A probabilistic approach to the simulation of non-linear stress-strain relationships for oriented strandboard subject to in-plane tension

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Abstract. This paper presents the results from an experimental test program conducted on commercially available oriented strandboard (OSB) panels and statistical analyses of the results. Standardised testing was used to determine the short-term behaviour of OSB/3 panels subjected to tension loading. A variety of thicknesses sourced from three different producers were used. Analysis of the results indicate that a quadratic expression in the form of $\sigma = a\varepsilon^2 + b\varepsilon$ provides the best description of the relationship between stress ($\sigma$) and strain ($\varepsilon$) up to the point of failure. It has also been shown that the coefficients $a$ and $b$ of the quadratic regression equations are negatively correlated to each other. Anderson-Darling goodness-of-fit tests were conducted on the results for tension strength and modulus of elasticity (MOE). The results indicate that the tension strength and MOE come from populations that follow either normal or lognormal probability distributions.

Introduction

Oriented strandboard (OSB) is a two-phase wood-based composite material made from elongated wood strands. The strands are sliced from small-diameter low-grade logs with the longest dimension aligned parallel to the grain of the log. They are coated in a thermosetting resin binder and are formed into a three-layered mat that is hot pressed to cure the binder, bonding the strands together to form large panels. OSB is typically used as a structural sheathing material in a similar fashion to plywood. Its main drawback is that its complex structure combined with the natural variability of the raw materials make its mechanical behaviour difficult to predict. Various attempts have been made to predict the mechanical behaviour of OSB by making major simplifying assumptions, limited geometric configurations and loading conditions. However, a generalised engineering approach to predict the mechanical response of OSB under all loading conditions while accounting for the natural variability is still not readily available.

The preliminary output of a study seeking to develop a method of predicting the mechanical response of OSB and its variability is presented in this paper. The focus is on results from a test program conducted to examine the short-term tension behaviour of commercially available OSB panels using standardised testing arrangements as per BS EN 789:2004 [1]. A variety of thicknesses of OSB/3 panels, produced in accordance with BS EN 300:2006 [2] by three different manufacturers, were tested. The results have been used to establish stress-strain relationships to describe the short-term mechanical behaviour up to the point of failure and to determine appropriate probability distribution models to describe the natural variability of the parameters.

Literature Review

A review of the use of probability based methods in the forest products industry conducted by Taylor \textit{et al.} [3] described the effectiveness of these methods to accurately simulate the natural variability of structural wood systems. The Monte-Carlo method has proven to be a particularly
useful tool to model the natural variability in both the physical characteristics and mechanical behaviour of wood-based composites. The effectiveness of the Monte-Carlo method is however dependent on knowledge of the probability distribution of each variable in the system.

The first attempt to predict the mechanical response of wood strand composites was conducted by Hunt and Suddarth [4]. A 2-D linear elastic finite element model was developed to predict the tension modulus and shear modulus of particleboard. Single layer random particleboard was modelled as a regular grid of beam elements (representing the binder) infilled with plate elements (representing the particles). The mechanical properties of individual particles were determined experimentally in the parallel and perpendicular to grain directions. The Monte-Carlo method was used to simulate the structure of the panel by randomly assigning a particle orientation to each plate element independently. Comparison with experimental results showed the average predicted tension MOE differed from the experimental value by 2% to 3% while the average predicted shear modulus differed from the experimental value by 10% to 12%.

Wang and Lam [5] made use of several probability based techniques in the development of a 3-D non-linear stochastic finite element model capable of predicting the probabilistic distribution of the tension strength and MOE of multi-layered parallel aligned wood strand composites. The model input was generated through testing of individual wood strands with standardised cross-sectional dimensions of 2.7x17 mm at a gauge length of 152 mm to evaluate the tension strength, MOE and to determine the underlying probability distributions of each material property. Assemblies of strands with 2, 3, 4 and 6 layers were also tested at a gauge length of 457 mm and the results were used for comparison with model predictions. The Monte-Carlo method was used to randomly assign material properties to individual strands based on the underlying probability distribution. Analysis of the results from testing wood strands also showed that a correlation existed between tension strength and tension MOE. This was one of the first attempts to preserve the relationship between two input variables during Monte-Carlo simulation using the standard bivariate normal distribution procedure developed by Lam et al. [6] and Wang et al. [7] to model the mechanical response of wood-based composites. A probability based technique was also used to simulate the size effect using the Weibull weakest link theory. Excellent agreement was achieved between the simulated and experimental probability distributions for tension strength and MOE of the multiply laminates.

Clouston and Lam [8] incorporated the probability based techniques developed by Wang and Lam into 2-D non-linear stochastic finite element model to predict the mechanical response of angle-ply wood strand laminates subjected to multiaxial stress conditions. This was one of the first attempts to model wood-strand composites with varying ply orientations and to model non-linear compression behaviour. Excellent agreement was observed between the predicted and experimental probability distributions of ultimate strength and MOE in tension, compression and bending. Further studies by Clouston and Lam [9, 10] developed the model into a 3-D non-linear stochastic finite element model for predicting the probabilistic distribution of strength, stiffness and failure load of angle-ply laminates subjected to tension, compression and bending. Clouston [11] and Arwade et al. [12] further developed this model to enable it to predict the strength and MOE of large section parallel strand lumber (PSL) members loaded in tension, compression and bending.

Past studies have largely concentrated on predicting the mechanical properties of wood-based composites based on the mechanical properties of the raw materials with model verification being achieved through experimental testing of small scale, laboratory produced panels. This study is focusing on predicting the mechanical properties of existing, large scale, commercially available panels based on the physical properties that can be controlled during panel production. It has been shown previously that both solid timber and timber-based composites behave elastically when loaded in tension [13-15] up to the point of failure. Therefore, it has been assumed in this study that OSB behaves elastically up to the point of failure when loaded in tension.
Testing

Materials. The materials tested were commercially available OSB/3 panels manufactured in accordance with BS EN 300:2006 [2]. Three different panel thicknesses (11 mm, 15 mm and 18 mm) produced by Manufacturer A, one panel thickness (15 mm) produced by Manufacturer B and one panel thickness (15 mm) produced by Manufacturer C were tested. Panels were produced by Manufacturer A using Sitka spruce and Scots pine wood strands bound with Methylene di-Phenyl di-Isocyanate (MDI) resin stacked in a 0-90-0 lay-up pressed in a daylight press. Panels were produced by Manufacturer B using Scots pine and Lodgepole pine wood strands bound with Melamine Urea Phenol Formaldehyde (MUPF) resin in the surface layers and Polymeric di-Phenyl Methane di-Isocynate (PMDI) resin in the core stacked in a 0-90-0 lay-up pressed in a daylight press. Panels were produced by Manufacturer C using pine wood strands bound with Melamine Urea Phenol Formaldehyde (MUPF) resin in the surface layers and di-Phenylmethane di-Isocynate (pMDI) resin in the core layer stacked in a 0-90-0 lay-up pressed in a continuous press.

Specimen Preparation. A total of 32 cutting plans were prepared for each panel thickness in accordance with the guidelines in BS EN 789:2004 [1] of which 15 were selected at random for cutting. The remainder were retained for future study. Cutting plans (see Fig. 1) are designed to ensure that the panel can be cut to form one test piece per material property in both directions. Test pieces cut with their longer dimension aligned parallel to the longer dimension of the panel are designated longitudinal (LONG) while test pieces cut at 90° to the longer dimension of the panel are designated lateral (LAT). The surface strands were aligned parallel to the longer dimension for all panels tested. An additional set of eight 11mm thick panels from Manufacturer A were cut to produce 5 tension test pieces in each direction. Four pieces were tested in each direction from each of these additional panels with the remainder being retained for further study. All test pieces were conditioned at 20°C and 65% relative humidity prior to testing. Tension test pieces were cut to the basic shape as described in BS EN 789:2004 [1] using slightly modified dimensions shown in Fig. 1 as per two previous studies [16, 17]. Details of the number of test replications in each material property direction are given in Table 1.

Fig 1. Sample Cutting Plan and Tension Test Piece Details (Dimensions in mm)
Table 1. Sample Sizes

<table>
<thead>
<tr>
<th>Thickness</th>
<th>Manufacturer A</th>
<th>Manufacturer B</th>
<th>Manufacturer C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Longitudinal</td>
<td>Lateral</td>
<td>Longitudinal</td>
</tr>
<tr>
<td>11mm</td>
<td>56**</td>
<td>39</td>
<td>-</td>
</tr>
<tr>
<td>15mm</td>
<td>15</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>18mm</td>
<td>15</td>
<td>15</td>
<td>-</td>
</tr>
</tbody>
</table>

* Includes results of previous study by O’Toole [16]

Testing. Testing was performed using a Dartec universal hydraulic testing machine with hydraulic grips and a 250 kN load cell capable of reading load to an accuracy of 1% (see Fig. 2(a)). Two 5 mm, full bridge LVDT’s with an accuracy of ± 1% were mounted to the test piece using custom made mounting blocks spaced 120 mm apart and bolted together through the test piece using M3 bolts (see Fig. 2(b)) as per the requirements of BS EN 789:2004 [1]. Load and displacement were continuously monitored using a National Instruments NI CDAQ-9172 data acquisition system and LabVIEW 8.2 software. Load was applied using a constant strain rate set such that the test pieces failed within 300 ± 120 s as specified in BS EN 789:2004 [1]. Moisture content was determined using the “oven dry method” as per in BS EN 322:1993 [18] to ensure consistency in the conditioning process.

Fig 2. Tension Test Setup

Fig. 2(a) – Tension test piece setup in Dartec 250 kN universal testing machine

Fig. 2(b) – Full-bridge LVDT’s mounted to the test piece using custom made mounting blocks

Results

Strength and Elastic Stiffness. Tension strength and stiffness properties were calculated for each test piece as per BS EN 789:2004 [1]. A linear regression analysis on the section of the load-deflection curve from 0.1 to 0.4 times the failure load was performed and tension stiffness was calculated using Eq. 1. Tension strength is calculated using Eq. 2 and the cross sectional dimensions at the failure location.

\[
E_t = \frac{(F_2 - F_1)l_1}{(u_2 - u_1)A}. 
\]

(1)

\[
f_t = \frac{F_{\text{max}}}{A}. 
\]

(2)
Where: \( F_2 = \) load at 0.4 \( F_{\text{max}} \); \( F_1 = \) load at 0.1 \( F_{\text{max}} \); \( u_2 = \) displacement corresponding to \( F_2 \); \( u_1 = \) displacement corresponding to \( F_1 \); \( F_{\text{max}} = \) failure load; \( l_1 = \) gauge length; \( A = \) cross-sectional area.

Summary statistics (including mean, 5th percentile and coefficient of variation (COV)) for tension strength and MOE are presented in Table 2 below for each panel manufacturer, thickness and material property direction.

### Table 2. Tension Test Results Summary

<table>
<thead>
<tr>
<th>Thickness</th>
<th>Mean (N/mm²)</th>
<th>5th Percentile (N/mm²)</th>
<th>CoV (%)</th>
<th>Mean (N/mm²)</th>
<th>5th Percentile (N/mm²)</th>
<th>CoV (%)</th>
<th>Mean (N/mm²)</th>
<th>5th Percentile (N/mm²)</th>
<th>CoV (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-11mm</td>
<td>10.85</td>
<td>7.63</td>
<td>20.31</td>
<td>4089</td>
<td>3148</td>
<td>15.07</td>
<td>9.28</td>
<td>6.39</td>
<td>20.32</td>
</tr>
<tr>
<td>A-15mm</td>
<td>13.19</td>
<td>9.67</td>
<td>15.44</td>
<td>4458</td>
<td>3904</td>
<td>8.72</td>
<td>10.51</td>
<td>9.35</td>
<td>8.43</td>
</tr>
<tr>
<td>A-18mm</td>
<td>10.86</td>
<td>8.59</td>
<td>15.51</td>
<td>3775</td>
<td>3147</td>
<td>11.47</td>
<td>8.98</td>
<td>6.14</td>
<td>20.21</td>
</tr>
<tr>
<td>B-15mm</td>
<td>10.32</td>
<td>8.95</td>
<td>11.10</td>
<td>3684</td>
<td>3236</td>
<td>10.02</td>
<td>8.94</td>
<td>7.44</td>
<td>9.98</td>
</tr>
<tr>
<td>C-15mm</td>
<td>10.57</td>
<td>7.12</td>
<td>18.78</td>
<td>4376</td>
<td>3568</td>
<td>13.14</td>
<td>6.76</td>
<td>5.60</td>
<td>12.62</td>
</tr>
</tbody>
</table>

Regression Analysis. OSB is traditionally regarded by design codes as being a linear elastic material when loaded in tension under the serviceability limit state. Initial inspection of the results indicated that the relationship between stress and strain is linear at low strain but that non-linearity exists at strains above a certain level. Inspection of the plot of stress vs. strain shown in Fig. 3 shows that the relationship is linear up to a point but deviates at strains higher than 0.002. Linear and quadratic regression analyses showed that for all specimens, the \( R^2 \) values for the quadratic model were superior to those for the linear model. This confirmed that a quadratic model is better at describing material behaviour over the full strain range. A quadratic stress-strain equation in the form \( \sigma = a\varepsilon^2 + b\varepsilon + c \) was fitted to the stress-strain data for each specimen tested. The constant term \( c \) approximates to 0 for all specimens, allowing the equations to be simplified to the form \( \sigma = a\varepsilon^2 + b\varepsilon \).

Test specimens were grouped according to manufacturer, thickness and direction. An average stress-strain curve was generated for each group using the stress-strain data from each test specimen in the group by calculating the mean stress along lines of constant strain [8-10]. Fig. 3 shows a typical average stress-strain curve and the associated 95% confidence interval for 11 mm thick panels produced by Manufacturer A loaded in the longitudinal direction. Confidence intervals are used to estimate population parameters based on sample statistics [19, 20]. This allows us to state with 95% certainty that the average longitudinal tension stress-strain curve for all 11 mm thick OSB panels produced by Manufacturer A will fall somewhere within the region shown in Fig 4. This can also be said for all the other manufacturers, panel thicknesses and material directions.

It was decided to investigate the strength of the relationship (if any) between the coefficients \( a \) and \( b \) of the quadratic stress-strain relationships. The quadratic regression equations for each test specimen were grouped according to manufacturer, panel thickness and direction and linear regression analyses were conducted between the coefficients for each group. Fig. 4 shows the results of a typical regression analysis for the 11 mm thick panels produced by Manufacturer A loaded in the longitudinal direction. The results show that a negative linear correlation exists between coefficient \( b \) and coefficient \( a \). The \( R^2 \) values indicate that the strength of the relationship in some cases is quite weak (\( R^2 = 0.165 \) for Manufacturer A, 11 mm longitudinal) and in some cases is quite strong (\( R^2 = 0.830 \) for Manufacturer B, 15 mm lateral).
The literature review suggested that strength and stiffness properties of wood-based composites tend to follow either the normal or lognormal probability distribution models [8-11]. A preliminary analysis using Minitab 15.0 statistical software confirmed this finding. It was decided to focus on these two probability distributions and to develop a computer program to automatically determine the more suitable probability distribution to describe the results from the experimental test program.

The program was written using the Microsoft Visual Basic for Applications (VBA) for Microsoft Excel 2000. The computer program output included probability plots to facilitate visual inspection of the goodness of fit between the empirical distribution function (EDF) of the experimental data and cumulative distribution function (CDF) for each probability distribution being examined. The Anderson-Darling test was used to definitively determine the more suitable probability distribution model to describe the data. The Anderson-Darling test is a quadratic one-tailed statistical hypothesis test. The goodness of fit between the EDF and the CDF for each probability distribution can be represented by a single number (the Anderson-Darling statistic, $A^2$). The Anderson-Darling test is considered the most robust goodness-of-fit test for both small and large samples and is widely used in commercial statistical software packages [21, 22]. The data set is ranked in ascending order and the Anderson-Darling statistic is calculated using Eq. 3 and then modified to take into account the effect of sample size using Eq. 4.

$$A^2 = -N - S$$ \hspace{1cm} (3)

$$A_{adj}^2 = A^2 \left( 1 + \frac{0.75}{N} + \frac{2.25}{N^2} \right)$$ \hspace{1cm} (4)

Where: $N =$ sample size; $S$ is given by Eq. 5 below; $F(Y_i) =$ CDF of probability distribution evaluated at observation $Y_i$; $F(Y_{N+1-i}) =$ CDF of probability distribution evaluated at observation $Y_{N+1-i}$.
\[ S = \sum_{i=1}^{N} \frac{2i-1}{N} \left[ \ln F(y_i) + \ln(1 - F(y_{N+i-1})) \right]. \]  

(5)

In the Anderson-Darling test, the null hypothesis states that the data comes from a population that follows a specific probability distribution model e.g. the null hypothesis is that the longitudinal tension strength of 11mm thick OSB panels produced by Manufacturer A follows a normal distribution. The \( A^2 \) value can be used to calculate a corresponding \( P \)-Value using sets of formulae derived by D’Agostino and Stephens [21]. The \( P \)-Value is the probability that the accepting the null hypothesis (i.e. the data comes from a population that follows the probability distribution being tested) is the correct decision. In other words, a high \( P \)-Value indicates a strong probability that the data set comes from a population that follows the probability distribution being tested. A level of significance (\( \alpha \)) is normally chosen prior to performing the Anderson-Darling test. An \( A^2 \) value that produces a \( P \)-Value less than \( \alpha \) leads to the immediate rejection of the null hypothesis i.e. the particular probability distribution currently being tested is poor at describing the data and should be rejected. A significance level of \( \alpha = 0.05 \) has been used throughout this study.

Fig. 5 and 6 show typical cumulative probability plots for the tension strength (\( f_t \)) and tension MOE (\( E_t \)) results, respectively, for the 11mm thick panels produced by Manufacturer A loaded in the longitudinal direction. The plots include the EDF for the sample results plotted on top of the CDF for normal and lognormal probability distributions. A summary table containing the sample size, the \( A^2 \) value and the corresponding \( P \)-Value is included on each chart. Visual inspection indicates that both probability distribution models describe the data quite well, making it difficult to make a decision based on visual comparison. However, in the case of the tension strength, the \( P \)-Value for the lognormal distribution is 0.8834 whereas the \( P \)-Value for the normal distribution is 0.5933, indicating that the lognormal probability distribution is the better fit. Likewise, for the tension MOE, visual inspection indicates that both probability distributions describe the data well but the \( P \)-Value for the normal distribution is 0.4783 as opposed to 0.3169 for the lognormal distribution, indicating that the normal distribution is the better fit. This process has been repeated for all panel types, thicknesses and material property directions and a summary is presented in Table 3.

**Fig. 5 Probability Plots (Manufacturer A, 11 mm, Strength, Longitudinal)**
As can be seen in Table 3, the $P$-Values for the lateral tension strength for the 15 mm thick panels produced by Manufacturer B are less than 0.05 for both the normal and lognormal probability distribution. It is likely that the inconclusive result was a consequence of an insufficient sample size to fully capture the underlying probability distribution and further testing would eliminate this problem. In all other cases, $P$-Values for tension strength and MOE for both probability distributions are greater than 0.05 and it is therefore not possible to outright reject one or the other. Since a higher $P$-Value indicates a stronger probability that accepting the null hypothesis is the correct decision, it can be concluded that the probability distribution with the higher $P$-Value is the probability distribution that best describes the data. The conclusions columns in Table 3 summarise the chosen probability distribution for each parameter based on the $P$-Value.

The results summarised in Table 3 make it difficult to state definitively, for example, if tension strength always follows a lognormal probability distribution or if tension MOE always follows a lognormal probability distribution.
normal probability distribution. However, visual inspection of the probability plots indicates that the sample results for strength and stiffness can be represented well by either a normal or lognormal probability distribution.

Conclusions

The experimental test program and statistical analyses of the results indicate that the short-term tension stress-strain behaviour of OSB can be described by a quadratic expression up to the point of failure. Linear and quadratic regression analyses were performed on the stress-strain data obtained from each test replication. Comparing the $R^2$ values indicated that a quadratic expression is more suitable to describe the short-term stress-strain behaviour up to the failure point. Furthermore, it has been shown using linear regression analyses that in the quadratic stress strain relationships ($\sigma = ae^2 + be$), the coefficients $a$ and $b$ are negatively correlated to each other. Average stress-strain relationships have been established for each panel type, thickness and material property direction along with the associated 95% confidence intervals. The Anderson-Darling test has been used effectively to determine the underlying probability distribution for each set of results with a definitive conclusion been made in all but one case. Visual comparison of probability plots indicates that the tension strength and tension MOE can be reasonably well represented by either a normal or lognormal probability distribution.

References


