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Performance Comparison of Max-Chart and AEWMA-Max Chart in Monitoring Temperature

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Abstract. Traditional control charts often struggle to simultaneously detect both small and large shifts in process parameters. This study proposes the AEWMA-Max method, an improved variant of the Adaptive Exponentially Weighted Moving Average Max (AEWMAM) chart, to address this challenge in the context of monitoring temperature fluctuations in vannamei shrimp ponds. The research employs a data set containing 26 out-of-control and 64 in-control temperature measurements. AEWMA-Max utilizes a dynamically adjusted smoothing parameter (L) to achieve optimum sensitivity for both subtle and significant deviations from the desired temperature range. Through extensive simulations and comparisons with existing control charts, L = 2.642 was identified as the most effective value for this specific application. The proposed AEWMA-Max method demonstrably outperforms traditional charts in detecting both small and large temperature shifts, offering enhanced process control and potentially improved shrimp survival rates.

Introduction

Ponds are one of the most popular places for cultivation in Indonesia. Ponds are artificial ponds that are usually found in coastal areas and are used as a means of aquaculture [1]. One of the aquaculture commodities that farmers are interested in is the *vannamei* shrimp *(Litopenaeus Vannamei*).

Vannamei Shrimp *(Litopenaeus Vannamei*) is a type of shrimp that has been developed with intensive technology, and the development is carried out only in brackish water media, *vannamei* has a high tolerance for temperature to live [2]. One of the determining factors for the success of aquaculture in ponds is the water temperature in the pond. Temperature is a water quality parameter that has an important role in the resistance and growth of *L. vannamei* [3].

Statistical Process Control (SPC) is a technique statistic That Lots used to ensure that something process fulfills certain criteria. SPC themselves is used To identify shift processes quickly and minimize the level of product damage [4]. SPC is a production method to achieve maximum efficiency, productivity, and quality to produce competitive products. The control chart is one of the main techniques in SPC), and is a very useful process monitoring technique [5].

The control chart, originally known as the Shewhart control chart, was a method devised by Walter Andrew Shewhart to increase the robustness of the current transmission system [6]. One of the statistical methods that is often used in solving problems with the quality control of the production process is the control chart. The control chart is a tool used for process monitoring where the process is running from time to time so that you can see the quality of the results of a production process, regardless of whether it is statistically controlled [7]. Control charts are divided into two, namely univariate and multivariate control charts. The univariate control chart is a control chart that uses one quality characteristic. The multivariate control chart is a control chart where more than one quality characteristic is used [8].

The monitoring process is carried out partially, namely on the mean and variability [9]. Several studies on univariate simultaneous control charts to monitor the mean and process variability have been carried out by [10], namely the Max chart which combines the control chart $\bar{X}$ and S into a

control chart to monitor the mean and process variability together. The max chart method uses the maximum function for two statistics [11].
The development of the control chart begins by differentiating the two variations that are more effective in detecting small process shifts, namely the Cumulative Sum (CUSUM) control chart by Page in 1954 and the Exponentially Weighted Moving Average (EWMA) control chart by SW Roberts in 1959. Shewhart control chart with data observed at the current time can detect a shift in the process mean or a large variation, while detecting a shift in the process mean or a small variation using the EWMA charts [12]. However, if in production there are small shifts in the mean value, then the Shewhart control chart is not good enough to detect it. Therefore, an EWMA control chart is needed which can overcome the problem of variation in production results when there is a small shift in the mean value of production.
A control chart based on Adaptive EWMA-Max (AEWMAM) is a method of development by combining two methods, namely the Max Chart method [10] with the Adaptive EWMA method of research [13]. On the other hand, the AEWMA control chart is also undergoing development. Considering the AEWMA control chart only works well on small mean shifts but not on large shifts[14]. AEWMA-Max is superior to other existing simultaneous charts in that by detecting small shifts in process parameters it is also good at detecting large shifts [15].
Based on the description of the explanation of the idea above, the researcher proposes to find the best combination of parameters by comparing control charts based on Adaptive exponentially weighted moving average max (AEWMAM) Charts and Max-Charts with real data (temperature) on the *vannamei* shrimp ponds of CV Air Mas Jaya. This chart is expected to be efficient in controlling the mean and variance simultaneously so that it has better performance.

Related Work

There are several studies in the literature review that have discussed theory development before and up to the AEWMA-Max chart, here are examples:
The Shewhart control chart with the observed data at the current time is able to detect shifts in the process mean or large variations, while detecting shifts in the process mean or small variations using the AEWMA charts [16].
Several studies on univariate simultaneous control charts to monitor the mean and process variability have been carried out by [17], namely the Max chart which combines the control chart $\bar{X}$ and S into a control chart to monitor the mean and process variability together.
In research[16] the application of data to the AEWMA method is good for use on data that has changing characteristics due to the relationship with external factors with a fairly high intensity. In his research, the data used is turbine temperature, turbine failure, and the number of failures in wind generators.
AEWMA schemes are superior to conventional EWMA in the reduction of many adverse effects at fixed States, and the evaluation of multiple graphs using fixed sizes [13]. Research [18] developed an Adaptive Exponentially Weighted Moving Average-Max (AEWMA-Max) control chart which is an extension of the Max chart where the AEWMA technique is applied to Max chart statistics. AEWMA-Max is superior to other existing simultaneous charts in that by detecting small shifts in process parameters it is also good at detecting large shifts[19]. On the other hand, the AEWMA control chart is also undergoing development. Considering the AEWMA control chart only works well on small mean shifts but not on large shifts.

Exponentially Weighted Moving Average (AEWMA)

A control chart based on Adaptive EWMA-Max (AEWMAM) is a method of development by combining two methods, namely the Max Chart method [10] with the Adaptive EWMA method of research [13]. The control chart is able to work well on small and large shifts that are the basis of Adaptive the AEWMAM statistical formula which is defined as follows:

$$AEWMAM_i = \left(\left(1 - \left(\frac{\phi_{hu}}{(e_i)}\right)\right) \times AEWMAM_{i-1}\right) + \left(\left(\frac{\phi_{hu}}{(e_i)}\right) \times M_{i-1}\right),$$
$$i = 1,2,3, \dots, m \tag{1}$$

$$\phi_{hu}(e) = \begin{cases} e + (1-\lambda)k & , if\ e < -k \\ \lambda e & , if\ |e| \le k \\ e - (1-\lambda)k & , if\ e > k, \end{cases} \tag{2}$$

$$e_i = M_i - AEWMAM_{i-1} \tag{3}$$

where $\phi_{hu}(e)$ is a Huber score function which is one of the best functions in providing performance at different shift sizes, and does not completely ignore the previous observation process [13]. The Upper Control Limit (UCL) and the lower control limit on the AEWMAM Control chart can be defined as follows.

$$UCL = \mu_M + h\sigma_M \tag{4}$$

$$\mathrm{h} = k\sqrt{\frac{\lambda}{(2-\lambda)}} \tag{5}$$

Max Chart

Control chart Max-$\bar{X}S$ or Max Chart is a univariate simultaneous control chart that has the function of monitoring process averages and variability in one chart. Given that X denotes certain characteristics of a process, μ is the average of the process, and σ is the standard deviation of the process. Given X_{ij}, i = 1, 2, 3, … and j = 1, 2, …, n_i is the measurement result of X with sample size n_i and i denotes the sample number index $X_{i1}, \dots, X_{in_i}$ is a random sample from a normal distribution with mean $\mu + a\sigma$ and standard deviation $b\sigma$ with $a = 0$ dan $b = 1$ by indicating the process is in-control, besides the process has shifted.

Given and $\bar{X}_i = \frac{(X_{i1} + \dots + X_{in})}{n}$ is the average of the i-th sample and $S_i^2 = \frac{\sum_{j=1}^{n}(X_{ij} - \bar{X})^2}{(n-1)}$ is the variance of the i-th sample. Therefore, the simultaneous control chart is formulated as follows:

$$U_i = \frac{(\bar{X}_i - \mu)}{\frac{\sigma}{\sqrt{n}}}, \tag{6}$$

$$V_i = \Phi^{-1}\left\{H\left(\frac{(n_i-1)S_i^2}{\sigma^2}; n-1\right)\right\}, \tag{7}$$

Where $\Phi(z) = P(Z \le z)$ for $Z \sim N(0,1)$, the standard normal distribution, $\Phi^{-1}(.)$ is the inverse function of $\Phi(.)$, and $H(w, v) = P(W \le w|v)$ for $W \sim \chi_v^2$, the chi-squared distribution with degrees of freedom v.

U_i and V_i are independent because $\bar{X}_i$ and S_i^2 are also independent, when $a = 0$ and $b = 1$, $U_i \sim N(0,1)$ and $V_i \sim N(0,1)$. The advantages of transforming $\bar{X}_i$ into Ui and S_i^2 becoming Vi are: (1) the distribution of U_i and V_i is independent for the sample size n_i when $a = 0$ dan $b = 1$, so that the case of variable sample size can be handled easily; (2) U_i and V_i have the same distribution so that a single control chart can be formed to monitor process averages and variability. In particular, the statistics for the simultaneous control chart are defined as follows:

$$M_i = max(|U_i|, |V_i|) \tag{8}$$

The value of the test statistic M_i becomes large when the process shifts away from μ and/or when the process variability increases or decreases. On the other hand, the value decreases M_i when the mean and process variability remain close to their respective target values. So that the control limits are as follows

$$UCL = \left\{\chi^2_{\sqrt{1-\alpha},1}\right\}^{\frac{1}{2}}. \tag{9}$$

Thus, the Upper Control Limit (UCL) on the Max chat can be easily determined for various types I error probabilities (α).

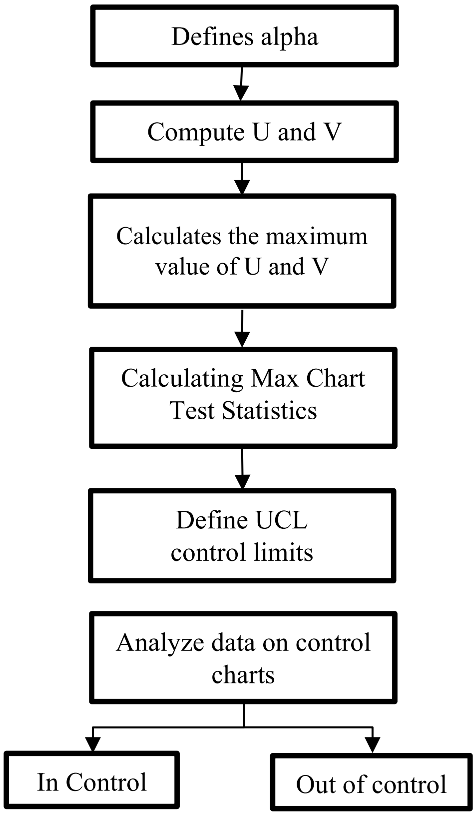


Fig. 1 Detection scheme flowchart using Max Chart

Applying AEWMA-Max Chart on pond temperature data

Bangkalan is one of the regencies in East Java Province, specifically on Madura Island. This temperature data collection is in the Klampis sub-district, Bator sub-district, where geographically, the Klampis sub-district is located on the coast which is very suitable for *vannamei* shrimp cultivation. One type of shrimp that has the potential to be developed is *vannamei* shrimp (*Litopenaeus vannamei*). This cultivation develops with intensive technology but is limited to the upper middle class (capital intensive) [2]. One of the determining factors for the success of aquaculture in ponds is the temperature of the water in the pond [3]. Temperature is a water quality parameter that has an important role in the development and growth of *L. vannamei*. There are 2 measurements of temperature in one day, in the morning and the evening. Spreading *vannamei* shrimp seeds until they are ready for harvest takes approximately 4 months.

Given X_1,X_2,…,X_i denotes the quality characteristics of a process with μ denotes the mean of the process and σ denotes the standard deviation of the process where i denotes the sample number index. AEWMA statistics with smoothing constant λ are shown in the following equation[20].

$$Z_i = (1 - \lambda)Z_{i-1} + \lambda X_i \tag{10}$$

with $0<\lambda\leq 1$. When the process is in-control, the mean and variance of the AEWMA statistics are

$$E(Z_i) = \mu_0 \tag{11}$$

$$Var(Z_i) = \sigma^2 \frac{\lambda[1-(1-(1-\lambda)^{2i}]}{2-\lambda} \tag{12}$$

where μ_0 and σ^2 denote the mean and variance of the target Xi respectively. If these two values are unknown, they can be estimated using a preliminary sample.

AEWMA-Max chart is an expansion control chart from Max chart by applying the AEWMA technique to Max chart statistics. The AEWMA-Max statistic is defined in the following equation[18].

$$AEWMAM_i = \left((1-(\frac{\phi_{hu}}{(e_i)})) \times AEWMAM_{i-1}\right) + \left((\frac{\phi_{hu}}{(e_i)}) \times M_{i-1}\right), \tag{13}$$

where G_0=1.128379 is the initial value. U_i and V_i are independent, when δ=0 and γ=1 then $U_i \sim N(0,1)$ and $V_i \sim N(0,1)$.
So that the cumulative Distribution Function (CDF) from Max Chart statistics, which $M_{(n_i)}$ is as follows

$F(y) = P\left(M_{(n_i)} \leq y\right)$
$= P(|U_i| \leq y, |V_i| \leq y)$
$= P(|U_i| \leq y)P(|V_i| \leq y)$
$= [2\mathbf{\Phi}(y) - 1]^2, y \geq 0$

Furthermore, the Probability Density Function (PDF) of statistics $M_{(n_i)}$ is the derivative of the F(y; σ_{U_i}) shown in the equation.

$f\left(y; \sigma_{U_i}\right) = 4\Phi(y)[2\mathbf{\Phi}(y) - 1]$

Using numerical computation, the mean and variance of are as follows$M_{(n_i)}$

$$E\left(M_{(n_i)}\right) = \frac{\sum_{i=1}^{n} x_i}{n} \tag{14}$$

$$V\left(M_{(n_i)}\right) = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1} \tag{15}$$

Using numerical computation, the mean and variance of Mi are as follows

$E(M_i)=\int_0^\infty yf(y)dy$, =1,128379,
$Var(M_i)=\int_0^\infty y^2 f(y)dy$, =0,363381.

E(Yi) can be distributed with E($M_{(n_i)}$) and $\sqrt{Var(Y_i)}$ can be substituted with $\sqrt{Var(M_{(n_i)})}$, $\sqrt{\frac{\lambda\,[1-(1-\lambda)^{2i}]}{2-\lambda}}$so the control limits for the AEWMA-Max chart are as follows.

$$UCL_i = E(G_i) + L\sqrt{Var(G_i)} = E\left(M_{(n_i)}\right) + \sqrt{V\left(M_{(n_i)}\right)}L\sqrt{\frac{\lambda\,[1-(1-\lambda)^{2i}]}{2-\lambda}}$$

For increasingly large i, the control limits will approach steady-state values, ie

$$UCL = 1.128379 + h.0.60281. \tag{16}$$

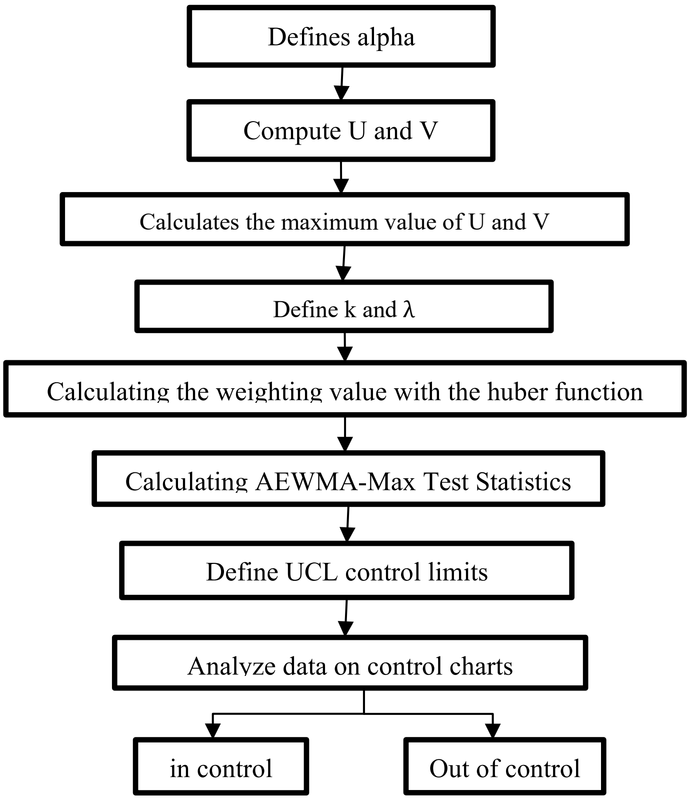


Fig. 2 Detection scheme flowchart using AAEWMA-Max

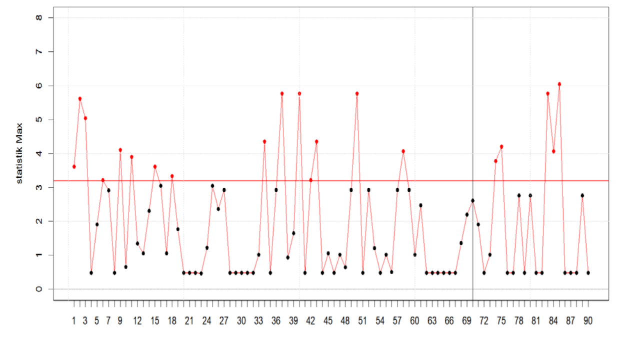
Performance evaluation of Max Chart and AEWMA-Max Chart

In this study, the application assists the process of data input and data processing. Data processing in Excel, namely inputting temperature data for *vannamei* shrimp ponds at CV Air Mas Jaya, which has been running for about 2 months. As well as data processing in the R program to issue the best combination between Max-Chart and AEWMA-Max on shrimp pond temperature data and find out how out of control the temperature data is in CV Air Mas Jaya *vannamei* shrimp ponds. In the simulation scenario on Max-Chart and AEWMA-Max, on AEWMA-Max there is a comparison of the best parameter combinations. The following is the parameter table for AEWMA-Max:

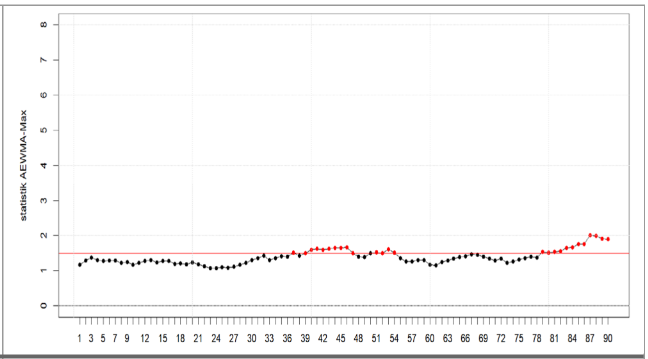
Table 1 Lambda and L parameter values used in the AEWMA-Max method

λ	0.1	0.2	0.3	0.4
k	2.642	2.934	3.092	3.212

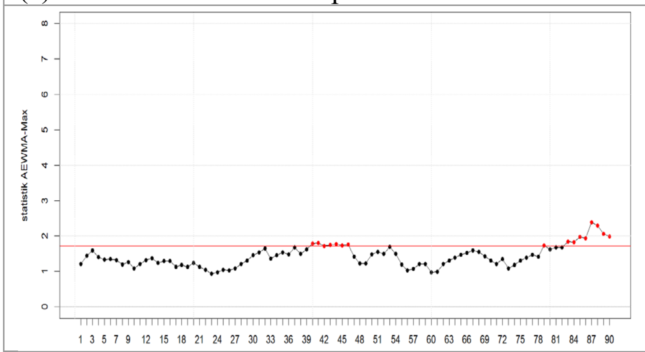
In all scenarios, the best method for monitoring the process is by implementing temperature data which can be seen from the amount of out-of-control data on the control chart graph. Here are the best results on the temperature data in the image below.



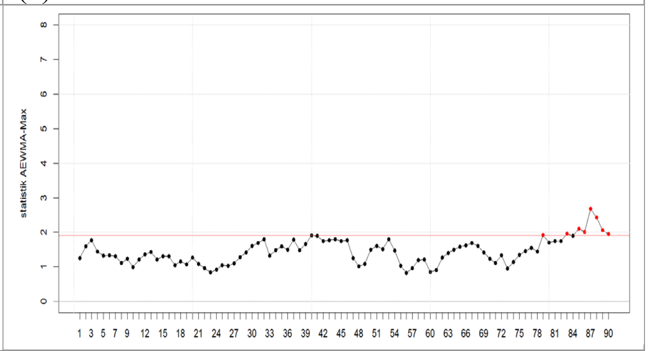
(a) Max-Chart on temperature data



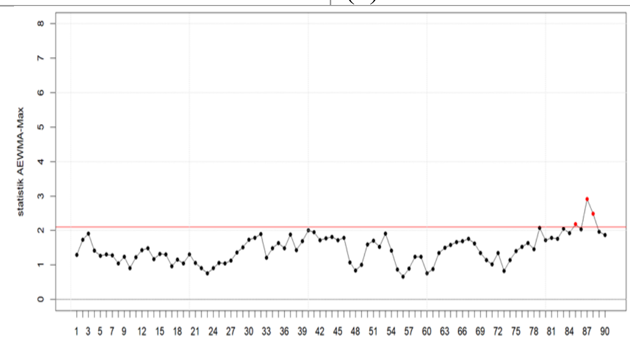
(b) AEWMA-Max with $\lambda = 0.1$ and k=2.642



(c) AEWMA-Max with $\boldsymbol{\lambda} = \mathbf{0.2}$ and k=2.934



(d) AEWMA-Max with $\lambda = 0.3$ and k=3.092



(e) AEWMA-Max with $\boldsymbol{\lambda} = \mathbf{0.4}$ and k=3.212

Fig. 3 AEWMA-Max control chart and Max Chart for temperature data

Table 2 Amount of data Out of Control and In Control on AEWMA-Max control chart and Max Chart

MAX-Chart	Out of Control	20
	In Control	70
AEWMA-MAX (λ=0.1 and L=2.642)	Out of Control	26
	In Control	64
AEWMA-MAX (λ=0.2 and L=2.934)	Out of Control	16
	In Control	74
AEWMA-MAX (λ=0.3 and L=3.092	Out of Control	22
	In Control	53
AEWMA-MAX (λ=0.4 and L=3.212)	Out of Control	3
	In Control	87

The explanation in the table above is as follows, in the temperature data with the Max-Chart method there are 20 data out of control and 70 data in control. In the AEWMA-Max method with λ =0.1 and L=2,642, 26 data were out-of-control and the number of data controlled was 64. In the AEWMA-Max method with λ =0.2 and L=2.934, the uncontrolled data is 16 and the number of controlled data is 64. In the AEWMA-Max method with λ =0.3 and L=3.092, the data out of control is 22, and the amount of data in control is 53. AEWMA-Max with λ =0.4 and L=3.212, the uncontrolled data is 3 and the number of controlled data is 87.

The temperature data on shrimp farms CV Air Mas Jaya *vannamei* tend to decrease in value, then the Max-Chart control chart is not appropriate because the amount of data out-of-control too much and too little data in-control, while the Aewma-Max control chart matches the best combination of parameters with λ =0.1 and L = 2.642, with 26 data out-of-control and 64 data in-control. AEWMA-Max with λ =0.1 and L = 2.57 is the best combination of the other two combinations because it has the largest and smallest out-of-control data on the control data, it can be seen that this method is the most suitable and sensitive method for temperature data in shrimp ponds *vannamei* CV Air Mas Jaya.

Conclusion

In the cultivation of *Litopenaeus vannamei* shrimp, water temperature in brackish water ponds plays a crucial role in their resilience and growth. This study sought to optimize temperature monitoring by comparing two control charts: Adaptive Exponentially Weighted Moving Average Max (AEWMAM) and Max-Chart. Our analysis identified the AEWMAM-Max chart with α = 0.7 and k = 2.642 as the most effective, with 26 out-of-control signals effectively highlighting potential deviations from optimal temperature ranges for shrimp health and growth. This finding contributes to improved precision and timeliness of intervention in shrimp aquaculture, potentially leading to enhanced resilience and productivity in *Litopenaeus vannamei* cultivation.

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