

Dynamic Modeling and Analysis of Rotor-Bearing System with Localized Defect in Rotating Machinery

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Keywords: Machinery diagnostic, Bearing faults, Vibration analysis, Signal Analysis.

Abstract. The numerical model is developed to study the vibration response due to the localized defect of ball bearing in rotating machinery. In order to simulate the dynamic response, the equations of motions are developed based on the rotor-bearing system where two identical rotors mounted on symmetric flexible shaft and supported by ball bearings are considered in this model. The presence of defect is introduced on a bearing outer raceway and lubrication effect between bearing components is also included. The numerical results are obtained by applying Runge–Kutta method to solve governing equations of motions. It has been observed that the vibration spectrum of the ball pass frequency outer race and its harmonics for the defect bearing is relatively higher than the good one. Moreover, this dynamic model can effectively enhance the understanding of vibration responses for good and defective bearing.

Introduction

The rolling element bearing is a major component in rotating machinery. The continuous dynamic and excessive load in the rolling element bearing can cause a localized defect. The localized defect in bearing may be developed during manufacturing process or generated during machine operation. This defect may lead to the failure of other components or even rotor-bearing system. Generally, the rotor-bearing systems generate vibrations even in good condition through their dynamic behaviors and their component interactions. Thus, the dynamic model of rotor-bearing system has an important role for condition monitoring and fault diagnosis of rotating machinery. Many researchers [1-10] have tried to develop the dynamic model to simulate vibration response with localized defect in bearing. For instance, Choudhury and Tandon [1] considered a local defect in a rotor-bearing by using three degrees of freedom in dynamic model. This research concentrated on an NJ 204 cylindrical roller bearing with a normal clearance test. The output frequency components in the empirical spectra are found to be the same as in the results from dynamic model. Patel et al. [2] used the Runge-Kutta method to solve coupled equations of motions of deep groove ball bearing based on single and multiple defects on the races. The results show healthy bearing behavior that presents frequencies with harmonic phenomena. Patil et al. [3] used a 6305 deep groove ball bearing to analyze a dynamic model for investigating a localized defect by considering the contact model between the ball and the races as a non-linear spring condition. The conclusion illustrated a mathematical model for the ball bearing vibration based on the defect situation that provided the vibrated amplitude levels for the outer race defect higher than the inner race defect. Liu et al. [4] researched a localized defect of ball bearing by using a piecewise response function. The results identify the relationship between the impulse response and the size of the local defect. Kankar et al. [5] applied the numerical integration technique Newmark- β with the Newton-Raphson method to solve the nonlinear differential equations for ball bearing surface defects. The outcomes predict a discrete spectrum with specific frequencies of waviness on the inner and outer races of a ball bearing. Cui et al. [7] analyzed the local fault of rolling-element bearing with five degrees of freedom, apply Hertz contact methodology, and relevant bearing techniques of kinematics and dynamics to solve a nonlinear vibration model. The results show a similar mechanical behavior between experimental and simulation signals. Cui et al. [8] are concerned

about the outer race defect size that impacts vibration response signals. The results show that the width of the bearing fault is related to the vibration signals. Shah and Patel [10] investigated dry and lubricated deep groove ball bearings to predict dynamic models in healthy and defective bearing operations. The results show good relation between theoretical and experimental vibration behaviors. Zhang et al. [11] concerns three parameters (radial load, defect size, and rotation speed) for localized defects of rolling element bearing. The outcome provided the proposed model that matches the experimental data very well. Liu et al. [12] applied the mass distribution and energy method to analysis the rotor- bearing-housing defects. The results show that this method is efficient for structural optimization and fault detective of the rotor- bearing-housing system.

However, most researchers have developed models considering dynamic behavior of bearing solely or with a lumped mass of a shaft. Thus, the objective of this study is the development of the dynamic model of a rotor-bearing system having a localized defect in bearing element. The model in this work has considered a dynamic response of the whole flexible rotor-shaft system where a multimass rotor is divided into discrete mass stations. Then, the general equations of motions have been numerically computed using Runge-Kutta method in MATLAB. The vibration responses of defect free bearing and defect bearing have been compared and analyzed. The expected defect frequency and its harmonics have been noticed in the vibration spectrum of defect bearing.

Simulation Modeling

To study the vibration characteristic of rotating machinery due to the localized defect in rolling element bearing, the numerical model is developed by base on the bearing test rig, shown in Fig.1. The model consists of two identical rotors, a symmetric shaft, and two ball bearings. The rotor shaft is supported by two ball bearings where the left end is totally in good condition whereas the test bearing is placed on the right end. For bearing model, it is assumed that the inner race is tightly held to the shaft while outer race is rigidly fixed and stationary in bearing housing.

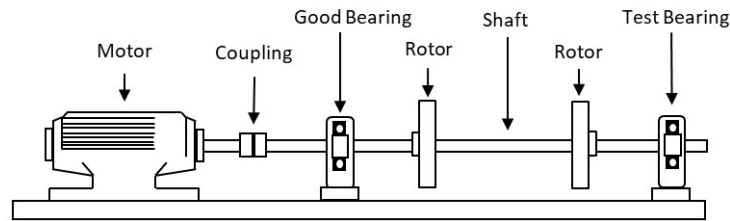


Fig.1 Schematic diagram of the bearing test rig

Numerical model of a rotor-bearing system

The equations of motions for a rotor-bearing system can be written in matrix form as

$$[M]\{\ddot{U}\} + [C]\{\dot{U}\} + [K]\{U\} = \{F(t)\} \quad (1)$$

where $[M]$ is the mass matrix of shaft, $[C]$ is the shaft damping matrix, and $[K]$ is shaft stiffness matrix incorporating an average bearing principal stiffness term. $\{U\}$ is the generalized displacement vector of rotor and $\{F(t)\}$ is the external force and mass imbalance excitation.

Nonlinear model of rolling element bearing

According to the Hertzian contact deformation theory [13], the non-linear relation load-deformation is given by

$$F = k_b \delta^n \quad (2)$$

where k_b is the load-deflection factor or constant for Hertzian contact elastic deformation, δ is the radial deflection or contact deformation and n is the load-deflection exponent which equals to 3/2 for ball bearing. The load-deflection factor k_b depends on the contact geometry between ball and each raceway and the material elasticity. The total factor between two raceways is the sum of the approaches between the rolling elements and each raceway.

$$k_b = \left[\frac{1}{((1/k_i)^{(1/n)} + (1/k_o)^{(1/n)})} \right]^n \quad (3)$$

k_i and k_o are inner and outer raceways to ball contact stiffness, respectively.

The radial deflection of the j^{th} ball takes into consideration the loading status is a function of the displacements of the inner race in the horizontal and vertical directions (x,y), the angular positions of the rolling elements Θ_j and the clearance C_r is given by

$$\delta_j = (x \cos \theta_j + y \sin \theta_j) - C_r \quad (4)$$

The angular positions of the j^{th} rolling element, Θ_j , can be expressed as a function of cage angular speed ω_c , number of rolling elements n_b , time period t , and the initial reference angular position Θ_0 .

$$\theta_j = \frac{2\pi(j-1)}{n_b} + \omega_c t + \theta_0 \quad (5)$$

The total nonlinear contact force be calculated from the sum of the contact restoring forces from each of the rolling elements. Thus, the contact forces exerted to flexible shaft in the x and y directions for a ball bearing with n_b balls are written as:

$$F_x = k_b \sum_{j=1}^{n_b} \gamma_j \delta_j^{3/2} \cos \theta_j \quad (6)$$

$$F_y = k_b \sum_{j=1}^{n_b} \gamma_j \delta_j^{3/2} \sin \theta_j \quad (7)$$

The contact deformation occurs only when the ball is located in the loading zone, and thus contact force is produced. Therefore, a switch function γ_j is introduced as follows:

$$\gamma_j = \begin{cases} 1, & \delta_j > 0 \\ 0, & \text{otherwise} \end{cases} \quad (8)$$

Outer race defect of rolling element bearing

When local defect occurs on the outer race of the faulty bearing, the defect can be defined as angular span of $\Delta\theta_d$, and initial angular position of θ_d as shown in Fig.2(a). In addition, this angular position of defect does not change with respect to shaft rotation due to stationary outer race. In effect, the rectangular spall is modeled as fault on outer race surface with the width W and the height of H as shown in Fig.2(b). When the j^{th} rolling element is in the site of defect, the additional deflection, C_d , can be computed by using geometric relation.

The defect switch β_j is defined the contact state of rolling elements over the defect as follows:

$$\beta_j = \begin{cases} 1, & \phi_d < \phi_j < \phi_d + \Delta\phi_d \\ 0, & \text{otherwise} \end{cases} \quad (9)$$

where the defect on the outer raceway is located between ϕ_d and $\phi_d + \Delta\phi_d$ which normally occurs in the load zone of the bearing. The deformation when the j^{th} rolling element enters the defect zone is expressed as

$$\delta_j = (x \cos \theta_j + y \sin \theta_j) - C_r - \beta_j C_d \quad (10)$$

The nonlinear contact force with the defect is calculated and substituted into dynamic equation of the nonlinear bearing contact force.

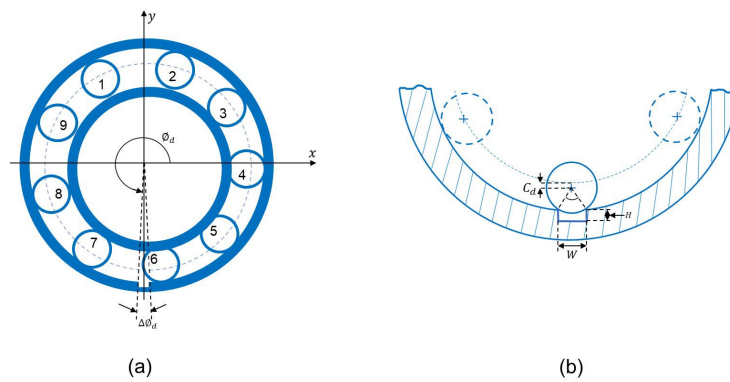


Fig.2 Schematic diagram of ball bearing: (a) angular position of defect on outer race and (b) additional deflection due to defect on outer race

Simulation Setup

The vibration responses of rotor-bearing system are obtained by solving the governing equations of motions and bearing contact restoring forces iteratively for each time step. The planar displacements, velocities, and accelerations are computed by the Runge-Kutta numerical method. In this simulation, the governing equations of motions have been iteratively solved by using ODE 45 solver in MATLAB with the time step (Δt) of 1 μ s.

Numerical Results

The schematic diagram of the bearing test rig shown in Fig.1 is used for studying the vibration responses. In this study, the numerical model has been developed including a shaft, two identical rotors, and two ball bearings. The 1.094-kg shaft has a length of 445 mm and a uniform cross section of 25.4-mm diameter. Each rotor has a mass of 0.778 kg, a diameter of 160 mm, and a thickness of 15.88 mm. The bearing parameters are given in Table 1.

Table 1 Parameters of the ball bearing

Parameters	Units
Inner raceway diameter (d_i)	31 [mm]
Outer raceway diameter (d_o)	46.35 [mm]
Ball diameter	7.9 [mm]
Contact angle (α)	0° (assumed)
Number of balls	9
Radial clearance (c_r)	10 [μ m]

To identify fault effect, the numerical vibration responses are computed for the bearing with defect and without defect. The simulations are performed under the same condition where the shaft rotating speed is kept at 1500 rpm (25 Hz). The vibration responses of the defect free bearing are shown in Fig.3(a) and 3(b) that the vibration response in both horizontal and vertical directions have periodic characteristics in the time domain.

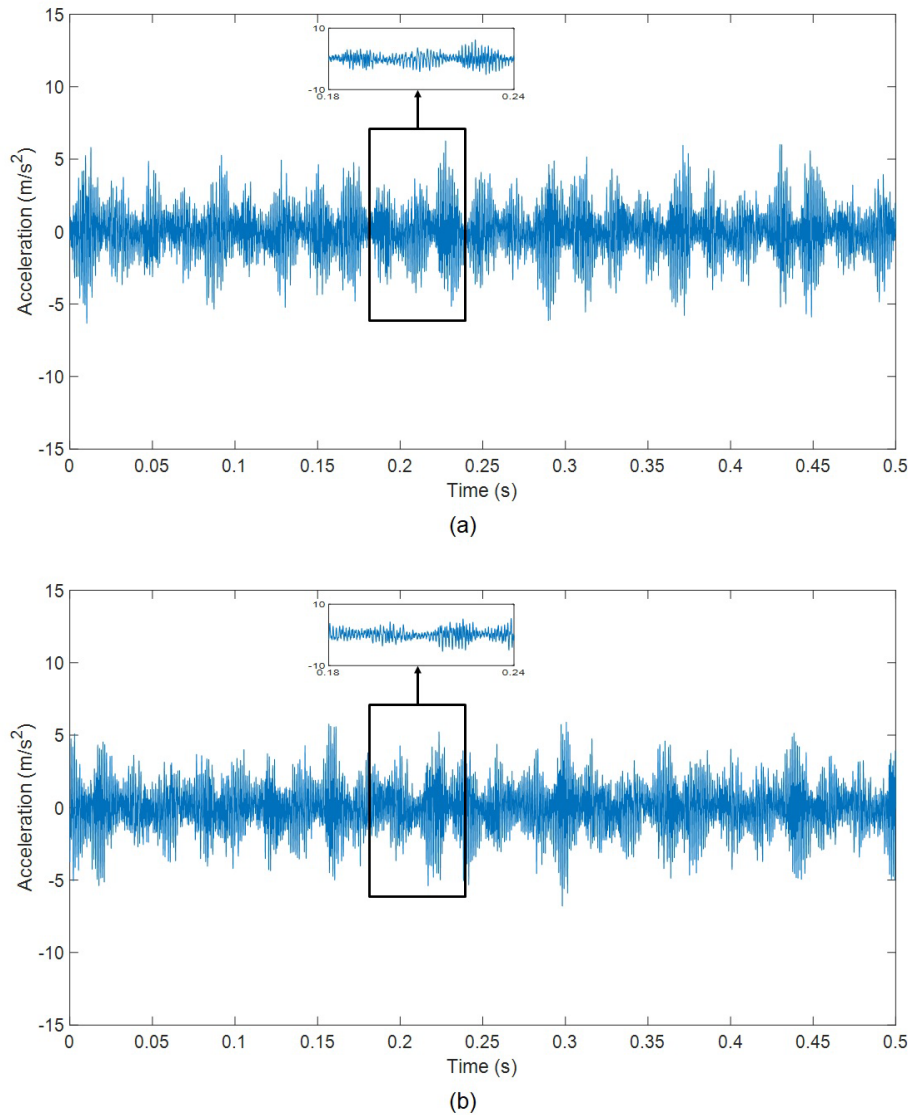


Fig.3 Vibration responses of numerical model with good bearing: (a) in horizontal direction (x) and (b) in vertical direction (y)

Similarly, Fig.4 shows the vibration responses (horizontal and vertical directions) for the test bearing with the outer race defect. Comparing Fig.4 with Fig.3, the overall amplitudes of vibration responses in time domain for defect bearing are higher than those with healthy one.

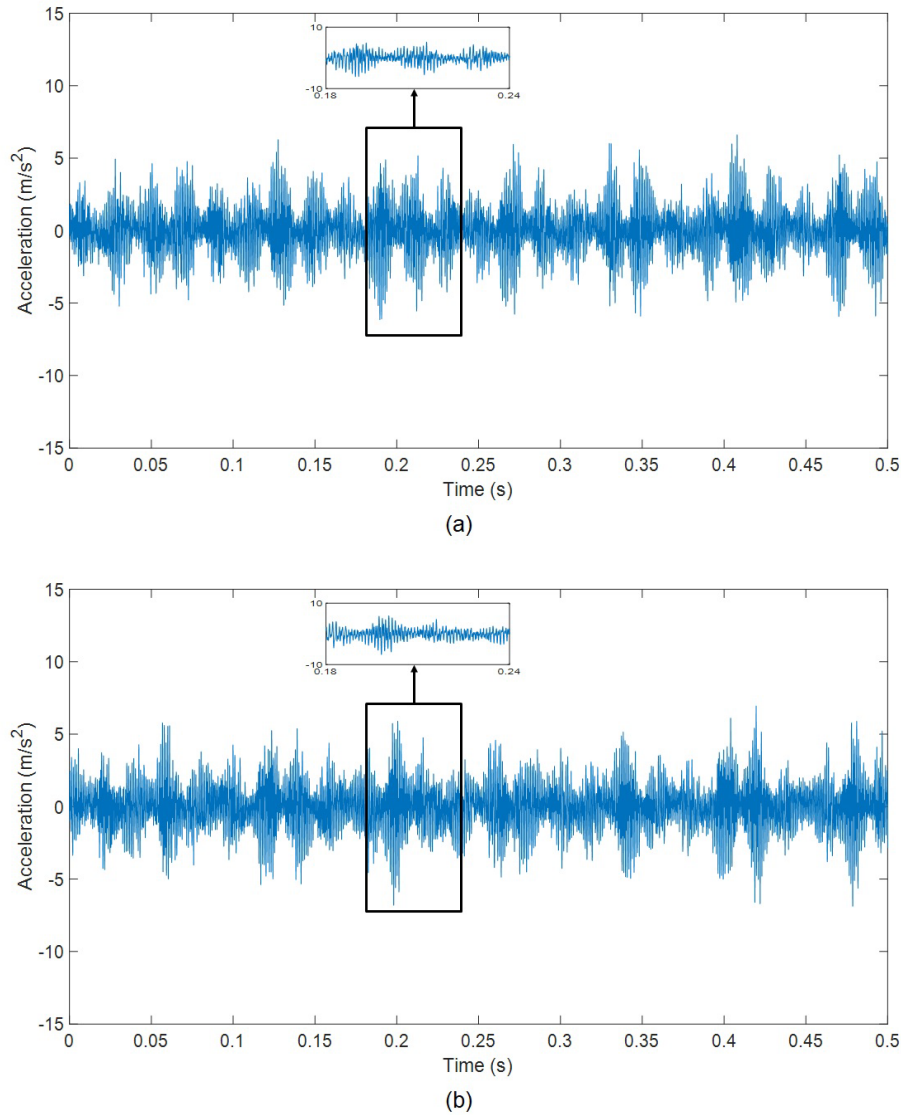


Fig.4 Vibration response of numerical model with defect bearing: (a) in horizontal direction (x) and (b) in vertical direction (y)

Hence, the results in time domain can be identified between good and defect bearing from the amplification of vibration responses. However, in order to investigate the defect components, the vibration responses need to be transformed to frequency domain. The frequency spectra of vibration responses for good bearing and defect bearing are shown in Fig.5(a) and 5(b), respectively. The characteristic components of shaft rotating speed are clearly observed at 24.91 Hz (f_s) in both Fig.5(a) and 5(b). Remarkably, the fault characteristic frequency can be calculated as 89.53 Hz which is same as ball pass frequency outer (f_{BPFO}). The spectra as shown in Fig. 5(b) are clearly visible at 89.53, 179, 269 Hz, 358, and 448 Hz which are corresponding to ball pass frequency outer (f_{BPFO}) and its harmonics frequency ($2*f_{BPFO}$, $3*f_{BPFO}$, $4*f_{BPFO}$, $5*f_{BPFO}$). The amplitudes at these frequencies are remarkably increasing when the localized defect is included in the simulation model. The excitation force due to the change of contact deformation exerts to the shaft component when a ball bearing element rolls over the defect. Thus, these periodic impacts would enhance the amplitudes of vibration response for defect bearing.

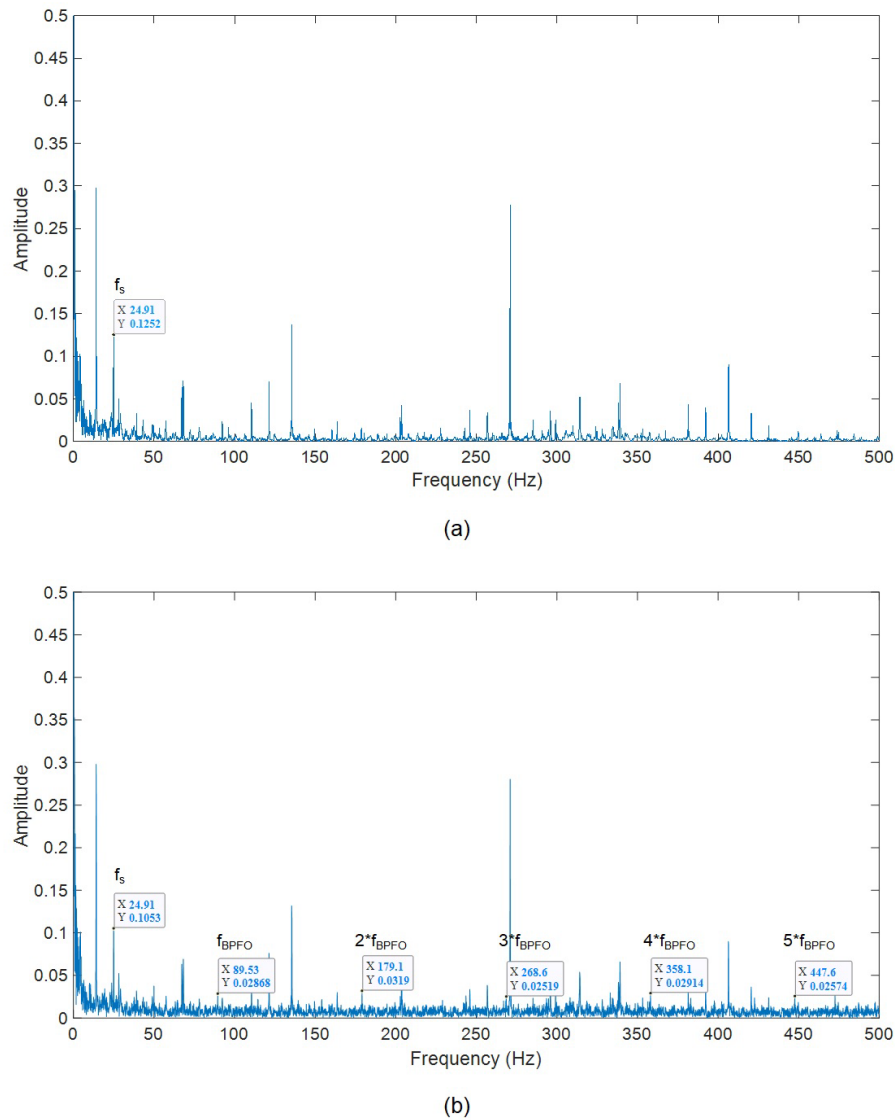


Fig.5 Frequency spectrum of vibration response: (a) good bearing and (b) defect bearing

Summary

This study develops the numerical simulation model which can be used to simulate the vibration responses of rotating machinery. The generalized equations of multimass rotor-bearing system including the effects of mass imbalance and nonlinear contact forces of bearings are generated. The nonlinear contact force of bearing is based on Hertzian contact deformation theory. The planar vibration responses have been obtained by solving the coupled equations of motions using the Runge-Kutta method in MATLAB. Then, the vibration results of good and defect bearing are compared. For the bearing without defect, the shaft rotating frequency (f_s) along with other frequencies can be observed in vibration spectrum while it is noticed that the characteristic defect frequency (f_{BPFO}) with its harmonics is visible in the case of outer race defect. Theoretically, this frequency is widely used as defective indicator of outer race ball bearing in vibration analysis. From the results, the numerical model is able to simulate the vibration responses of rotating machinery incorporated with good and defect bearing. Moreover, the understanding of vibration responses and defect characteristic can help preventing the catastrophic breakdown of the rotating machinery.

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