Key Engineering Materials
ISSN: 1662-9795, Vol. 1048, pp 63-71
doi:10.4028/p-Hm69aW

Submitted: 2025-12-20
Revised: 2026-02-13
Accepted: 2026-02-25
Online: 2026-04-14

Calibrating a Lode-Parameter-Dependent Damage Evolution Equation to Cold Extrusion Experiments

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Keywords: Ductile Damage, Lode-parameter, Parameter Identification, Cold Forging

Abstract. Forming processes significantly influence the product properties of a formed workpiece. Next to the effects of work hardening and residual stresses, the influence of ductile damage determines the final performance of a formed component. Thus, precise damage models are crucial for designing new forming process sequences. In general, this is achieved by modelling the evolution of damage as a function of hydrostatic and deviatoric stress, characterized by the stress triaxiality and the Lode-parameter. However, calibrating damage models to the effects of triaxiality and the Lode-parameter is not trivial, since experiments usually represent a combination of both influences. A recent experimental approach by the authors offers the possibility to vary the Lode-parameter in extrusion experiments while keeping the triaxiality constant. This paper aims to use this data of the isolated deviatoric effect on damage to calibrate a damage evolution equation. The model is calibrated to void area fraction measurements obtained by scanning electron microscopy of extruded case-hardening steel 16MnCrS5. For validation, the model predictions for non-constant Lode-parameter histories are compared to corresponding experiments. The model and experiments are in good agreement.

Introduction

Ductile damage does not only cause the onset of macroscopic fracture, but also affects the product performance of a formed component. By controlling the evolution of damage during a forming process, the impact toughness, fatigue strength and elastic stiffness can be increased [1]. To avoid excessive use of material when designing a formed component, the effect of ductile damage on product properties has to be taken into account. This way, the component can be tailored to its service loads without applying generous safety factors. To facilitate the damage-controlled design of components, precise damage models are inevitable. In the scientific literature, a vast variety of damage models exists [2]. Most models can be formulated as functions of the stress triaxiality η given by

$$\eta = \frac{\sigma_1+\sigma_2+\sigma_3}{3\sigma_{flow}} = \frac{\sigma_h}{\sigma_{flow}}, \tag{1}$$

with the principal stresses σ_i and the flow stress σ_{flow} [1] of the material. The triaxiality is a dimensionless measure of the hydrostatic stress and is known to be a decisive factor for void growth [3, 4]. Damage is also dependent on the deviatoric part of the stress state. Thus, models also consider the so-called Lode-parameter [5] given by

$$L = \frac{2\sigma_2 - \sigma_1 - \sigma_3}{\sigma_1 - \sigma_3}, \tag{2}$$

[1] The flow stress is used for the definition of the triaxiality, since only stress states under plastic flow are relevant to the evolution of ductile damage.

in their damage evolution equations. Many models focus on predicting macroscopic fracture instead of modelling the evolution voids. Thus, conventional damage models often times struggle to depict the evolution of damage in terms of void area fractions measured by scanning electron microscopy (SEM) [6]. The calibration with measured void area fractions improves the prediction quality of the void evolution. However, the data basis used for calibration with void area fractions is usually scarce, since the SEM-measurement of large areas is cumbersome. Thus, also the quality of data is crucial. Conventional tests for calibration of damage models such as tensile tests with various specimen geometry or Nakajima tests show specific load paths in terms of triaxiality and Lode-parameter. However, when comparing two of these specimens, usually both, the triaxiality and the Lode-parameter differ between the two specimens. This makes it hard to extract information on the effect of either the triaxiality or the Lode-parameter on damage evolution. Thus, a large number of specimens is needed to capture the whole triaxiality-Lode-parameter-space.

Recent investigations show that cold extrusion experiments are capable of adjusting the triaxiality without affecting the Lode-parameter [7] and vice versa [8]. The obtained void area fraction data from these experiments is used in this paper to calibrate a damage evolution equation for the case-hardening steel 16MnCrS5. To model the experimental data, a triaxiality-dependent damage evolution equation proposed earlier by the authors is combined with an additional Lode-parameter-dependent equation proposed by Lou et al. [9].

Methodology

This section describes the experimental methods used in [7] and [8] to isolate the effects of triaxiality and the Lode-parameter on damage. In addition, the proposed numerical model and the used parameter identification scheme are presented.

Isolating the effect of triaxiality on damage. In forward rod extrusion, a cylindrical billet, encased in a closely-fitting cylindrical container, is pushed at room temperature by a punch through a conical die. After forming, the extrudate is ejected by an ejector pin. Depending on the area reduction of the die, the diameter of the initial billet is reduced. The equivalent plastic strain $\bar{\varepsilon}$ on the central axis of the extrudate is given exactly by

$$\bar{\varepsilon} = \ln\left(\frac{d_0^2}{d_1^2}\right) = 2\ln\left(\frac{d_0}{d_1}\right). \quad (3)$$

The deviatoric stress state on the central axis in the forming zone equals for an isotropic material the deviatoric stress state in a uniaxial tensile test. The deviatoric tension equals a Lode-parameter of L = -1 (Fig. 1).

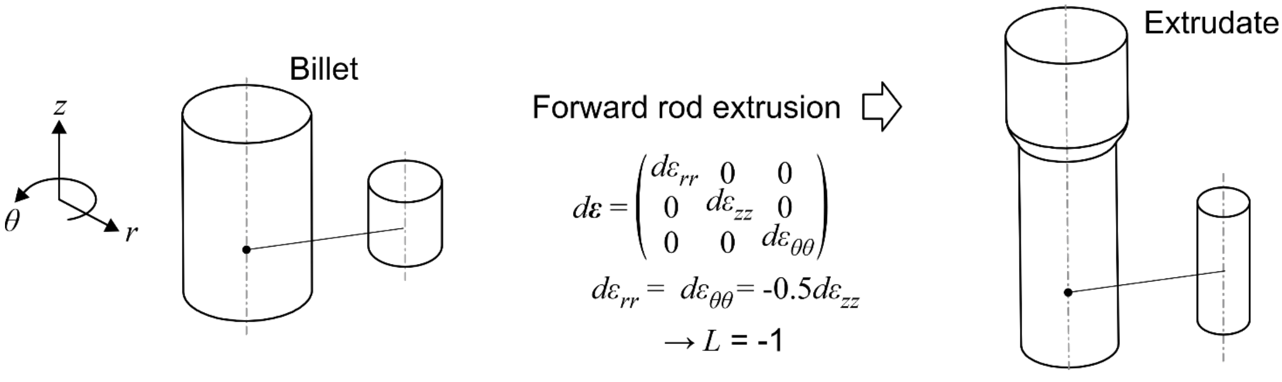

Fig. 1. Deformation of a material point on the central axis of forward rod extrusion and resulting Lode-parameter, adopted from [8]

Depending on the die cone angle 2α, a hydrostatic stress state is superposed to the deviatoric tension. For small extrusion strains and large die cone angles, high triaxialities are induced (Fig. 2). By changing the cone angle, only the hydrostatic stress is affected, while the equivalent plastic strain and the Lode-parameter stay constant [1]. Alternatively, the triaxiality can be adjusted by applying counter pressure via the ejector pin [10]. Thus, forward rod extrusion allows to study the effect of triaxiality isolated from other influencing factors.

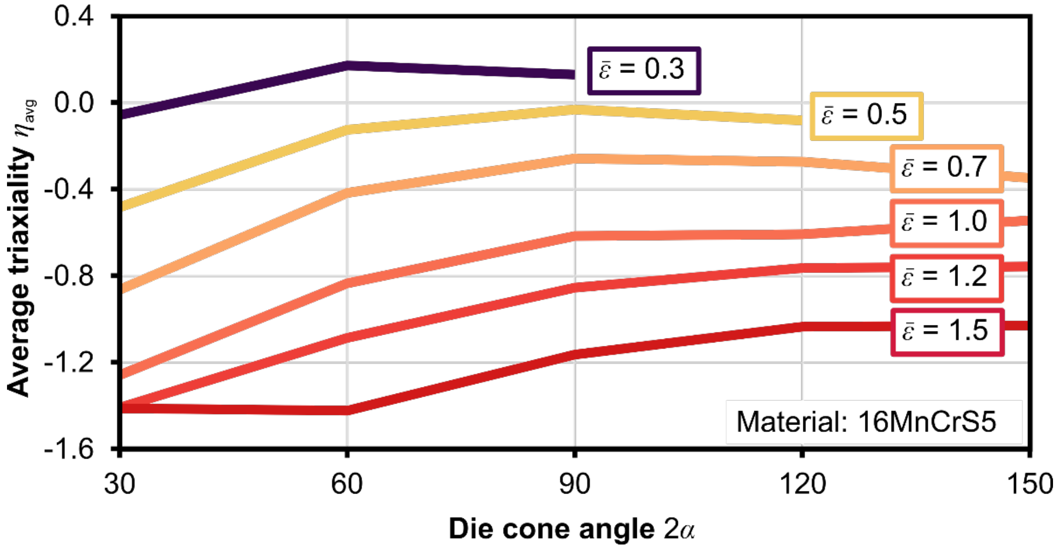

Fig. 2. Effect of extrusion strain and die cone angle in forward rod extrusion on average triaxiality on the central axis in the forming zone, adopted from [1]

Isolating the effect of the Lode-parameter on damage. In forward hollow extrusion, a thick-walled tube is pushed at room temperature by a punch through a conical die. A mandrel ensures that the inner diameter of the tube remains constant during forming, while the conical die reduces the outer diameter of the tube-shaped billet. The mandrel suppresses the strain component in the circumferential direction on the inner radius of the tube. This plane strain state results in a Lode-parameter of $L = 0$ (Fig. 3).

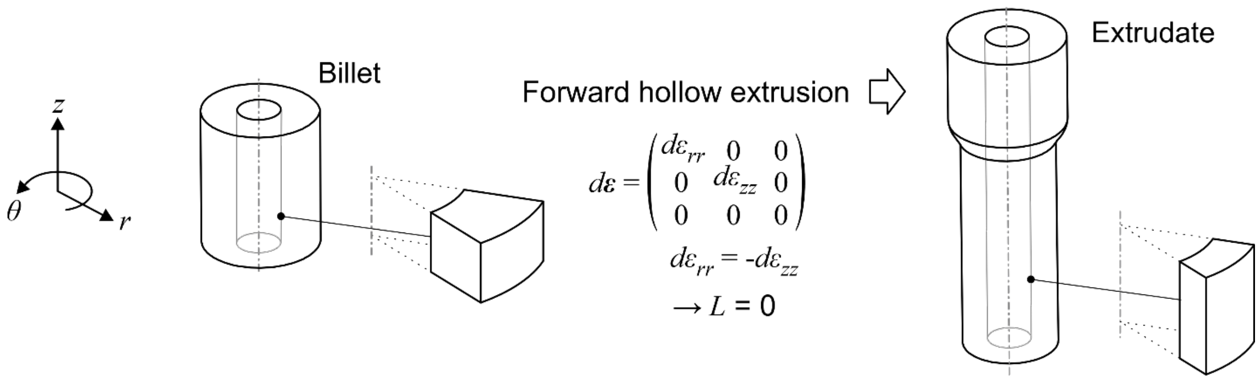

Fig.3. Deformation of a material point at the inner radius of forward hollow extrusion and resulting Lode-parameter, adopted from [8]

Compared to forward rod extrusion with $L = -1$, both extrusion variants induce different Lode-parameters in the forming zone. By carefully selecting the process parameters for both methods, the equivalent plastic strain and the triaxiality history can be kept the same in both process variants, so that the forming history differs only in terms of the Lode parameter [8]. This allows the influence of deviatoric stresses to be examined separately. To match the equivalent plastic strain in both process variants, the diameter reduction in forward hollow extrusion is adjusted accordingly. The correct diameter is obtained from FEM simulations. To match the triaxiality history in both processes, the triaxiality in forward rod extrusion is reduced by adding counter pressure as described above. The triaxiality in rod extrusion is offset by a constant value compared to the triaxiality in hollow extrusion (Fig. 4). By reducing the triaxiality by counter pressure in forward extrusion by this constant offset, the triaxiality is matched in both extrusion variants. The needed counter pressure is given by

$$p_c = \Delta\eta \cdot \sigma_{\mathrm{vM}}, \tag{4}$$

where $\Delta\eta$ is the triaxiality offset between forward hollow extrusion and forward rod extrusion without counter pressure and σ_{vM} is the equivalent stress in the forming zone of forward rod extrusion obtained from FEM simulations.

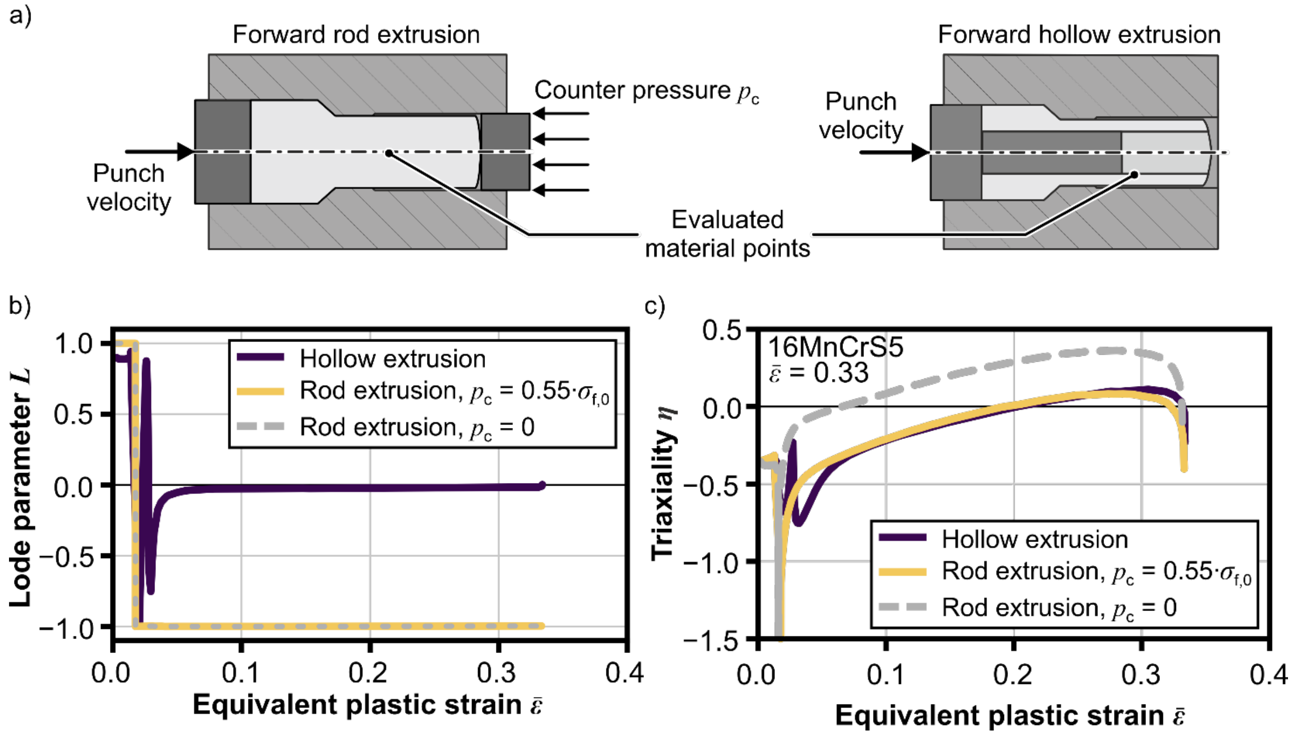

Fig. 4. Setting of Lode-parameters of L = -1 in forward rod extrusion and L = 0 in forward hollow extrusion, with equal equivalent plastic strains and triaxiality histories in both experiments, adopted from [8]

Void evolution equation. To model the damage evolution observed in the cold extrusion experiments, a triaxiality and Lode-parameter-dependent evolution equation is proposed. Inspired by the GTN-model [11], an individual evolution equation for void nucleation and void growth is used:

$$D(\bar{\varepsilon}_1) = D_0 + \int_0^{\bar{\varepsilon}_1} [f_{nuc}(\eta, L) + f_{vol}(\eta)] d\bar{\varepsilon}. \tag{5}$$

As described in [12], void coalescence occurs in the used 16MnCrS5 only immediately before macroscopic fracture. With the motivation to model damage for formed components (that do not fracture during forming) in mind, no additional term for void coalescence is used. The volumetric term is adopted from [12]. The term reads

$$f_{\mathrm{vol}} = C_{\mathrm{vol}} \sinh(1{,}5\eta) D(\bar{\varepsilon}), \tag{6}$$

with a material parameter C_{vol} and the void area fraction $D(\bar{\varepsilon})$ at the current time step. The hyperbolic sine function governs void growth for positive triaxialities as well as void shrinkage for negative triaxiality. The multiplication with the current void area fraction ensures an asymptotic behavior of the total void area fraction towards zero for negative triaxialities.
Since the Lode-parameter shows a pronounced effect on void nucleation in the used material [8], the nucleation term originally proposed in [12] is substituted by a Lode-parameter-sensitive evolution equation proposed by Lou et al. [9].

That equation reads:

$$f_{\mathrm{Lou}}(\eta, L) = \begin{cases} \frac{1}{C} \cdot \left(\frac{2\tau_{\max}}{\sigma_{\mathrm{vM}}}\right)^{C_L} \cdot \left(\frac{1+3\eta}{2}\right)^{C_\eta} & , \quad 1+3\eta \geq 0 \\ 0 & , \quad 1+3\eta < 0 \end{cases} \tag{7}$$

with the material parameters $1/C$, C_L and C_η. Since the equation is used in this paper to model the void nucleation, the notation of the parameter $1/C$ is changed to C_{nuc}. Using the Lode-parameter L, the proposed expression reads:

$$\Leftrightarrow f_{\mathrm{nuc}}(\eta, L) = \begin{cases} C_{\mathrm{nuc}} \cdot \left(\frac{2}{\sqrt{L^2+3}}\right)^{C_L} \cdot \left(\frac{1+3\eta}{2}\right)^{C_\eta} & , \quad 1+3\eta \geq 0 \\ 0 & , \quad 1+3\eta < 0\,. \end{cases} \tag{8}$$

Parameter identification. The damage evolution equation is calibrated to void area fraction measurements of rod [7] and hollow extrusion [8] experiments to capture the effects of triaxiality and the Lode-parameter on damage evolution. The measurements form both works are obtained by the same methodology presented in [7]. This method uses large area backscattered electron (bse) imaging to detect potential voids. With additional energy dispersive x-ray spectroscopy (edx) the method can distinguish between voids and non-metallic inclusions that may show the same grayscale value as voids. To also include information on a Lode-parameter of $L = 1$, also an upsetting test is considered in the parameter identification scheme. To compare the experimental data to the proposed evolution equation, the load paths of the considered experiments are determined in FEM-simulations. All experiments are simulated using the commercial FEM-solver Abaqus/Standard with implicit time integration to obtain the load paths. The model uses axisymmetric 4-noded solid elements. The billet with an initial diameter of 30 mm is discretized by a mesh with an initial element edge length of 0.2 mm. The elastic die is discretized with an element edge length of 0.4 mm in the die shoulder area. The isotropic workpiece is modelled by von Mises plasticity with flow curves at room temperature, 200°C and 400°C obtained from upsetting tests, to account for plastic heating. The contact between billet and die is modelled by Coulomb friction with a friction coefficient of $\mu = 0.04$.

For each experiment, the triaxiality and the Lode-parameter is extracted from the simulation as a function of the equivalent plastic strain. With these load paths at hand, the model prediction is calculated by numerically integrating Eq. 5 with the midpoint rule. The model parameters are obtained by minimizing the error between void area fraction measurements D_i^{exp} and the corresponding model prediction $D_i^{\mathrm{mod}}(\boldsymbol{x})$ with the parameter set $\boldsymbol{x} = [C_{\mathrm{vol}}, C_{\mathrm{nuc}}, C_L, C_\eta]$. The total error is quantified by the error function

$$f_{\mathrm{error}} = \sqrt{\frac{1}{n}\sum_{i=1}^{n} w_i\left(D_i^{\mathrm{exp}} - D_i^{\mathrm{mod}}(\boldsymbol{x})\right)^2}\,, \tag{6}$$

with the weighting factors w_i. The error function is minimized with an evolutionary algorithm implemented in the Python library SciPy. Each call of the error function calculates the model predictions for the current parameter set. Since the model prediction is directly computed from the extracted load paths, it is not necessary to run the FEM model in each iteration. The chosen weighting factors are found in Table 1. One third of the total weight is shared by three forward rod extrusion experiments with an extrusion strain of $\bar{\varepsilon} = 0.5$, but various die cone angles. These three experiments capture the effect of the triaxiality. Another third of the total weight is dedicated to two forward hollow extrusion experiments to capture the effect of the Lode-parameter. The remaining third of the total weight is assigned to an additional rod extrusion experiment with $\bar{\varepsilon} = 1$ to add information at large strains and the upsetting test to add information for $L = 1$.

Table 1. Experiments used for parameter identification of the proposed damage evolution equation

Experiment no.	1	2	3	4	5	6	7
Extrusion type	Rod	Rod	Rod	Rod	Hollow	Hollow	Upsetting
Eq. plastic strain $\bar{\varepsilon}$	0,5	0,5	0,5	1	0,33	0,5	0,3
Die cone angle 2α	30°	60°	90°	90°	90°	90°	-
Lode-parameter L	-1	-1	-1	-1	0	0	1
Average triaxiality η_{avg}	-0,41	-0,06	-0,01	-0,59	-0,13	-0,26	-0,33
Void area fraction D^{exp} in % [data origin]	0,006 [7]	0,023 [7]	0,024 [7]	0,001 [7]	0,037 [8]	0,017 [8]	0,008 [new data]
Weighting factor w_i in %	11,11	11,11	11,11	16,67	16,67	16,67	16,67

Model validation. The calibrated damage evolution equation is validated by additional void area data, measured outside of the central axis of a forward rod extrusion specimen. Hering et al. [7] measured void area fractions on five equidistant locations along the radius of an extrudate (Fig. 5). These locations show highly non-proportional Lode-parameter histories. Since this type of data is not included in the parameter identification scheme, it is well suited to benchmark the prediction quality of the proposed model. The extrusion experiment is modelled in FEM-simulations to obtain the load paths at the investigated positions of the specimen.

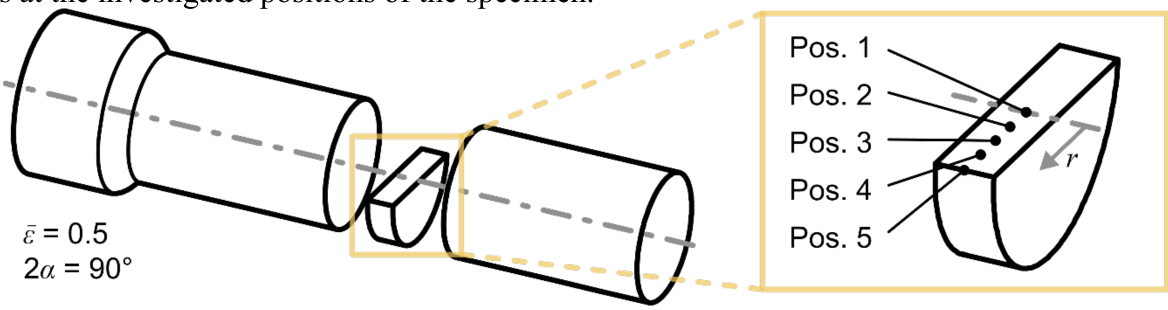

Fig.5. Void area measurement locations in validation experiment (data obtained in [7])

Findings

A parameter set for the proposed evolution equation is found by the described parameter identification scheme. The obtained minimum yields an error function value (Eq. 6) of $f_{\text{error}} = 2.1{\cdot}10^{-5}$. The identified parameter set is given in Table 2. Comparing the model predictions with the obtained parameter set to the experimental data used for the calibration, shows a good agreement (Fig. 6).

Table 2. Parameter set found by minimizing the error function (Eq. 6)

Parameter	C_{vol}	C_{Nuk}	C_L	C_η
value	0.899	$1.12{\cdot}10^{-3}$	$1.04{\cdot}10^{1}$	2.04

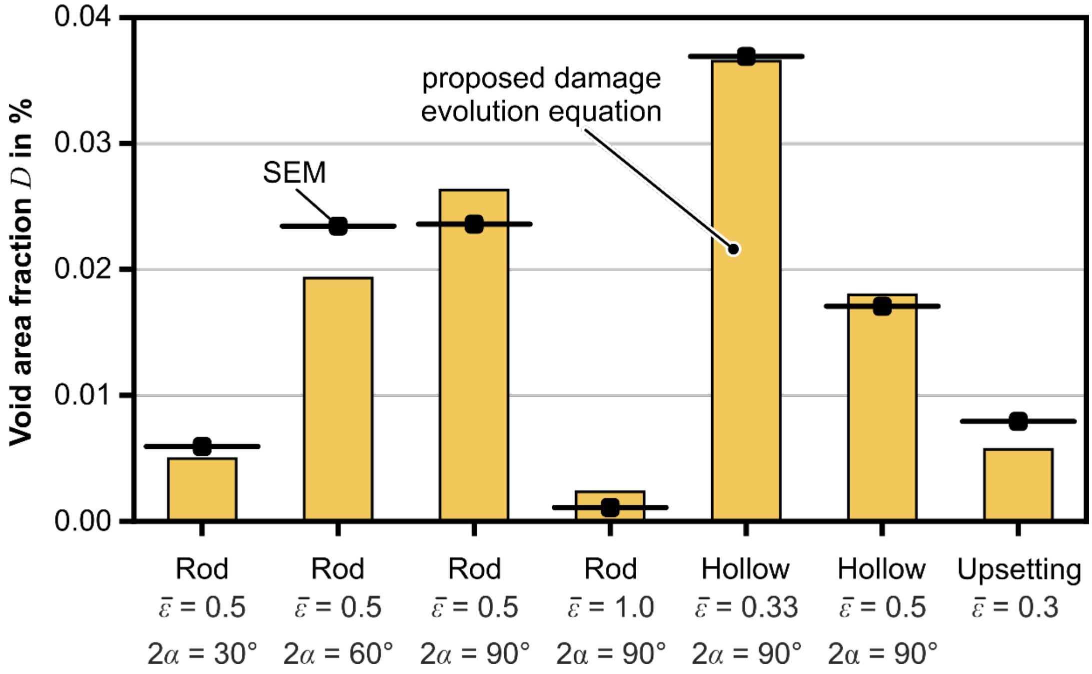

Fig.6. Fit of proposed evolution equation to the calibration data

To benchmark the proposed evolution equation, it is compared to the validation data that has not been used in the calibration. In addition, it is also compared to model predictions of the evolution equation proposed in [12]. This evolution equation is purely dependent on the stress triaxiality and neglects the Lode-parameter influence. Fig. 7 shows the comparison of the model predictions to the SEM measurements.

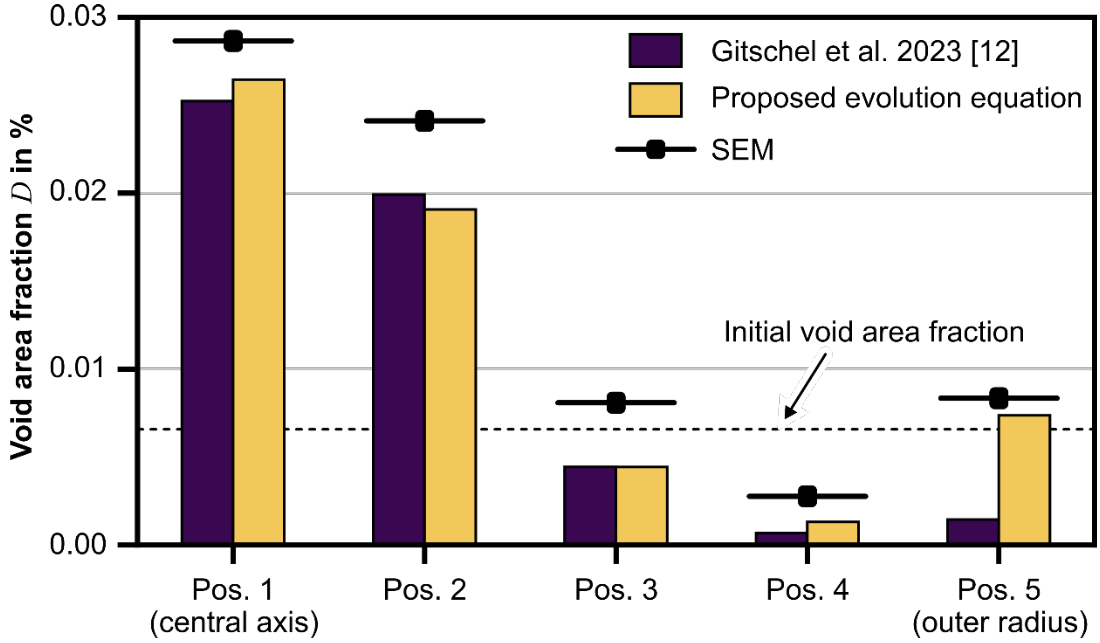

Fig. 7. Comparison of evolution equations to validation data, i.e., data that was not used for model calibration. See Fig. 5 for measurement positions.

At positions 1 to 3, both evolution equations show good agreement with the experimental data. The good agreement is due to the similarity of the load paths of these positions with the calibration load paths. At position 4, the load path in the validation experiment deviates significantly from the calibration load paths. The strongly negative triaxiality at position 4 leads to a reduction in the initial void area fraction. The equation proposed in [12] reproduces the void shrinkage. However, it overestimates the decrease in the void area fraction. The Lode-parameter-dependent evolution equation proposed in this paper provides a better prediction. At position 5, a qualitatively similar behavior of the models can be observed. The experiment shows only a minimal increase in the void area fraction compared to the void area fraction of the initial material. The equation proposed in [12] predicts a sharp decrease in the void area fraction compared to the initial state. The Lode-parameter-dependent evolution equation is in good agreement with the experimental data at position 5. In general, both models underestimate the void area fraction in the validation experiments. In future works, this observation should be assessed by additional validation experiments to check if this is a systematic shortcoming of the models or if this can be attributed to experimental errors.

Summary

Cold extrusion experiments offer a unique opportunity to generate calibration data for damage models. While the geometry of tensile specimen affects the triaxiality and the Lode-parameter at the same time, extrusion experiments allow to adjust either of the stress measures without affecting the other. Void area fraction measurements by SEM-investigations on such extrusion experiments are used to calibrate a triaxiality- and Lode-parameter-dependent void evolution equation. The proposed model uses a term to describe void growth and shrinkage and an additional term to model void nucleation. The void growth term is based on former works by the authors. For the Lode-parameter-sensitive void nucleation term, an equation from literature is adopted. The proposed damage evolution equation captures the calibration data well. Also, non-monotonic Lode-parameter histories are captured well in validation experiments. The uncoupled modelling of damage and plasticity in the presented approach allows using the damage evolution equation with any plasticity model. Also the presented parameter identification scheme based on void area measurement on cold extruded material can be used to calibrate other Lode-parameter-sensitive damage models. In future works, the experimental plan can be extended to multi-stage experiments to also account for load path reversal effects.

Acknowledgements

The authors kindly thank the German Research Foundation (DFG) for the financial support of project A02 in the Collaborative Research Centre CRC/Transregio 188 “Damage Controlled Forming Processes” (Project number 278868966 – TRR 188).

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