

Estimation of Uncertainty of Coordinate Measurements According to the Type B Method

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Abstract. The paper describes a new method for analytical estimation of the uncertainty component introduced by the CMM, including temperature influence. The uncertainty is estimated separately for each characteristic (dimension or geometrical deviation) given in the geometrical specification. The uncertainty is calculated directly, i.e. no analysis of the accuracy of determination of particular geometrical elements is performed. The fundamental condition enabling analytical estimation of the uncertainty is assumption that uncertainty of coordinate measurement depends on the *differences of coordinates* of characteristic points used to calculate particular deviation.

Introduction

The significance of estimation of measurement uncertainty needs no justification. So far, precisely formulated has been the methodology of estimation of measurement uncertainty with the use of calibrated workpiece identical or similar to the workpiece for which the uncertainty is to be estimated. This methodology was published as ISO/TS 15530-3:2004. Due to high costs of calibrated workpieces, it can only be implemented for large scale production, especially in the automotive industry.

Other well known possibility of uncertainty estimation in coordinate measurements is the use of computer simulation. Methodological basis of this conception was formulated in Physikalisch-Technische Bundesanstalt (PTB) in Braunschweig. On this basis, in different research centres (including PTB), a proper software was developed. This conception does not raise any serious doubts, but so far it is used only in research laboratories.

The VDI/VDE 2617 Blatt 11 describes an analytical method for estimation of uncertainty in coordinate measurements but it does not give a sufficiently effective solution. The method seems to be interesting one but has not good mathematical backgrounds. Especially the geometrical errors of CMM influencing the measurement uncertainty of respective features are chosen arbitrary.

The author of this paper was successful in finding method fully compatible with the type B method presented in GUM. This model is also in accordance to the general trends in the range of geometrical product specification [1, 2]. The assumptions for the model, on the 2D examples, were published in [3]. In the next section, the key condition which the model has to fulfil to enable the derivation of the algorithm of analytical estimation of measurement uncertainty is presented. In the following chapters, model of simple measuring task for 3D measuring machines is presented.

Generality Condition

Using the measurement model which meets the generality condition is the most important property thanks to which an effective analytical method of estimation of coordinate measurement uncertainty can be defined. It will be shown that if the measuring task in coordinate measurement was treated as indirect measurement in which the measurands are the distances measured along the axes, i.e. the *differences of coordinates* in particular axes, then the model of measuring task enabling the effective estimation of uncertainty by means of analytic techniques could be built [4]. It is easy to justify this statement. If in the measurement model subtracting of values having systematic errors

takes place, then there occurs often at least partial compensation of these errors. If the measurand x is calculated as the difference between x_1 and x_2 and both measurands contain systematic errors, respectively e_{x1} and e_{x2} :

$$x = x_2 - x_1, \quad x_2 = x_2^* + e_{x2}, \quad x_1 = x_1^* + e_{x1} \quad (1)$$

then the measurand x contains the systematic error amounting to the difference of these errors

$$x = x^* + (e_{x2} - e_{x1}) \quad (2)$$

and there is a chance for at least partial compensation of the errors. In such cases the uncertainty of difference u_{x2-x1} should be directly estimated.

Finding a good estimation of measurement uncertainty of difference of two measurands does not have to be an easy task. In coordinate measurements, it will prove necessary to define functions expressing the maximum values which the differences of particular geometrical errors can assume. More information on the generality condition presents [4].

Measurement of Distance between Point and Plane

As a conclusion arising from the analysis of the practical measuring tasks it can be pointed out that a lot of them can be modelled as a measurement of distance point-plane. One of the examples of such a measuring task is a measurement of parallelism of axes for the workpiece depicted on Fig. 1.

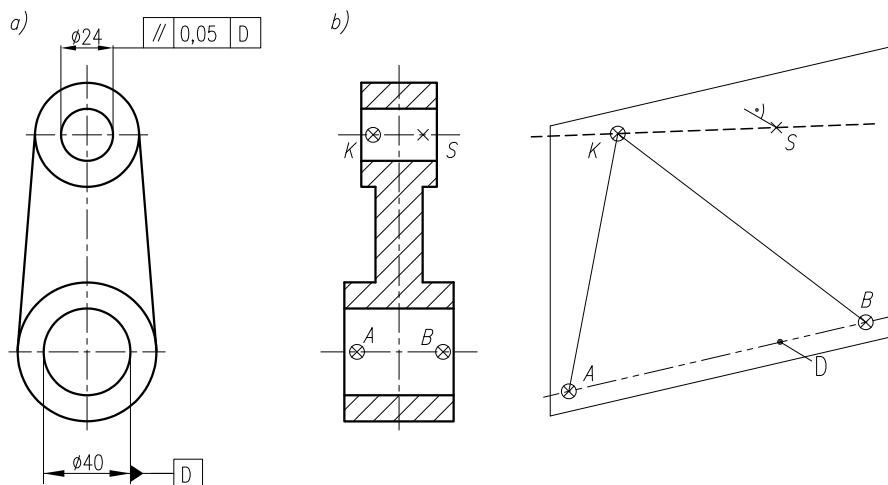


Fig. 1. Measurement of axes' parallelism: a) specification, b) measurement model with the characteristic points and the parallelism deviation defined as the distance of point S from the plane

In presented model the plane is determined by three points: $A(x_A, y_A, z_A)$, $B(x_B, y_B, z_B)$ and $K(x_K, y_K, z_K)$. The distance between the point $S(x_S, y_S, z_S)$ and the plane ABC (parallelism) is calculated as

$$l = |\mathbf{v} \cdot \mathbf{r}| \quad (3)$$

where \mathbf{v} – unit normal vector of the plane ABK , \mathbf{r} – vector of any point of the plane to the point S .

The unit normal vector of the plane ABK is

$$\mathbf{v} = \frac{(x_B - x_A, y_B - y_A, z_B - z_A) \times (x_K - x_A, y_K - y_A, z_K - z_A)}{\|(x_B - x_A, y_B - y_A, z_B - z_A) \times (x_K - x_A, y_K - y_A, z_K - z_A)\|}. \quad (4)$$

In the remaining part of this paper the shortened notation will be used: instead of $x_B - x_A$ it will be used x_{BA} , instead of $y_B - y_A$ it will be used y_{BA} , etc. So after the transformation the above formula will have a form:

$$v = \frac{(y_{BA}z_{KA} - y_{KA}z_{BA})i + (x_{KA}z_{BA} - x_{BA}z_{KA})j + (x_{BA}y_{KA} - x_{KA}y_{BA})k}{\sqrt{(y_{BA}z_{KA} - y_{KA}z_{BA})^2 + (x_{KA}z_{BA} - x_{BA}z_{KA})^2 + (x_{BA}y_{KA} - x_{KA}y_{BA})^2}}. \quad (5)$$

As vector r can be used vector AS , BS or KS . In first case the distance l_1 can be calculated by following formula

$$l_1 = |v \cdot (x_{SA}, y_{SA}, z_{SA})|. \quad (6)$$

For the analysis of measurement uncertainty, one has to assume that the distance l_1 is a function of nine differences of coordinates measured directly

$$l_1 = f(x_{BA}, x_{KA}, x_{SA}, y_{BA}, y_{KA}, y_{SA}, z_{BA}, z_{KA}, z_{SA}). \quad (7)$$

Type B standard measurement uncertainty is calculated as

$$u_{l1} = \sqrt{\left(\frac{\partial l_1}{\partial x_{BA}} \cdot u_{xBA}\right)^2 + \left(\frac{\partial l_1}{\partial x_{KA}} \cdot u_{xKA}\right)^2 + \left(\frac{\partial l_1}{\partial x_{SA}} \cdot u_{xSA}\right)^2 + \dots + \left(\frac{\partial l_1}{\partial z_{SA}} \cdot u_{zSA}\right)^2} \quad (8)$$

$$u_l = \min\{u_{l1}, u_{l2}, u_{l3}\}. \quad (9)$$

Measurement Uncertainty of Differences of Coordinates of Two Points

The geometrical model of measuring machine enables to derive the formulae for the components of error vector in point $A(x_A, y_A, z_A)$. It has to be taken into consideration that, during the measurement, the readings from the CMM scales x'_A, y'_A, z'_A depend also on the used stylus tip parameters x_{tA}, y_{tA}, z_{tA} . Appropriate relationship is following:

$$x_A = x'_A - x_{tA}, \quad y_A = y'_A - y_{tA}, \quad z_A = z'_A - z_{tA}. \quad (10)$$

The components of measurement error in point A depend on geometrical errors of CMM and are following:

$$e_{xA} = -(y_{tA} - m)xwy - (h - z_A)xwz + xpx(x'_A) - (h - z_A)xry(x'_A) - (y_{tA} - m)xrz(x'_A) + ytx(y'_A) + z_A \cdot yry(y'_A) - (y_{tA} - m)yrz(y'_A) + ztx(z'_A) - (h - z_A)zry(z'_A) - y_{tA} \cdot zrz(z'_A) \quad (11)$$

$$e_{yA} = x_A \cdot xwy + (h - z_A)ywz + xty(x'_A) + (h - z_A)xrx(x'_A) + x_{tA} \cdot xrz(x'_A) + ypy(y'_A) - z_A \cdot yrx(y'_A) + x_A \cdot yrz(y'_A) + zty(z'_A) + (h - z_A)zrx(z'_A) + x_{tA} \cdot zrz(z'_A) \quad (12)$$

$$e_{zA} = -x_{tA} \cdot xwz + (y_{tA} - m)ywz + xtz(x'_A) + (y_{tA} - m)xrx(x'_A) - x_{tA} \cdot xry(x'_A) + ytz(y'_A) + (y_{tA} - m)yrx(y'_A) - x_A \cdot yry(y'_A) + zpz(z'_A) + y_{tA} \cdot zrx(z'_A) - x_{tA} \cdot zry(z'_A) \quad (13)$$

where h, m – design parameters of CMM [3].

To calculate the component standard measurement uncertainties of differences of coordinates u_{xBA} one has to notice, that respective measuring error e_{xBA} is:

$$\begin{aligned}
e_{xBA} = & [xpx(x'_B) - xpx(x'_A)] + [ytx(y'_B) - ytx(y'_A)] + [ztx(z'_B) - ztx(z'_A)] - [xry(x'_B) \cdot \\
& \cdot (h - (z_B + z_{tB})) - xry(x'_A) \cdot (h - (z_A + z_{tA}))] + [xrz(x'_B) \cdot (y_{tB} - m) - xrz(x'_A) \cdot (y_{tA} - m)] + \\
& + [yry(y'_B) \cdot (h - (z_B + z_{tB})) - yry(y'_A) \cdot (h - (z_A + z_{tA}))] - [yrz(y'_B) \cdot (y_{tB} - m) - yrz(y'_A) \cdot \\
& \cdot (y_{tA} - m)] - [zry(z'_B) \cdot (h - (z_B + z_{tB})) - zry(z'_A) \cdot (h - (z_A + z_{tA}))] - [zrz(z'_B) \cdot y_{tB} - zrz(z'_A) \cdot y_{tA}] \\
& + [xwy \cdot (y_{tB} - m) - xwy \cdot (y_{tA} - m)] + [xwz \cdot (h - (z_B + z_{tB})) - xwz \cdot (h - (z_A + z_{tA}))]
\end{aligned} \quad (14)$$

The formulae for e_{yBA} i e_{zBA} are analogical.

According to generality condition in above formulae the particular errors are grouped in pairs to enable separate estimation of the values achieved by the differences of the geometrical errors of the same type. It is an indirect measurement, in which the measuring errors of the difference of coordinates are the sum of a few components. For correct estimation of component uncertainty e_{xBA} the 11 components of error (similarly in the errors e_{yBA} and e_{zBA} further 22 components) have to be distinguished (Eq. 14):

$$e_1 = xpx(x'_B) - xpx(x'_A) \quad (15)$$

$$\dots \quad e_{11} = xwz \cdot (h - (z_B + z_{tB})) - xwz \cdot (h - (z_A + z_{tA})) \quad (16)$$

In the following, the functions, arguments of which are coordinates' differences l (l – distance measured by CMM scales of particular axis), describing the maximum values which the differences of particular geometrical errors can achieve are defined (on the example of xpx_{MAX}):

$$xpx_{MAX}(l) = \max_x |xpx(x) - xpx(x+l)| \quad (17)$$

The example chart of function describing the geometrical error and of the respective function describing the maximum difference of errors are shown on Fig. 2.

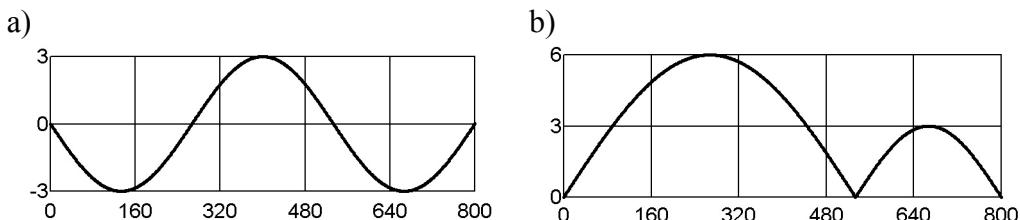


Fig. 2. The example function describing the maximal influence of the geometrical error: a) function of the geometrical error, b) the respective function of the maximum differences of errors

The defining of the above functions is one of the essential elements of the presented methodology of estimation of measurement uncertainty. Using the above functions, the maximal values (according to type B method) for the errors (on the example xpx) were estimated:

$$|xpx(x'_B) - xpx(x'_A)| \leq xpx_{MAX}(|x'_B - x'_A|) \quad (18)$$

Further, the respective components of the type B standard uncertainty are expressed as products of extreme values of errors and k_i coefficients arising from the probability distribution of the particular error.

$$u_1 = k_1 \cdot xpx_{MAX}(|x'_B - x'_A|) \quad (19)$$

Example Results

The general input data for the software are the design parameters of the CMM as well as information on the geometrical errors and probing system error. The characteristic point coordinates are given for each measuring task. For each characteristic point the parameters of the used stylus tip are needed.

Similarly to the described task (parallelism of axes in plane normal to common plane), using the “point-plane distance” model, following tasks were solved: parallelism of axes in common plane, parallelism of axis to plane, parallelism of plane to axis, parallelism of planes, flatness, perpendicularity of planes, perpendicularity of axes, perpendicularity of axis to plane, perpendicularity of plane to axis, angularity, and some cases of position deviation.

The uncertainty of measurement of straightness, coaxiality, parallelism of axes with cylindrical tolerance zone, perpendicularity of axis to plane are evaluated according to “point-axis distance” model.

Conclusion

The fundamental condition enabling analytical estimation of the uncertainty is assumption that uncertainty depends on the differences of coordinates of characteristic points used to calculate particular characteristic. It means for example, that for measurement uncertainty evaluation of parallelism of a hole’s axis and a plane it is necessary to use the differences of coordinates of characteristic points both the axis of cylinder and the plane.

The starting point for preparing the uncertainty budget is a formula expressing particular characteristic as a function of differences of characteristic points coordinates. For calculation of uncertainty of each difference in the formula the 33 component errors connected with geometrical errors of CMM and 3 with probing system are taken into account.

Calculation of maximal values that the particular components errors can have is possible thanks to the function defined by the author which is composed on the basis of the function describing the geometrical error and the argument of which is the difference of coordinates.

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