and the effectiveness of the proposed approach was demonstrated through an experimental evaluation.

However, while different researchers have considered different variants of earliness and tardiness-based scheduling problems, literature is sparse where a multicriteria scheduling problem with four due date-based performance measures was reduced to two non-regular bicriteria problems. In this work, two non-regular performance measures; Conditional Mean Tardiness (CMT) and Conditional Mean Earliness (CME) are minimized with a due window **subject** to a static constraint. The problem is assumed to be deterministic, and the machine is assumed to be available continually and can only process one job at a time.

Problem Definition

Using the Garaham notation [17], the problem of interest can be represented as

$$\text{Min } \alpha \sum_{i=1}^{n} \boldsymbol{T_i}\,(\boldsymbol{x}) + \boldsymbol{\beta} \sum_{i=1}^{n} \boldsymbol{U_i}\,(\boldsymbol{x}) + \gamma \sum_{i=1}^{n} \boldsymbol{E_i}\,(\mathrm{x}) + \delta \sum_{i=1}^{n} \boldsymbol{N_i}\,(\boldsymbol{x}) \quad (17)$$

Subject to $x \in \mathcal{S}$

*W*here:

S is the set of feasible solutions, $\boldsymbol{x}$ ***is*** the decision vector, $\boldsymbol{x} = (x_1, x_2, x_3 ----x_n)$

Where:

$\alpha, \beta.\gamma$ *and* δ are the are relative weights of tardiness ($\boldsymbol{T_i}$), earliness (E_i), number of tardy jobs, (U_i) and proportion of early jobs ($\boldsymbol{N_i}$) to be optimized respectively

$\alpha + \beta + \gamma + \delta = 1$

This problem has been reduced to

$$\text{Min } (\boldsymbol{A_1} \frac{\sum_{i=1}^{n} Tj}{Uj}\,(\mathrm{x}) + \boldsymbol{A_2} \frac{\sum_{i=1}^{n} Ej}{\boldsymbol{Z_i}}\,(x)) \quad (18)$$

Subject to $x \in \mathcal{S}$

*W*here:

S is the set of feasible solutions, $\boldsymbol{x}$ ***is*** the decision vector, $\boldsymbol{x} = (x_1, x_2, x_3 ----x_n)$

Where

$\boldsymbol{A_1}$ ***and*** $\boldsymbol{A_2}$ are the are relative weights of $\frac{\sum_{i=1}^{n} Tj}{Uj}$ and $\frac{\sum_{i=1}^{n} Ej}{\boldsymbol{Z_i}}$ respectively

$\frac{\sum_{i=1}^{n} Tj}{Uj}$ is defined as the conditional mean tardiness and,

$\frac{\sum_{i=1}^{n} Ej}{\boldsymbol{Z_i}}$ is called the conditional mean earliness

Therefore, the problem consists of scheduling n jobs in the set I= {J_1, J_2, …, Jn} on a single machine with the due window and zero release dates. The target is to minimize the conditional mean tardiness and conditional mean earliness simultaneously to achieve an approximate JIT schedule. Fig. 1 illustrate that the two performance measures (the conditional mean tardiness and conditional mean earliness) is an indication of deviation of Just In -Time (JIT) schedule. The parameters involved in the problem are completion time, due date, number of tardy jobs and number of early jobs

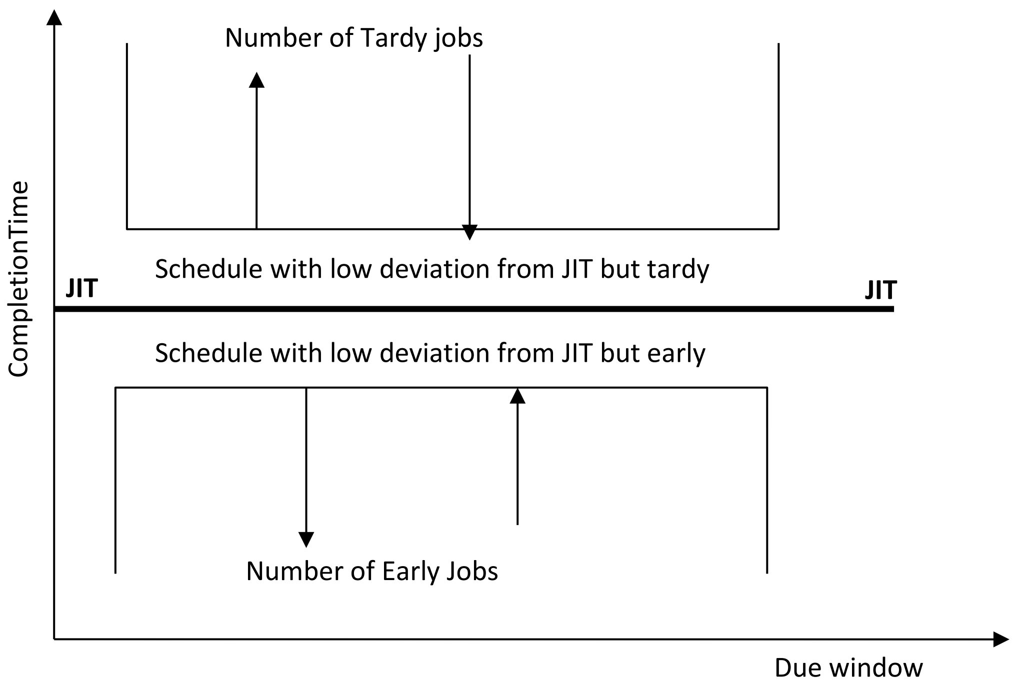

Fig. 1. Variation of JIT Schedule with problem parameters

JIT schedule is an ideal schedule where the due date is equal to the completion time. It is impossible to always achieve JIT, thus, the following conditions denoted by Eq. 19 – Eq. 22 is inevitable

$$\sum_{i=1}^{n} T_i \geq 0 \tag{19}$$

$$\sum_{i=1}^{n} E_i \neq 0 \tag{20}$$

$$\sum_{i=1}^{n} U_i \geq 0 \tag{21}$$

$$\sum_{i=1}^{n} N_e \geq 0 \tag{22}$$

The upper deviation (jobs completed after the due dates) from the equilibrium JIT is measured by total tardiness and the number of tardy jobs. The conditional mean tardiness measures the average amount of tardiness for the completed jobs which are found to be tardy. The lower deviation (jobs completed before the due date) is measured by total earliness and the number of early jobs. The conditional mean earliness is defined as the mean earliness for a set of early jobs.

The Proposed Solution Method

Two solution methods named, GOA 1 and GOA 2 are proposed for solving the problem. The algorithms for the solution methods are outlined as follows.

GOA 1

This algorithm modified the critical ratio rule to solve the problem.

Initialization

JobSet A = [J_1, J_2, J_3, . . . Jn] set of given jobs. JobSet B = [0], set of scheduled job

STEP I: Compute the due date allowance ($D_j^L - D_j^e$) associated with the due window for all the jobs in the JobSet A

STEP 2: Compute the modified critical ratio (MCR) defined as ($\frac{D_j^L - D_j^e}{P_i}$) associated with all the JobSet A in JobSet A

STEP 3: Arrange the JobSet A in increasing order of $\frac{D_j^L - D_j^e}{P_i}$ and put the same in JobSet B

STEP 4: Compute the Linear Composite Objective Function (LCOF) of the JobSet B

STEP 7: Stop .

GOA 2

This algorithm uses the same priority index as the Modified Due Date (MDD) rule, but with a different concept for job selection for processing. Interested readers can contact [12] for further reading on MDD Rule

Just like the MDD rule, this algorithm at any time computes the priority index, which is given by:

$$\Pi i = \{t + p_i, D_j^L, d_o, D_j^e\} \tag{23}$$

Where t is the starting time of the next unscheduled job $i(i \in U)$ which can either be the completion time of the job in position i-1 or the release date of job i, p_i is the processing time, and d_i is the due date.

The step of the algorithm is outlined as follows :

Initialization

JobSet A = [J_1, J_2, J_3, . . . Jn] set of given jobs. JobSet B = [0], set of scheduled job

Initialization

JobSet A = [J_1, J_2, J_3, . . . Jn] set of given jobs. JobSet B = [0], set of scheduled job

STEP I: Compute priority index, Πi= $\{t + p_i, D_j^L, d_o, D_j^e\}$for all the jobs in the jobSetA set t =0

STEP 2: For i = 1 – n, schedule job I if $t + p_i, = D_j^E$ if there is a tie break the tie using the EDD, if the tie still uses SPT, if a tie still exists break arbitrarily. If JobSeT B is null (Empty) Go to STEP 3, else Go To STEP 7.

STEP 3: then, For I = 1 – n, schedule job i if $D_j^E <= t + p_i < d_o$, if there is a tie break the tie using the EDD, if a tie still exists use SPT, if a tie still exist break arbitrarily. If JobSeT B is null (Empty) Go to STEP 4, else Go to STEP 7,

STEP 4: For I = 1 – n, schedule job i if $d_o <= t + p_i <= D_j^L$,if there is a tie break the tie using the EDD, if a tie still exists use SPT, if the tie exists again break arbitrarily. If JobSeT B is null (Empty) Go to STEP 5, Else Go To STEP 7

STEP 5: For I = 1 – n, schedule job i if $t + p_i <= D_j^L$,if there is a tie, break the tie using the EDD, if a tie still exists use SPT, if the tie exists again break arbitrarily. If JobSeTB is null (Empty) Go to STEP 6, Else Go To STEP 7

STEP 6: For I = 1 – n, schedule job i if i has the lowest $t + p_i$ there is a tie use EDD, if tie exist again break arbitrarily, Else Go To STEP 7

STEP 7: For I = 2 – n, set t = C_{i-1} where C_{i-1} is the completion time of the last scheduled job in Job set B.

STEP 8: Repeat Steps 1 to 7 until all the jobs have been scheduled

STEP 9: Compute LCOF for the schedule obtained in the JobSetB

Experimentation

Single processor scheduling problems with due window were simulated using the desktop tool module (editor) on MATLAB R2010 programming language. The number of jobs (*n*), the processing times (*pi*), and the due windows (*di*) time interval were generated using the Süer et al., [18] and [19]. The processing time *P* follows the uniform distribution *U*(1, 10). For the due window, each job entering the system is allocated to ascertain the due date D_j using Total Work Content (TWK) method as explored by Süer et al [18].

$$D_j = R_i + KP_i \tag{24}$$

The earliest due date (D_j^e), and the latest due dates (D_j^L) computed from the original due date (Dj). The due window is denoted by Eq. 25 - Eq. 27.

$$D_j^e = D_i - R \times A_j \tag{25}$$

$$D_j^L = D_i + R \times A_j \tag{26}$$

Where

A_j is the flow allowance assigned to job j at time zero (R_i = 0.). It is set at (20 % - 40% of D_j)

$$D_j^{L/E} = D_j \pm (A_j \times D_j) \tag{27}$$

In this work, the three due windows were explored

Results and Discussion

The two proposed heuristics named GOA 1 and GOA 2 and some existing solution methods; the Smallest Critical Ratio (SCR), Modified Due Date (MDD), and Earliest Due Date (EDD) from the literature were implemented to obtain the values of Condition Mean Earliness (CME), Condition

Mean Tardiness (CMT) as well as the unnormalized Linear Composite Objective Function (LCOF). Table 2 to 11 shows the results obtained. The CME and the CMT results were explored to analyze the performance of all the solution methods concerning the ideal JIT schedule. The ideal JIT schedule is assumed the zero values of CME and CMT. The LCOF values were not used for results analysis because it is not normalized (See Akande et al., [20]) on Normalization of Composite Objective Function for Multicriteria Scheduling Problems with Zero Release Dates).

Table 2. Results of the performance measures for the solution methods for 5 x1 problem size

	Earliest Due Date			Original Due Date			Latest Due Date		
Solution Method	$\sum_{i=1}^{n} CMT_i$	$\sum_{i=1}^{n} CME_i$	LCOF	$\sum_{i=1}^{n} CMT_i$	$\sum_{i=1}^{nCM} E_i$	LCOF	$\sum_{i=1}^{n} CMT_i$	$\sum_{i=1}^{n} CME_i$	LCOF
SCR	6.95	12.00	7.02	0.00	15.40	10.49	10.90	34.81	14.56
MDD	6.26	1.10	4.57	5.00	4.33	3.46	0.00	9.27	6.99
GOA 1	6.95	12.00	7.01	0.00	15.40	10.49	7.65	10.32	6.73
GOA 2	0.00	12.00	8.44	0.00	22.60	12.27	0.00	35.07	15.85
EDD	0.00	11.41	4.25	0.00	19.20	9.19	0.00	34.87	14.65

Table 3. Results of the performance measures for the solution methodsfor 10 x 1 problem size

	Earliest Due Date			Original Due Date			Latest Due Date		
Solution Method	$\sum_{i=1}^{n} CMT_i$	$\sum_{i=1}^{n} CME_i$	LCOF	$\sum_{i=1}^{n} CMT_i$	$\sum_{i=1}^{n} CME_i$	LCOF	$\sum_{i=1}^{n} CMT_i$	$\sum_{i=1}^{n} CME_i$	LCOF
SCR	19.67	2.48	10.75	7.83	5.50	12.53	14.12	24.31	14.72
MDD	18.30	2.00	7.37	7.86	3.00	4.68	0.00	9.89	5.19
GOA 1	19.67	2.48	10.76	8.00	5.50	12.52	15.41	7.65	10.29
GOA 2	17.69	8.03	13.36	8.00	18.00	13.77	0.00	23.69	16.10
EDD	14.75	4.35	6.41	5.00	6.00	5.60	0.00	22.59	9.14

Table 4. Results of the performance measures for the solution methodsfor 15 x 1 problem size

	Earliest Due Date			Original Due Date			Latest Due Date		
Solution Method	$\sum_{i=1}^{n} CMT_i$	$\sum_{i=1}^{n} CME_i$	LCOF	$\sum_{i=1}^{n} CMT_i$	$\sum_{i=1}^{n} CME_i$	LCOF	$\sum_{i=1}^{n} CMT_i$	$\sum_{i=1}^{n} CME_i$	LCOF
SCR	29.12	1.53	16.70	33.10	3.80	16.23	18.64	15.78	18.91
MDD	20.90	0.00	11.89	17.80	3.80	8.19	2.25	8.61	5.68
GOA 1	29.12	1.53	16.72	33.10	3.80	16.25	27.78	2.76	16.26
GOA 2	25.74	9.62	17.25	21.25	21.71	17.94	12.15	26.76	17.51
EDD	15.64	3.93	11.40	17.00	4.00	8.39	0.00	14.84	6.82

Table 5. Results of the performance measures for the solution methods for 20 x 1 problem size

	Earliest Due Date			Original Due Date			Latest Due Date		
Solution Methods	$\sum_{i=1}^{n} CMT_i$	$\sum_{i=1}^{n} CME_i$	LCOF	$\sum_{i=1}^{n} CMT_i$	$\sum_{i=1}^{n} CME_i$	LCOF	$\sum_{i=1}^{n} CMT_i$	$\sum_{i=1}^{n} CME_i$	LCOF
SCR	43.20	4.65	23.00	29.37	5.00	22.34	37.20	6.16	22.33
MDD	30.46	0.55	16.23	15.33	0.00	13.89	20.39	4.67	11.04
GOA 1	45.19	1.60	23.00	29.37	5.00	22.43	45.27	0.65	23.22
GOA 2	32.70	7.64	21.44	20.80	23.44	22.87	26.67	16.60	22.74
EDD	31.03	0.81	15.69	14.35	1.67	14.36	19.17	6.10	11.24

Table 6. Results of the performance measures for the solution methodsfor 40 x 1problem size

	Earliest Due Date			Original Due Date			Latest Due Date		
Solution Methods	$\sum_{i=1}^{n} CMT_i$	$\sum_{i=1}^{n} CME_i$	LCOF	$\sum_{i=1}^{n} CMT_i$	$\sum_{i=1}^{n} CME_i$	LCOF	$\sum_{i=1}^{n} CMT_i$	$\sum_{i=1}^{n} CME_i$	LCOF
SCR	44.86	0.00	22.73	104.89	7.33	49.43	87.90	4.53	47.19
MDD	33.86	0.00	16.28	75.10	1.00	33.06	64.92	3.90	30.47
GOA 1	44.86	0.00	22.72	104.89	7.33	49.43	104.15	0.00	51.46
GOA 2	33.07	9.22	21.72	70.03	23.55	40.84	67.37	22.98	40.51
EDD	31.02	1.25	15.67	77.47	3.33	34.28	61.72	6.89	31.19

Table 7. Results of the performance measures for the solution methods **for 50 x 1** problem size

	Earliest Due Date			Original Due Date			Latest Due Date		
Solution Methods	$\sum_{i=1}^{n} CMT_i$	$\sum_{i=1}^{n} CME_i$	LCOF	$\sum_{i=1}^{n} CMT_i$	$\sum_{i=1}^{n} CME_i$	LCOF	$\sum_{i=1}^{n} CMT_i$	$\sum_{i=1}^{n} CME_i$	LCOF
SCR	123.09	0.00	64.54	101.27	0.00	62.95	106.13	5.60	59.11
MDD	94.69	0.00	49.53	65.78	1.00	44.26	61.96	3.47	39.63
GOA 1	123.09	0.00	64.53	101.33	0.00	62.97	121.57	1.50	63.91
GOA 2	86.01	9.86	49.51	63.58	18.14	50.29	67.76	19.19	49.03
EDD	88.43	0.50	47.19	65.92	1.00	45.35	65.27	3.25	40.47

Table 8. Results of the performance measures for the solution methods for 100 x 1 problem size

	Earliest Due Date			Original Due Date			Latest Due Date		
Solution Method	$\sum_{i=1}^{n} CMT_i$	$\sum_{i=1}^{n} CME_i$	LCOF	$\sum_{i=1}^{n} CMT_i$	$\sum_{i=1}^{nCM} E_i$	LCOF	$\sum_{i=1}^{n} CMT_i$	$\sum_{i=1}^{n} CME_i$	LCOF
SCR	230.64	0.00	137.33	275.61	3.00	136.54	264.88	3.30	133.18
MDD	171.76	0.55	107.07	195.53	0.00	98.23	192.10	0.93	94.46
GOA 1	230.61	0.00	137.34	275.44	3.00	136.52	276.27	0.00	139.22
GOA 2	141.79	8.53	95.51	183.35	17.86	98.13	181.28	11.89	98.62
EDD	156.19	0.00	101.37	198.00	1.67	99.87	193.53	0.93	95.92

Table 9. Results of the performance measures for the solution methods for 150 x 1 problem size

	Earliest Due Date			Original Due Date			Latest Due Date		
Solution Method	$\sum_{i=1}^{n} CMT_i$	$\sum_{i=1}^{n} CME_i$	LCOF	$\sum_{i=1}^{n} CMT_i$	$\sum_{i=1}^{n} CME_i$	LCOF	$\sum_{i=1}^{n} CMT_i$	$\sum_{i=1}^{n} CME_i$	LCOF
SCR	423.49	0.00	211.22	423.52	0.00	208.17	403.03	2.85	205.39
MDD	321.43	0.00	166.28	314.95	1.00	151.87	282.49	0.38	148.79
GOA 1	423.52	0.00	211.22	423.52	0.00	208.18	417.88	0.00	212.10
GOA 2	267.10	9.30	142.49	208.44	17.21	144.48	250.19	30.46	144.32
EDD	305.44	0.00	157.46	318.69	1.00	153.78	286.76	1.15	150.94

Table 9. Results of the performance measures for the solution methods for 200 x 1 problem size

	Earliest Due Date			Original Due Date			Latest Due Date		
Solution Method	$\sum_{i=1}^{n} CMT_i$	$\sum_{i=1}^{n} CME_i$	LCOF	$\sum_{i=1}^{n} CMT_i$	$\sum_{i=1}^{n} CME_i$	LCOF	$\sum_{i=1}^{n} CMT_i$	$\sum_{i=1}^{n} CME_i$	LCOF
SCR	560.26	0.00	283.52	560.75	3.00	280.72	558.31	3.62	277.60
MDD	436.19	0.60	220.62	418.84	1.00	203.91	388.64	2.62	200.85
GOA 1	559.90	0.00	283.49	560.78	3.00	280.72	575.58	0.25	284.49
GOA 2	359.65	9.76	185.71	363.22	15.80	188.47	358.15	20.40	189.80
EDD	411.63	0.95	208.76	426.03	1.50	206.80	394.50	2.31	203.49

Table 10. Results of the performance measures for the solution methods for 300 x 1 problem size

	Earliest Due Date			Original Due Date			Latest Due Date		
Solution Method	$\sum_{i=1}^{n} CMT_i$	$\sum_{i=1}^{n} CME_i$	LCOF	$\sum_{i=1}^{n} CMT_i$	$\sum_{i=1}^{n} CmE_i$	LCOF	$\sum_{i=1}^{n} CMT_i$	$\sum_{i=1}^{n} CME_i$	LCOF
SCR	876.98	0.00	430.80	862.09	0.00	422.96	837.49	2.60	422.15
MDD	675.85	0.43	338.29	624.50	0.00	307.09	603.58	2.05	307.93
GOA 1	876.86	0.00	430.77	861.91	0.00	422.95	853.73	0.00	429.32
GOA 2	546.81	6.30	277.64	549.45	14.31	275.03	534.83	22.08	280.37
EDD	637.01	1.43	320.35	633.69	1.33	311.32	613.81	1.83	311.77

Table 11. Results of the performance measures for the solution methods for 400 x 1problem size

	Earliest Due Date			Original Due Date			Latest Due Date		
Solution Methods	$\sum_{i=1}^{n} CMT_i$	$\sum_{i=1}^{n} CME_i$	LCOF	$\sum_{i=1}^{n} CMT_i$	$\sum_{i=1}^{n} CME_i$	LCOF	$\sum_{i=1}^{n} CMT_i$	$\sum_{i=1}^{n} CME_i$	LCOF
SCR	1176.10	0.00	577.53	1161.00	6.00	573.87	1123.00	0.700	570.49
MDD	915.23	0.43	450.57	847.45	2.00	418.63	808.06	0.35	413.90
GOA 1	1176.30	0.00	577.40	1161.00	6.00	573.83	1143.00	0.00	578.18
GOA 2	735.70	4.94	367.08	727.28	15.35	369.70	708.97	21.77	370.60
EDD	864.28	1.60	426.76	860.51	2.75	424.33	820.42	0.35	419.39

Performance Evaluation

The results obtained (as depicted in Table 2 to 11) were analyzed for the three windows. The values of the CMT and CME were plotted against the problem sizes. The t-test using Paired Two Sample for Means at 99% was also used to quantify the significance of the differences between the solution methods and the ideal JIT schedule for different problem sizes.

Analysis of the Results Based on the Earliest Due Window

Figure 2 illustrates the plot of CMT against the problem sizes. The plot shows the GOA2 heuristic exhibits the minimum conditional tardiness deviation from the JIT schedule. GOA 1 and SCR plots align.

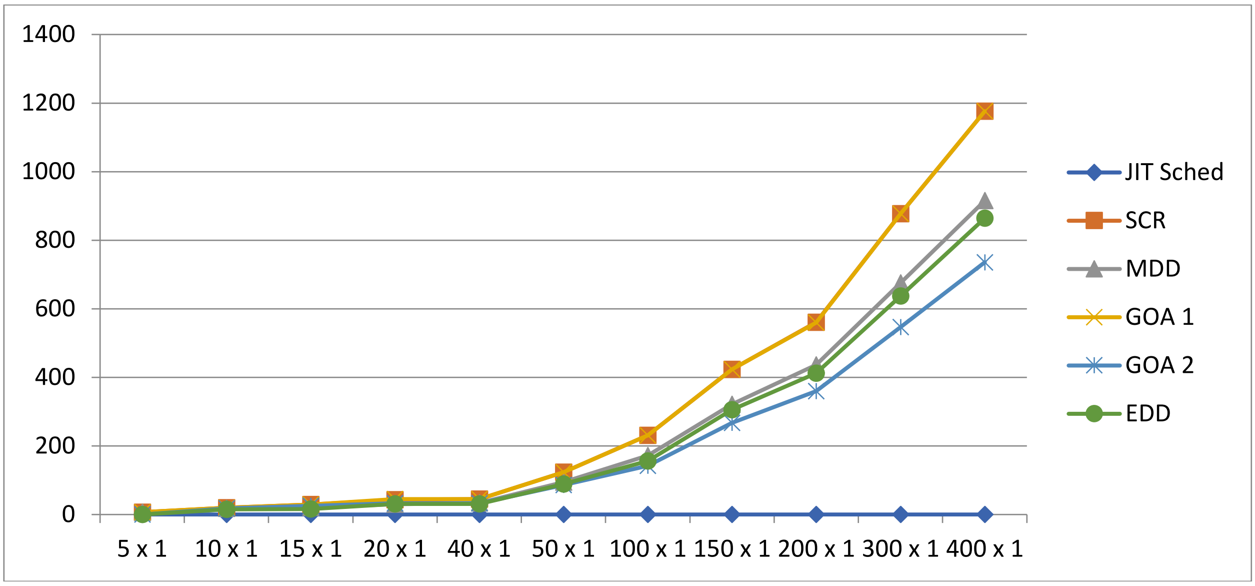

Fig. 2. The plot of Conditional Tardiness values against the problem sizes for the earliest Due window

NOTE In Fig. 2 . SCR completely align with GOA 1, thus the SCR plot seems invisible

The t-test also shows **that the GOA 2 heuristics is not significantly different from the imaginary JIT schedule for problem sizes: $5 \leq n \leq 15$. Table 12 shows the results of the t-test.**

Table 12. t-test results for the JIT schedule against GOA 2 for problem sizes; $5 \leq n \leq 15$

	JIT Schedule	*GOA 2*
Mean	0	14.47666667
Variance	0	173.3810333
Observations	3	3
Pearson Correlation	#DIV/0!	
Hypothesized Mean Difference	0	
Df	2	
t Stat	-1.90427	
P(T<=t) one-tail	0.098589	
t Critical one-tail	2.919986	
P(T<=t) two-tail	0.197178	
t Critical two-tail	4.302653	

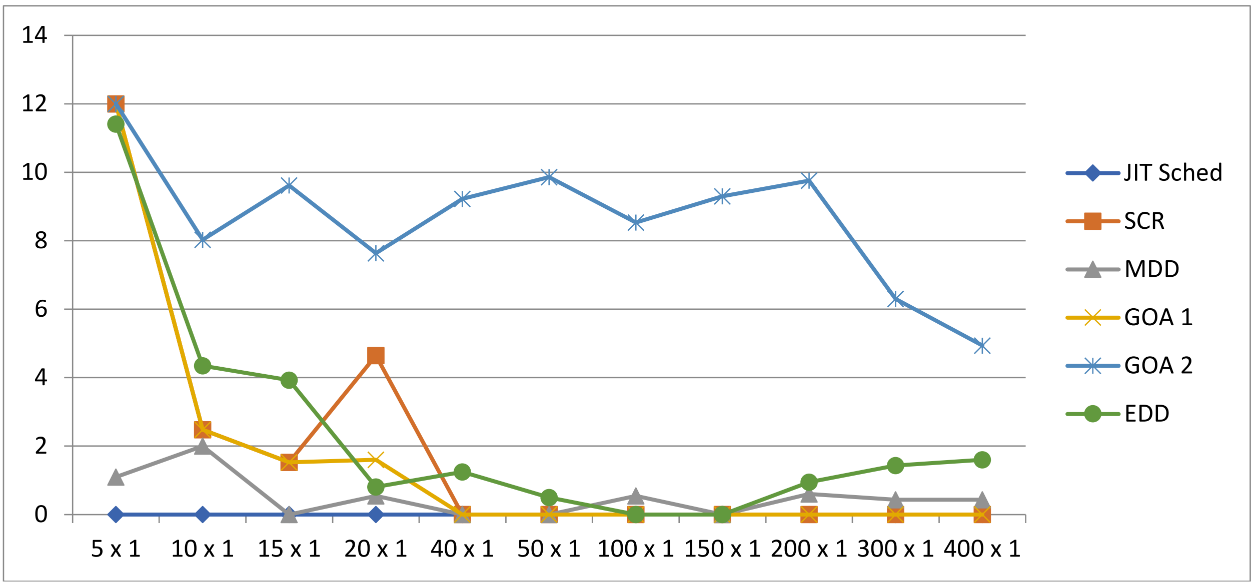

Fig. 3. The plot of Conditional Earliness against the problem sizes for the earliest Due window

Figure 3 illustrates the plot of conditional earliness against the problem sizes. The plot shows that for problem sizes$5\leq n \leq 15$, all the solution methods have a higher deviation from the optimal with the GOA 2, the highest, and MDD exhibits the lowest deviation. The t-test shows that EDD, SCR, and GOA 1 solution methods are not significantly different from the imaginary JIT schedule for problem sizes; $20\leq n \leq 400$. The approximate P-value (one-tail) for both the SCR and GOA 1 is **0.175.**

Analysis of the results based on the original due window

The plot of conditional tardiness against the problem sizes(Figure 4) for the original due window shows little deviation among all the heuristics as well as the optimal JIT schedule for problem sizes $5\leq n \leq 15$. As problem sizes, n increases, a higher deviation is noticed between the JIT schedule and other solution methods. At higher problem sizes $n\geq 100$, GOA 2 has the lowest deviation from the JIT. However, at $n\geq 300$, the EDD shows a positive deviation in performance.

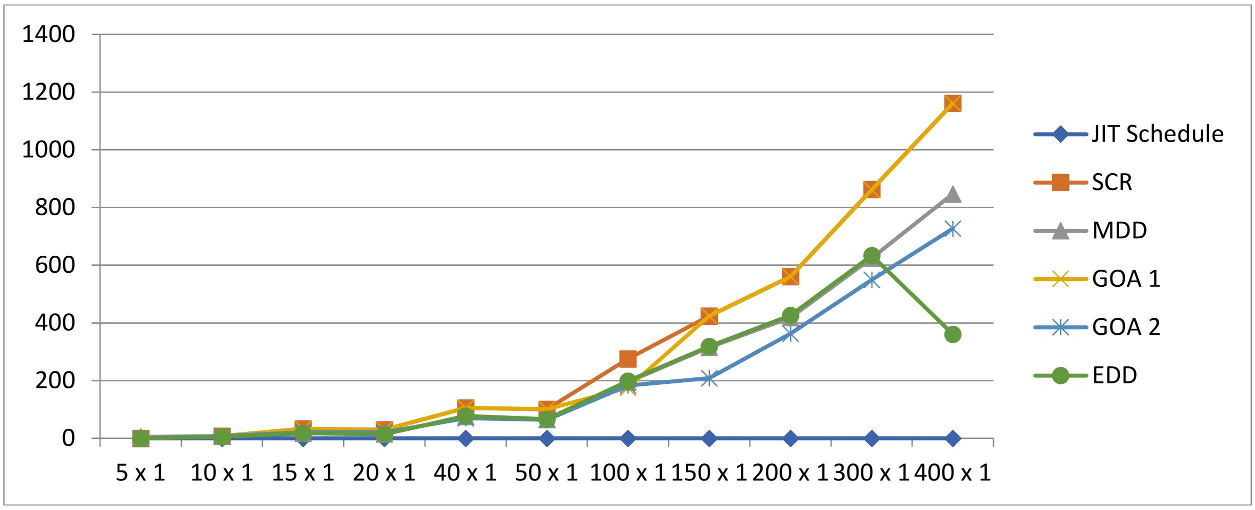

Fig 4. The plot of Conditional Tardiness values against the problem sizes for the original Due window

Furthermore, for problem sizes $5\leq n \leq 15$, the t-test shows that all the solution methods are not significantly different from the JIT schedule. The P values of the solution methods are shown in Table 13

Table 13. The P values for solution methods for problem sizes $5\leq n \leq 15$

Solution method	P values (One Tail)	Inference
GOA 2	0.1281	Not Significantly difference
EDD	0.1416	Not Significantly difference
MDD	0.0595	Not Significantly difference
SCR	0.1526	Not Significantly difference
GOA 1	0.1516	Not Significantly difference

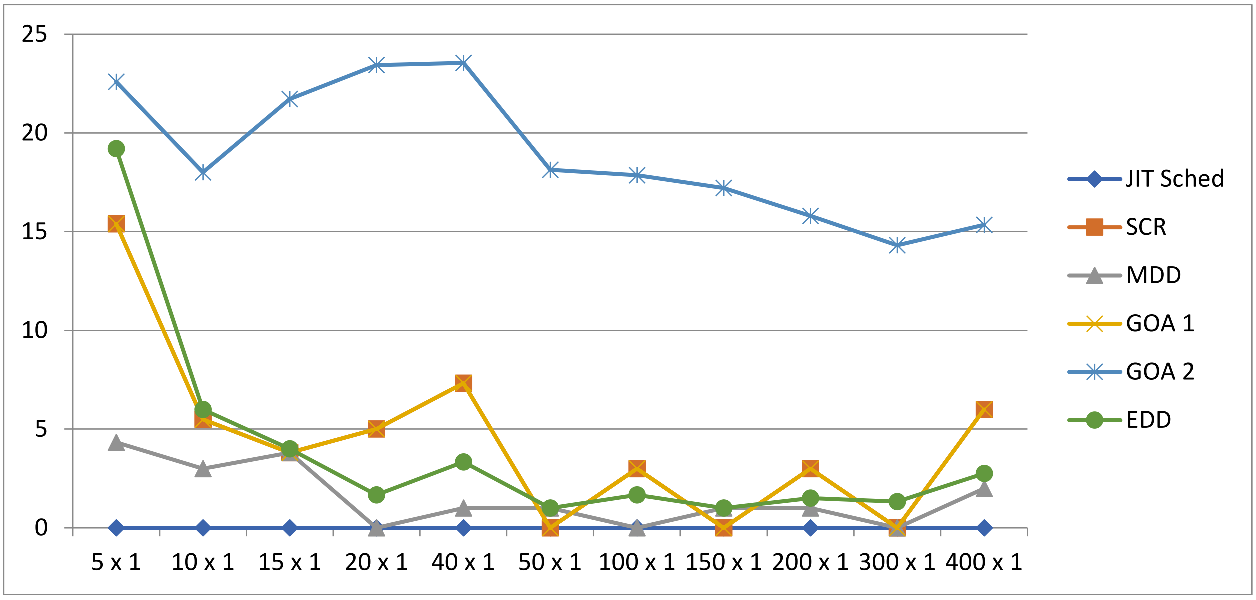

Fig. 5. The plot of Conditional Earliness against the problem sizes for the Original Due window

The plot of conditional earliness against the problem sizes for the original due window (Figure 5) shows an alternating in the performance of all the heuristics. GOA 2 shows the highest deviation for all the problem sizes. The t-test(using Paired Two Sample for Means) at 99% also reveals that all the solution methods are significantly different from the t-test with their respective P- values less than 0.05.

Analysis of the results based on the latest due window

The plot of conditional tardiness against the problem sizes reveals a small deviation for problem sizes $5 \leq n \leq 20$among all the solution methods including the imaginary JIT schedule. As $n > 20$, a more feasible deviation is observed between the JIT plot and other solution methods. However, GOA 2 heuristics shows the lowest deviation from the JIT schedule while the SCR, MDD, and GOA 1 solution methods align.Table 14 shows the P-values for all the solution methods against the imaginary JIT schedule when subjected to a t-test at a 90% significant difference for problem sizes $5 \leq n \leq 20$.

Table 14. P-values for all the solution methods against the imaginary JIT schedulefor $5 \leq n \leq 20$.

Solution Methods	P values	Inference
GOA 2	0.1116	Not Significantly difference
EDD	0.1955	Not Significantly difference
MDD	0.1674	Not Significantly difference
SCR	0.02	Significantly difference
GOA 1	0.0308	Significantly difference

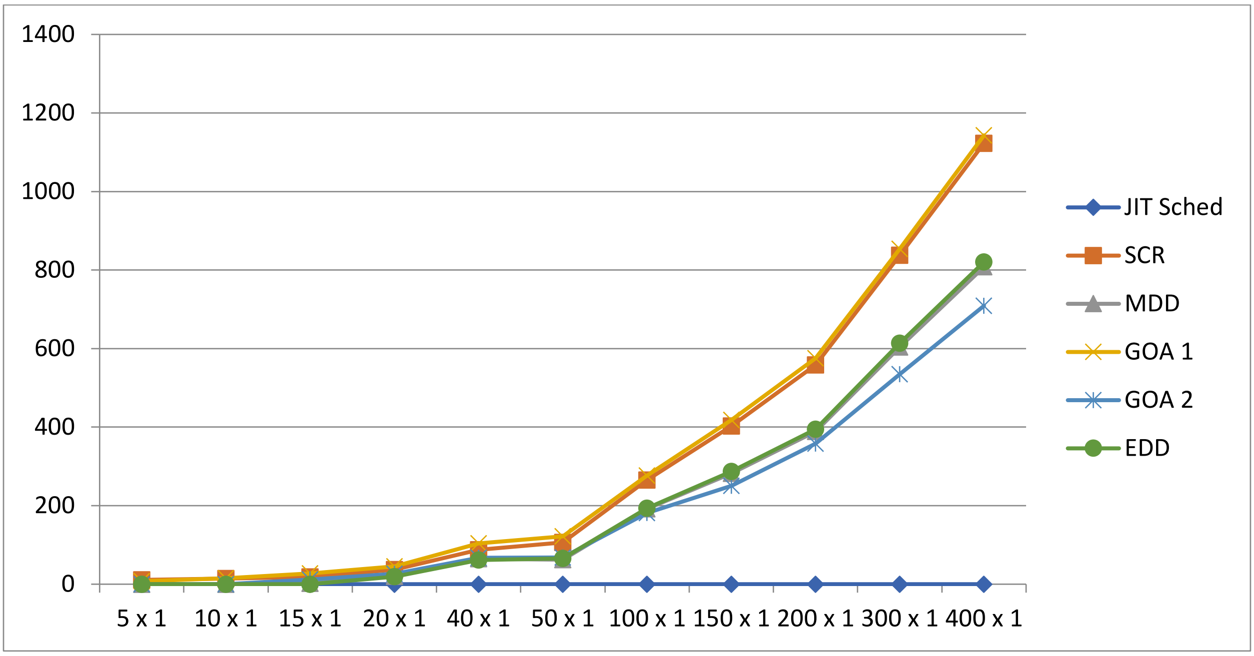

Fig. 6. The plot of Conditional Tardiness against the problem sizes for the latest Due window

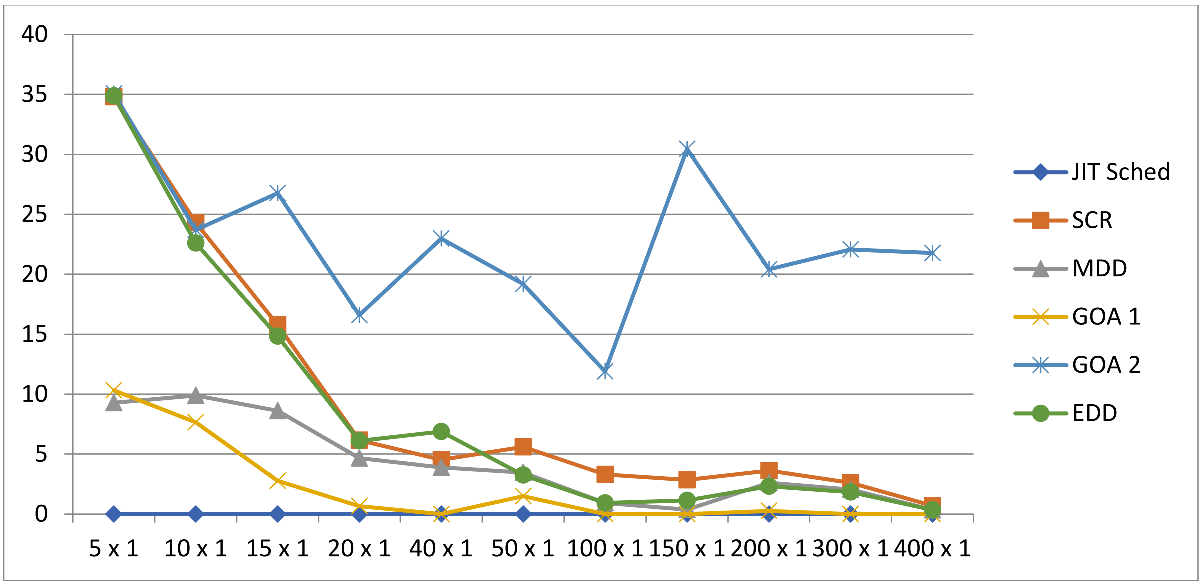

Fig. 7. The plot of Conditional Earliness against the problem sizes for the latest Due window

The plot of the conditional Earliness against the problem sizes shows that for the problem sizes; $n \leq 20$, there is a very high deviation between the imaginary JIT schedule and all other solution methods. However, when $n > 20$, the GOA 1 performance improves and almost aligns with the JIT schedule. The t-test also reveals that for problem sizes $20 \leq n \leq 400$, only the GOA 1 heuristic yields a result that is not significantly different from the JIT schedule. Table 15 shows the result of the t-test of GOA 1 against the JIT schedule.

Table 15. The result of the t-test of GOA 1 against the JIT schedule.

	JIT	*GOA 1*
Mean	0	0.3
Variance	0	0.287857
Observations	8	8
Pearson Correlation	#DIV/0!	
Hypothesized Mean Difference	0	
Df	7	
t Stat	-1.58153	
P(T<=t) one-tail	0.078885	
t Critical one-tail	1.894579	
P(T<=t) two-tail	0.15777	
t Critical two-tail	2.364624	

Conclusion and Recommendation

In this paper, we reduced a multicriteria scheduling problem that involves four due date-based performance measures to an equivalent bicriteria problem of minimizing conditional mean Tardiness and conditional mean earliness. Two proposed solution methods named GOA 1 and GOA 2 as well as some other dispatching heuristics were benchmarked against the ideal JIT schedule (a schedule with zero tardiness and zero earliness). The results show that the proposed heuristics yielded results that are not significantly different from the optimal in some instances. Further research can focus on implementing the results in terms of the Linear composite objective function after carrying out the normalization procedure for the two performance measures involved in the reduced bicriteria problem.

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