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Bulging of Isotropic Materials

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Abstract. There is a large body of literature on both the techniques for bulge testing and experimental results for various metallic materials (see the book of Banabic [1]). Generally, the experimental data for isotropic materials are interpreted using the von Mises yield criterion [2]. In this paper, we investigate the role played by the third invariant of the stress deviator, J_3, on the response under bulging of isotropic materials that have the same mechanical response in tension and compression. To this end, we use the yield criterion developed by Cazacu [3] and our implementation of this model in the F.E. code Abaqus [4]. For isotropic materials, this yield criterion involves a unique parameter, denoted α; in the case when $\alpha = 0,$ it reduces to the von Mises yield criterion while for $\alpha \neq 0,$ it involves dependence on J_3. The results of F.E. simulations of bulge tests for isotropic materials characterized by various values of the parameter α put into evidence new aspects concerning the stress states experienced by the respective materials under bulging.

1. Introduction

The bulge test is a simple test that allows for the assessment of the formability of metallic sheets. Generally, the experimental data reduction and F.E. analysis of bulging of isotropic materials is done assuming that the plastic behavior can be modeled with the von Mises yield criterion; for the classical analytical solution for the stresses and strains at the pole of a hemispherical bulge, see Young et al [5]. However, the assumption of plastic behavior governed by the von Mises yield criterion may lead to an overestimation of how much a specimen can bulge before it fails. In the case of isotropic materials that display tension-compression asymmetry and dependence on J_3 on yielding, Cazacu and Revil-Baudard [6] derived a new solution to the problem and a correction to the classical relation ([5]) that is generally used to extract the equivalent stress vs. equivalent strain curve from bulge data. In this paper, we provide analysis of the bulge test using the isotropic form of the Cazacu [3] yield criterion. While this criterion predicts the same yielding response in tension and compression, it can also capture a dependence of the yielding on J_3. Specifically, for isotropic materials, this yield criterion involves a unique parameter, denoted α, that can be determined based on the ratio between the yield stress in simple tension and that in shear. Only in the case when $\alpha = 0,$ it reduces to the von Mises yield criterion while for $\alpha \neq 0,$ there is dependence of yielding on J_3. For the purpose of simulating the bulging response, the elastic/plastic model using the Cazacu [3] yield criterion was implemented into the FE implicit solver Abaqus Standard (see Abaqus [4]) by developing a user material routine (UMAT). Simulations of bulging tests were performed for isotropic materials characterized by various values of the parameter α. The paper is organized as follows. In section 2, we present the constitutive model, the results of the FE simulations of bulge tests for various materials are given in section 3. A summary and concluding remarks are presented in section 4.

2. Constitutive Modeling

The common sign convention used in metal plasticity i.e., tensile stresses and strains are positive is employed. The total strain rate is the sum of an elastic part and a plastic part $\boldsymbol{D}^p$. To describe the linear elastic response, the isotropic Hooke's law is used:

$$\dot{\boldsymbol{\sigma}} = \boldsymbol{C}^e : (\boldsymbol{D} - \boldsymbol{D}^p) \tag{1}$$

where $\dot{\boldsymbol{\sigma}}$ is the Green-Naghdi rate of the Cauchy stress tensor $\boldsymbol{\sigma}$, $\boldsymbol{C}^e$ is the elastic fourth-order tensor; the symbol ":" denotes the contracted product between the two tensors. Note that a superposed dot denotes differentiation with respect to time. With the assumption of linear isotropic elasticity, the elastic tensor $\boldsymbol{C}^e$ takes the following form in any coordinate system:

$$C^e_{ijkl} = \left(\frac{E\nu}{(1+\nu)(1-2\nu)}\right)\delta_{ij}\delta_{kl} + \left(\frac{E}{2(1+\nu)}\right)\delta_{ij}\delta_{kl} \tag{2}$$

with $i,j,k,l = 1...3$, δ_{ij} being the Kronecker delta tensor, E the Young's modulus, and ν the Poisson's ratio. For metallic materials, the associated flow rule is:

$$\boldsymbol{D}^p = \dot{\lambda}\frac{\partial F}{\partial \boldsymbol{\sigma}} \tag{3}$$

where $\dot{\lambda}$ is the plastic multiplier, and F is the yield function. It is assumed that hardening is isotropic and governed by the equivalent plastic strain, denoted $\bar{\varepsilon}^p$, then F takes the following form:

$$F(\boldsymbol{\sigma}, \bar{\varepsilon}^p) = \varphi(\boldsymbol{\sigma}) - Y(\bar{\varepsilon}^p) = \bar{\sigma} - Y(\bar{\varepsilon}^p) \tag{4}$$

where φ is the yield criterion, $Y(\bar{\varepsilon}^p)$ is the hardening law and $\bar{\sigma}$ denotes the effective stress associated to the yield criterion and $\bar{\varepsilon}^p$ is the work-conjugate of $\bar{\sigma}$.
As previously mentioned, in order to account for the influence of J_3 on yielding of isotropic materials and consequently on their plastic behavior under bulging, we use the isotropic form of Cazacu [3] yield criterion for which the effective stress is given by:

$$\bar{\sigma} = B\left[(J_2)^4 - \alpha J_2 (J_3)^2\right]^{\frac{1}{8}} \tag{5}$$

The parameter α can be expressed in terms of the yield stress in uniaxial tension, σ_{T} and the yield stress in shear, τ_{Y} , as:

$$\alpha = \frac{27}{4}\left[1 - \left(\frac{\tau_Y\sqrt{3}}{\sigma_T}\right)^8\right] \tag{6}$$

In Eq. (5), B is a constant defined such that for uniaxial tension, the effective stress $\bar{\sigma}$ is equal σ_{T}, i.e.

$$B = \frac{3\sqrt{2}}{\left[48(27-4\alpha)\right]^{\frac{1}{8}}} \tag{7}$$

For the yield surface to be convex, the parameter α should belong to the range: $-27/5 \le \alpha \le 3$.
As an example, Fig. 1 shows the projections in the biaxial plane $(\sigma_{xx}, \sigma_{yy})$ of the theoretical yield loci according to the Cazacu [3] yield criterion corresponding to $\alpha = 0$, (Von Mises), α = -5, and $\alpha = 3$, respectively. A Swift law is used to model hardening, i.e.,:

$$Y(\bar{\varepsilon}^p) = K_0\left(\varepsilon_0 + \bar{\varepsilon}^p\right)^n \tag{8}$$

where K_0, ε_0, and n are material parameters. For example for the aluminum alloy AA 6016-T4, $K_0 = 498.8\,\text{MPa}$, $\varepsilon_0 = 0.0089$, and $n = 0.285$ while the elastic properties are: E = 69 GPa and $\nu = 0.3$ (see benchmark data and information on this material reported in [7]).

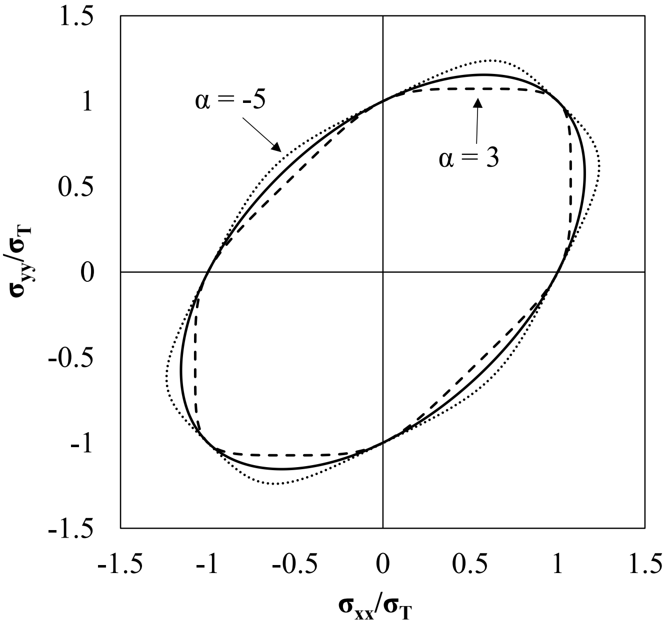

Fig. 1. Projections in the $(\sigma_{xx}, \sigma_{yy})$ plane of the yield loci corresponding to $\alpha = 0$ (von Mises), $\alpha = -5$, and $\alpha = 3$ according to Cazacu [3]. Stresses are normalized by the yield stress in uniaxial tension, σ_T.

3. FE Results of Bulge Tests

Numerical simulations of the bulge test were done with our in-house UMAT developed for the constitutive model given by Eq. (1)-(8). The circular blank considered has a radius of 100 mm and is 1 mm thick. It was meshed using Abaqus C3D8R solid elements (3D eight node linear brick elements with reduced integration), with three elements along the thickness direction. Due to the symmetry of the circular blank, only one-quarter of the specimen needs to be modeled; a total of 4557 FE elements were used. The die has a circular aperture, with an opening diameter of 140 mm and a fillet radius of 10 mm. It was meshed with Abaqus R3D4 rigid elements. Irrespective of the material, the bulging pressure was applied linearly with time.

Fig. 2 presents the evolution of the height of the pole with increasing pressure for materials characterized by $\alpha = 0$ (von Mises), α = -5, and $\alpha = 3$, respectively.

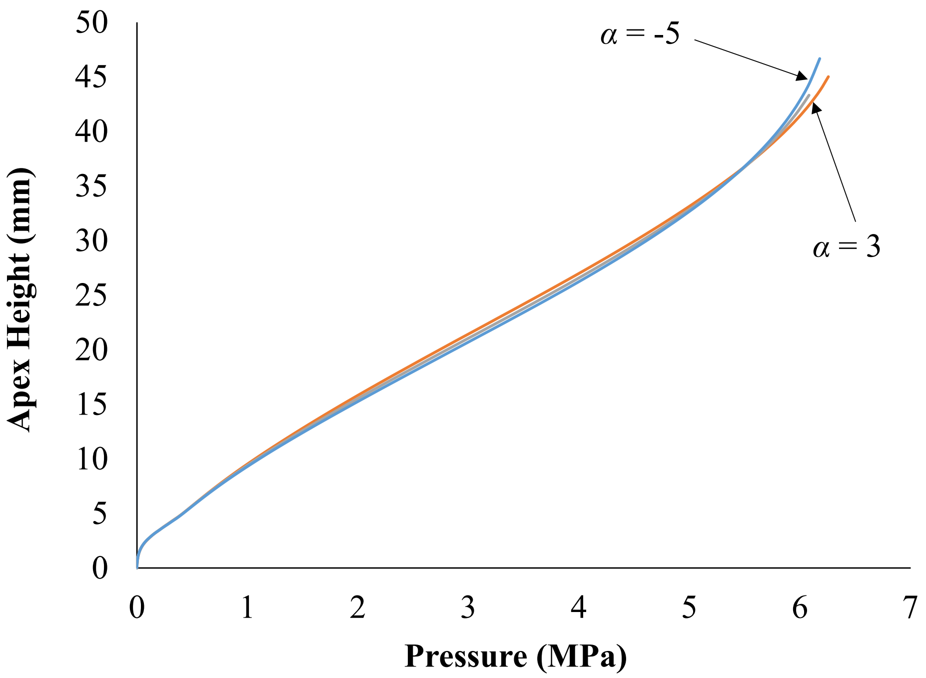

Fig. 2. Hydraulic pressure versus the pole height obtained with Cazacu [3] yield criterion for isotropic materials characterized by α = -5 (blue curve) α = 3 (yellow curve) respectively, and for a von Mises material (gray curve).The die aspect ratio is $a/b=1$ (hemispherical bulging).

Note that irrespective of the material, it takes about the same amount of pressure to achieve a certain apex height. This is to be expected given that the blank is circular and all three materials are isotropic, so they should have the same yield stress under equibiaxial loading.

Let us recall that in sheet metal forming applications where the material experiences stretching, the thickness at any region of the part is not allowed to be less than a specified threshold value, otherwise the part is not considered to be "safe." For this reason, it is crucial to accurately predict the reduction in thickness. Fig. 3 shows the thickness at the pole vs. height of the apex for the three isotropic materials. Note that the thickness at the pole depends on the parameter α that models the influence of J_3 on the plastic behavior. It is worth noting that for the same height of the bulge, the thickness at the pole is greater for the material characterized by $\alpha > 0$ than that of a von Mises material ($\alpha = 0$); on the other hand, for a material with $\alpha < 0$, the thickness is lower. Furthermore, for a certain thickness, the greater is the value of α, the higher is the bulge height meaning that to attain the same thickness the material has to bulge more. This suggests that for a material characterized by a negative value of α value, neglecting its dependence on J_3 and simply modeling its behavior with the von Mises criterion will lead to an overestimation of the thickness of the bulge,and consequently, the thinning of the material will be underestimated.

It is worth to further examine the isocontours of the equivalent plastic strain in the three bulges at the same pressure. Note that the level of equivalent plastic strain attained for the same pressure depends on the parameter α that models the influence of J_3 on the plastic behavior. The results shown in Fig. 4 indicate that for the material characterized by $\alpha > 0$ the maximum equivalent plastic strain in the specimen is lower than in a bulge obtained with a von Mises material (compare Fig 4(a) to Fig.4(c)) whereas for the material characterized by an α <0 the maximum equivalent plastic strain is higher (compare Fig 4(a) to Fig.4(b)). Those results are consistent with the trends shown in Fig.3.

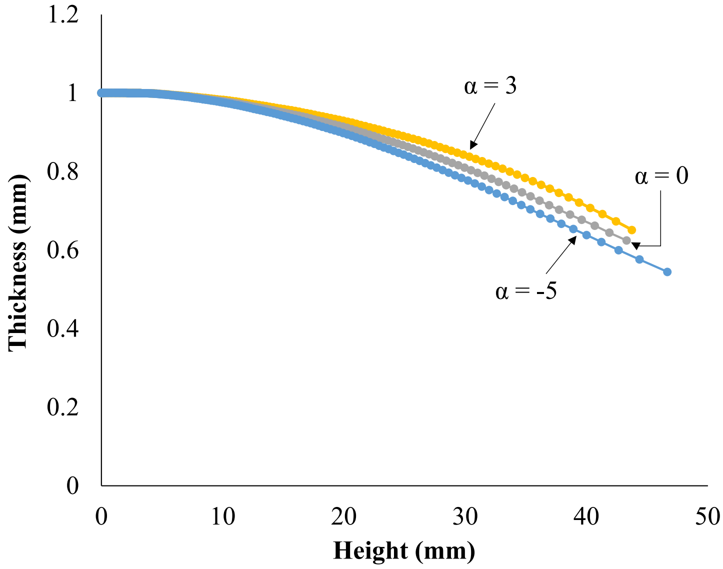

Fig. 3. Influence of J_3 on the thickness at the pole vs the apex height obtained with the isotropic form of Cazacu [3] yield criterion; die aspect ratio of $a/b=1$ (hemispherical bulging). Note that $\alpha=0$ corresponds to von Mises yielding behavior.

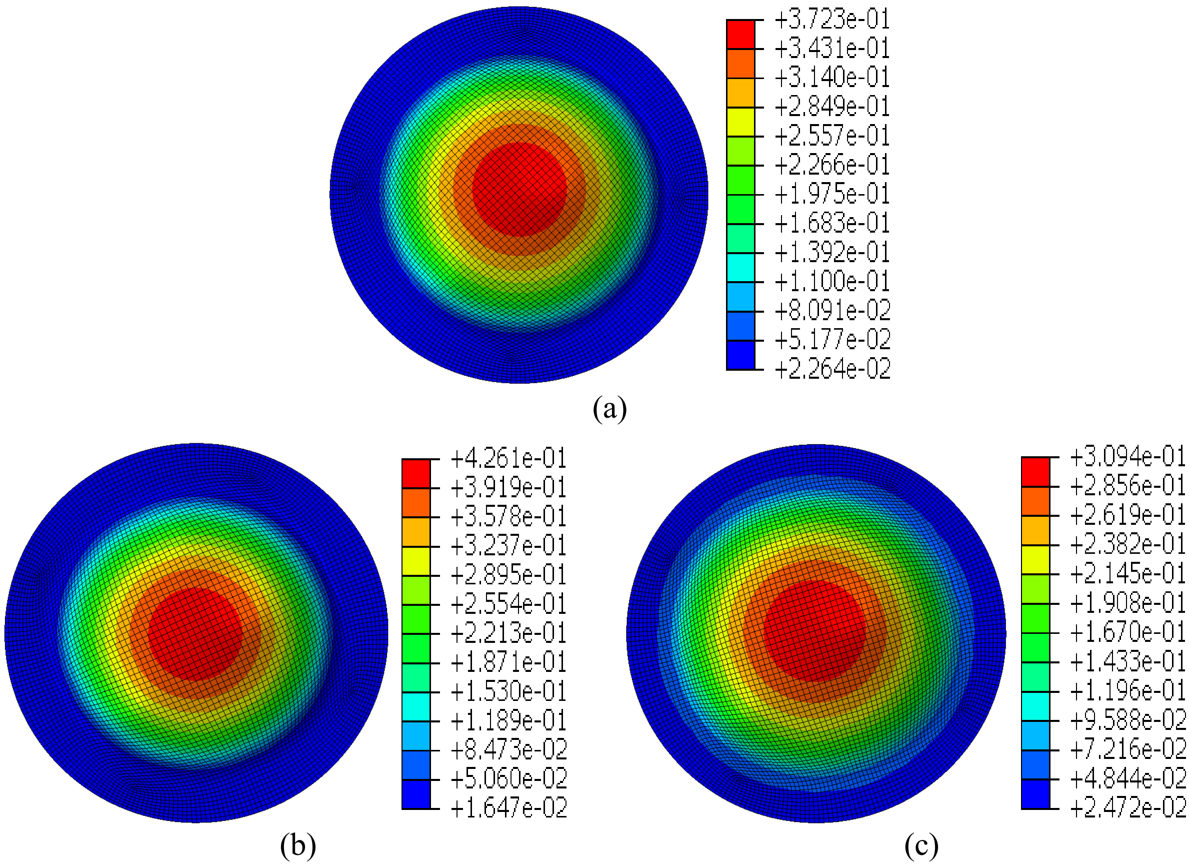

Fig. 4. Predicted isocontours of the equivalent plastic strain according to the isotropic form of Cazacu yield criterion [3] for about 4.73 MPa pressure for materials characterized by different values of the parameter α: (a) $\alpha=0$ (von Mises) (b) $\alpha=-5$, and (c) $\alpha=3$, respectively.

4. Conclusions

It is generally assumed that for isotropic materials, the effect of the third invariant on yielding does not affect the response during forming, so the von Mises yield criterion is used for data reduction and analysis of simple forming tests such as bulging under pressure. FE simulations using a yield criterion that enables to differentiate between isotropic materials on the basis of the influence of their yielding behavior on J_3, have shown that there is a correlation between the value of the parameter α that models that dependence on J_3 and the plastic strains that develop in the bulge and respectively the thinning behavior.

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