Method for the Estimation of Wear Resistance Curves in Sheet Metal Forming with Uncoated Tools

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Abstract. The wear resistance diagram (WRD) is a useful tool for quantitatively estimating the life span of a tribological system in sheet metal forming at different load levels. However, the WRD derived from the authors' previous research consists merely of scattered points, which make it difficult to reliably estimate the life span and its uncertainties at all load levels. Therefore, a wear resistance curve (WRC) that encompasses the life spans of all load levels is required for a broader scope of application of wear resistance estimation. In this paper, different models for the estimation of the S-N curve describing the fatigue behavior are tested to investigate their applicability in the estimation of wear resistance curves (WRC) in sheet metal forming. WRDs of different tribological systems are investigated and the results of curve fitting are evaluated. Besides the median of the estimated life spans, the WRCs of different confidence levels are also derived quantile calculation to estimate the load dependent uncertainties of tool life spans. Moreover, due to the time-consuming experiments to determine tool life, it is necessary to discuss possibilities for achieving a satisfactory estimation with the smallest possible number of tests. Therefore, the minimal number of tests and the suitable load levels for a satisfactory estimation will be discussed. After the study, several findings have been obtained:

- Compared to other models, Hwang and Han's model for estimating fatigue life of composite material shows good applicability and the highest accuracy in estimating the WRC of uncoated tools.
- In terms of damage development, the fatigue damage of the composite material and the damage caused by wear on uncoated tools have similarities, which explains the applicability of Hwang and Han's model for the estimation of WRC.
- Both higher and lower life wear data are the prerequisite for satisfactory wear estimation.

1. Introduction

In recent years, wear in sheet metal forming has become an important issue for several reasons. Firstly, the increasing demand for high-strength light-weight material offers an immense challenge, since the wear resistance of forming tools declines as a result of the high contact stress between tool and sheet metal [1]. Secondly, the ecological demand for environmentally friendly lubricants is increasing, leading to a reduction in lubricant quantities and the avoidance of environmentally harmful additives [2]. Thirdly, the surface of the forming tools should be manufactured as wear-resistant as necessary to avoid unnecessarily high expanditure for cost intensive surface treatment [3]. Against this background, it is required that the wear resistance of the forming tools is predictable.

Due to the system characteristic of a tribological system, wear prediction is still a challenge. Several methods have been proposed [4]. Archard [5] and Rabinowicz [6] have derived wear models for wear prediction. These models take into account the influence of contact pressure, sliding length and material hardness. The introduced model parameter wear coefficient k represents a challenge since its value deviates in dependence of the tribological system and the phase of wear development [7]. Besides wear models which describe the wear depth or volume also other parameters are used for wear prediction. Ashby and Lim proposed a concept for the prediction of the dominating wear

mechanism. The so called "wear mechanism map" is based on data gained in pin-on-disc tests [8]. This map offers a database for the analysis of influencing parameters such as sliding velocity and normalized pressure on the wear coefficient and mechanism. If the parameters of the load collective are known, the wear coefficient and the resulting wear mechanism for a specific tribological system can be estimated through the wear map.

In addition, in the field of forming technology, a method for wear prediction using the life span estimation through wear resistance diagrams (WRD) was developed [3,9]. These diagrams have been used to characterize the wear resistance of different tribological systems in laboratory tests [10]. The WRD illustrates the life span of tools under different contact stress, quantified by the sliding length reachable in the strip drawing test. The results of previous studies show that the life spans deviate drastically at different load levels, not only in terms of the absolute value of life span, but also in the variation of the life span under the same nominal conditions. At lower load levels, the obtained life span und its deviation increase significantly, while at higher load levels the life span is significantly lower and its deviation decreases distinctively [3]. The main barrier for the application of the WRD is the required amount of wear data, especially at lower tribological loads. So far, data have been obtained at high contact normal stresses with comparatively low total sliding length. Based on this data it is difficult to predict tool life under the loads commonly found in industrial forming processes. In order to extend the predictability of wear an extrapolation of the derived data is required. Therefore, a wear resistance curve (WRC) described by a mathematical model is proposed. Since the concept of WRD was inspired by the S-N curve of the material fatigue strength estimation [11], the transferability of the S-N curve estimation approaches to wear prediction is investigated in this study. Since the duration of the model test by means of strip drawing test is very lengthy [12], it is necessary to discuss possibilities to obtain a satisfactory estimate by performing as few tests as possible. In this paper, only the behavior and its established WRC of uncoated tools are discussed.

In section 2, established models for the S-N curve estimation are listed and pre-selected. The procedures for the calculation of the estimated fitting parameters will also be described in this section. After that, the selected models are investigated with raw data of WRD derived in previous wear resistance analyses of different tribological systems with the aid of strip drawing tests. The quality of fitting and the preliminary evaluation of different models are discussed in section 3. The models for variation estimation of the WRD are also discussed in this section. All selected models are evaluated according to their estimation accuracy for higher and lower levels as well as their uncertainty, which will be discussed in section 4. In section 5, the similarities of the damage development and mechanism between fatigue failure and wear are discussed to interpret the applicability of the models for S-N curve estimation on WRC estimation. Finally, the minimal number of tests and the suitable load levels for a satisfactory estimation will be discussed in section 6.

2. Models for S-N Curve Estimation

The concept of fatigue stress of metals was initiated by Woehler and intended for illustrating the endurance of metals through a series of material tests [13]. The curve which illustrated the stress and its resulting fatigue load cycles is called "S-N curve" (Woehler curve). This method of illustration is widely used to evaluate the fatigue strength of materials by means of its life span. For the illustration of fatigue strength under all stress levels, several mathematical models are derived which allow S-N curve estimation in different applications [14]. For example, the models of Basquin [15], Weibull [16], Henry and Dayton [17] and Stromeyer [18] are used to estimate the fatigue strength of single metals, while some other models, such as the models of Hwang and Han [19], Sendeckyi [20] and Poursatip and Beaumont [21], are used to estimate the fatigue strength of composite materials like glass fibre reinforced materials.

The models of S-N curve estimation are summarized in Table 1. In this paper, 10 models are taken into account.

Table 1 Models for S-N Curve estimation

	S-N Curve models	Number of fitting parameters	Material Parameters	Number of citing
Basquin[15]	$\sigma_{max} = \alpha(N_f)^{\beta}$	2	-	1963
Stromeyer[18]	$\sigma_{max} = \alpha(N_f)^{\beta} + \sigma_{\infty}$	2	σ_{∞}	162
Weibull[16]	$\sigma_{max} = (\sigma_{uT}) \cdot \exp[-\alpha(\log N_f)^{\beta}]$	2	σ_{uT}	4251
Henry and Dayton[17]	$\sigma_{max} = \dfrac{\alpha}{N_f} + \beta$	2	-	187
Sendeckyj[20]	$\sigma_{max} = \dfrac{\sigma_{uT}}{(1 - \alpha + \alpha N_f)^{\beta}}$	2	σ_{uT}	158
Hwang and Han[19]	$\sigma_{max} = \sigma_{uT}(1 - \dfrac{N_f^{\beta}}{\alpha})$	2	σ_{uT}	403
Kohout and Vechet[22]	$\sigma_{max} = \sigma_{\infty}[\dfrac{(N_f + \alpha)}{N_f + \gamma}]^{\beta} = \sigma_{uT}[\dfrac{(1 + N_f/\alpha)}{1 + N_f/\gamma}]^{\beta}$	3	$\sigma_{uT}, \sigma_{\infty}$	133
Kim and Zhang[23]	$\sigma_{max} = \sigma_{uT}(\dfrac{\alpha(\beta - 1)(N_f + N_0)}{\sigma_{uT}^{-\beta}} + 1)^{\frac{1}{1-\beta}}$	2	σ_{uT}	25
Poursartip and Beaumont[21]	$N_f = \alpha(\dfrac{\sigma_{max}}{\sigma_{uT}})^{-6.393} \cdot (\dfrac{\sigma_{uT} - \sigma_{max}}{\sigma_{uT}})$	1	σ_{uT}	130
D'Amore et al.[24]	$N_f = [\dfrac{\sigma_{uT} - \sigma_{max}}{\alpha' \cdot \sigma_{max}} + 1]^{1/\beta}$	2	σ_{uT}	78

In these models, the parameter σ_{max} stands for the stress amplitude and N_f the fatigue load cycles. For the application of the models for a specific material, the values for model parameters such as α, β and γ have to be determined. Additionally, data for material parameters such as σ_{∞} and σ_{uT} are needed for some of the models. The fatigue limit σ_{∞} is defined as the stress limit, under which an infinite number of fatigue load cycles can be borne by the material [25]. The ultimate tensile strength σ_{uT} corresponds to the tensile strength of the material and in view of the fatigue strength it can be defined as "the failure of material occurs within one cycle under the ultimate tensile strength". In the following section, the procedure for WRC-estimation by applying the models known from the S-N curve is presented.

2.1　Pre-selection of the models

Before estimating the WRC by using the models for S-N curves, the similarities between the WRC and the S-N curve are discussed.

The S-N curve shows the reachable number of load cycles under different stress amplitudes for a specific material, while the WRC defines the reachable life span of a tool in a tribological system under different levels of contact stress. The concept of "stress amplitude" in the S-N curve models corresponds to the "contact stress" of the WRC. Stress amplitude and contact stress represent the input of the model. The output parameters of the models are the life span of the material (S-N curve) and the life span of the tool before severe wear occurs (WRC). As a result of these similarities, the

parameter σ_{max} in the S-N curve can be substituted by the parameter p (contact stress) in WRC and the parameter N_f (fatigue load cycles) can be replaced by L (sliding length or stroke number). The substitution of parameters between S-N curve and WRC is illustrated in the following table:

S-N Curve	WRC
$N_f = f(\sigma_{max})$	$L = f(p)$

The model selection procedure for the WRC estimation consists of two steps. Firstly, if the models of S-N curve include parameters, which do not exist or are difficult to define in the concept of wear resistance analysis, the models are no longer considered as transferable. For example, the fatigue limit σ_∞ does not always exist even in fatigue strength analysis [26]. When analyzing wear resistance, this parameter is also difficult to define, since according to Archard's wear model, wear always occurs no matter how low the contact stress is [5]. Therefore, the models in Table 1 with the material parameter σ_∞ are not suitable for wear resistance estimation (eg. "Stromeyer" and "Kohout and Vechet") On the other hand, ultimate tensile strength σ_{uT} can be determined in wear resistance analysis. σ_{uT} can be defined as "the failure occurs after one load cycle" in the field of fatigue life study, while in the field of wear resistance analysis, it can be defined as "the severe wear occurs after a very short sliding distance". Christiany defines the "very short sliding distance" as 1 m in the strip drawing test [11]. Thus, a material parameter, the "ultimate contact stress p_{uT}" is derived with the definition "the contact stress, under which the severe wear occurs over a sliding distance of 1 m".

In this paper, the models for fatigue life estimation with widest application in academic field are pre-selected for applicability in the life span estimation. According to the citing frequency of the models listed in Table 1, the four models with the widest spread (determined by the number of citations) are considered in this work: "Basquin", "Weibull", "Henry and Dayton" and "Hwang and Han". The equations after parameter substitution and linearization are summarized in Table 2.

Table 2 Selected equations after parameter substitution and linearization for WRC estimation

S-N curve model	Original equation	Linearized equation
Basquin	$p = \alpha(L)^\beta$	$\log p = \log \alpha + \beta \log L$
Weibull	$p = p_{uT} \cdot \exp\left(-\alpha(\log L)^\beta\right)$	$\log\left[-\log\left(\dfrac{p}{p_{uT}}\right)\right] = \log \alpha + \beta \log(\log L)$
Henry and Dayton	$p = \dfrac{\alpha}{L} + \beta$	$pL = \alpha + \beta L$
Hwang and Han	$p = p_{uT}\left(1 - \dfrac{L^\beta}{\alpha}\right)$	$\log\left(1 - \dfrac{p}{p_{uT}}\right) = -\log \alpha + \beta \log L$

2.2 Determination of fitting parameters

After the model selection, the next step is to determine the fitting parameters α and β of the selected models. Since these parameters are estimated parameters, the fitting parameters are named as $\hat{\alpha}$ and $\hat{\beta}$ in this paper.

Curve fitting methods are used to analyze the transferability of models. Since the variation of tool life spans at lower load levels is relatively high due to the complexity of the tribological system [3], not only the median of life span but also its uncertainties are considered for a reliable life span estimation in this study. Compared to the curve fitting method "least square error", the method "Maximum likelihood estimation (MLE)" is more suitable for curve fitting in this research, since the determination of the fitting parameters can be related to the probability density function (pdf) from a sample set [27]. By using the pdf, it is possible to derive curves not only with 50 % confidence (median) but also with other confidence levels through quantile calculation. In this research, it is assumed that the distribution of the life span data for each load level follows the normal random distribution. For curve fitting with MLE, the equations should be linearized. The linearized equations are shown in Table 2.

When it comes to estimating the fitting parameters in a linear function (slope and intercept) using MSE, a linear regression is adopted. In linear regression, there is a basic hypothesis that the linear function is as follows:

$$y = \hat{a} + \hat{b}x + \varepsilon, \ \varepsilon \in N(0, \sigma^2) \tag{1}$$

in which ε is the random error, and $\hat{a}$ and $\hat{b}$ the to be determined fitting parameters. Taking the Basquin model in Table 2 as an example, the linearized equation can be written in $x = \log(L)$; $y = \log(p)$; $\hat{a} = \log(\hat{\alpha})$; $\hat{b} = \log(\hat{\beta})$. After the determination of $\hat{a}$ and $\hat{b}$, the fitting paramters $\hat{\alpha}$ and $\hat{\beta}$ can be calculated accordingly.

This equation is called unary linear regression model [28].

$$\varepsilon = y - \hat{a} + \hat{b}x \tag{2}$$

Supposing the variation of the tool life span follows the normal distribution, the likelihood function of collected samples $(x_1, y_1), \ (x_2, y_2), L, (x_n, y_n)$ for ε is: [29]

$$L = \prod_{i=1}^{n} \frac{1}{\sigma\sqrt{2\pi}} \cdot \exp\left[-\frac{1}{2\sigma^2}(y_i - \hat{a} - \hat{b}x_i)^2\right] \tag{3}$$

For the determination of the maximum value of the likelihood function (equation 1), it is to be assumed:

$$Q(\hat{a}, \hat{b}) = \sum_{i=1}^{n}(y_i - \hat{a} - \hat{b}x_i)^2 \tag{4}$$

being the minimum.

Taking partial derivative of $\hat{a}$ and $\hat{b}$ of equation (4) and solving the equations $\frac{\partial Q}{\partial \hat{a}} = 0$; $\frac{\partial Q}{\partial \hat{b}} = 0$, equation (5) can be obtained:

$$\begin{cases} n\hat{a} + (\sum_{i=1}^{n} x_i)\hat{b} = \sum_{i=1}^{n} y_i \\ (\sum_{i=1}^{n} x_i)\hat{a} + (\sum_{i=1}^{n} x_i^2)\hat{b} = \sum_{i=1}^{n} x_i y_i \end{cases} \tag{5}$$

$\hat{a}$ and $\hat{b}$ can be determined by solving equation (5). The result is given by equation (6):

$$\begin{cases} \hat{b} = \frac{n\sum_{i=1}^{n} x_i y_i - (\sum_{i=1}^{n} x_i)(\sum_{i=1}^{n} y_i)}{n\sum_{i=1}^{n} x_i^2 - (\sum_{i=1}^{n} x_i)^2} \\ \hat{a} = \frac{1}{n}\sum_{i=1}^{n} y_i - \frac{\hat{b}}{n}\sum_{i=1}^{n} y_i \end{cases} \tag{6}$$

After the determination of $\hat{a}$ and $\hat{b}$, the fitting parameters $\hat{\alpha}$ and $\hat{\beta}$ can be calculated according to Table 2.

For quantile estimation, the quantile line can be fitted by adjusting the fitting parameters $\hat{a}$ into $\widehat{a_q}$, supposing that the distribution obeys the Gauss distribution. For a $q\%$ quantile, the adjusted parameter $\widehat{a_q}$ is as following:

$$\widehat{a_q} = \hat{a} + \Phi^{-1}(q) \cdot \hat{\sigma} \tag{7}$$

The σ stands for the standard deviation of the random error ε (see equation 1).
The calculation of σ follows [30]:

$$\hat{\sigma}^2 = \frac{Q(\hat{a}, \hat{b})}{n-2} \tag{8}$$

Therefore, $\widehat{a_q}$ can be calculated by combining the equations (4), (7) and (8):

$$\widehat{a_q} = \hat{a} + \Phi^{-1}(q)\sqrt{\frac{\sum_{i=1}^{n}(y_i - \hat{a} - \hat{b}x_i)^2}{n-2}} \tag{9}$$

After the determination of $\widehat{a_q}$, the fitting parameter for quantile $\hat{\alpha}$ can also be determined through transformation according to the equations in Table 2.

3. Estimation of WRC for a Typical Tribological System in Sheet Metal Forming

In this section, four WRDs of four tribological systems with uncoated tools are discussed for WRC estimation. These include tribological systems with high and low wear resistance. The tribological systems tested are summarized in Table 3. The wear data of the tribological systems No. 1,2 and 4 are documented in the authors' previous publications [3,31]. The data amount and its load levels are also summarized in Table 3.

Table 3 The tested tribological systems with uncoated tool

No.	Tool	Sheet metal	Data amount	Load levels	Lubricant
1[3]	CP4M	DP980	15	3	
2[3]	CP4M	ZStE1000	9	3	PL61 of Zeller Gmelin Amount $1g/m^2$
3	1.2379 (X153CrMoV12)	1.4301 (X5CrNi18-10)	19	5	
4[31]	1.2379 (X153CrMoV12)	H630LA	12	3	

The first two tribological systems have a relatively high wear resistance (over 500 m at the contact stress of 590 N/mm^2), while the other two tribological systems have relatively low life spans (within 10 m at the contact stress of 200 N/mm^2). In this study, it is interesting to discuss the transferability of the pre-selected models on the WRC estimation for tribological systems with both high and low life spans.

The tribological system No.3 is a combination of cold working steel 1.2379 and stainless steel 1.4301 (X5CrNi18-10), which is also a critical tribological system with low wear resistance [32]. For this tribological system, 19 tests are deployed under five load levels. The life span of the tribological system is obtained through the measurement of thermoelectric current according to [31].

For WRC estimation, the parameter p_{uT} should be defined separately. As discussed in Section 2.1, p_{uT} is defined as the contact stress, from which the severe wear of the tribological system occurs with a sliding length of 1 m.

Table 4 Value of $\mathbf{p_{uT}}$ of the four tested tribological system

No. of tribological system	Value of $\mathbf{p_{uT}}$ (N/$\mathbf{mm^2}$)
1	880
2	910
3	550
4	518

3.1 The estimated WRC of tested system – median

The median estimation follows the procedure of fitting parameters' calculation according to equation (4). The estimated WRC for the median of the tested tribological systems 1 - 4 are shown in Figure 1. Not only the fitted curves but also the goodness of fitting in form of MSE (mean square error) is depicted. From the fitted curves, it can be found that there is some generality in the trend of the models from Basquin, Weibull and Henry, and Dayton. At higher load levels, the gradient of the curve is steep and most data of the WRD are located at the right of the derived curves, which means the tool lives are underestimated. In contrary, the gradient of the curves of the three fitting models becomes flatter at low load levels. At the lowest load level of the fitting data, most of the data is located on the left side of the derived curve, indicating an overestimated tool life at low load level.

Compared to the three other models, the model of Hwang and Han shows a different behavior. The gradient of the curves is flatter at higher load levels and steeper at lower load levels. The data of each

load level is relatively evenly distributed on both sides of the fitted curve, which means that the Hwang and Han model shows better prediction of tool life span for all load levels.
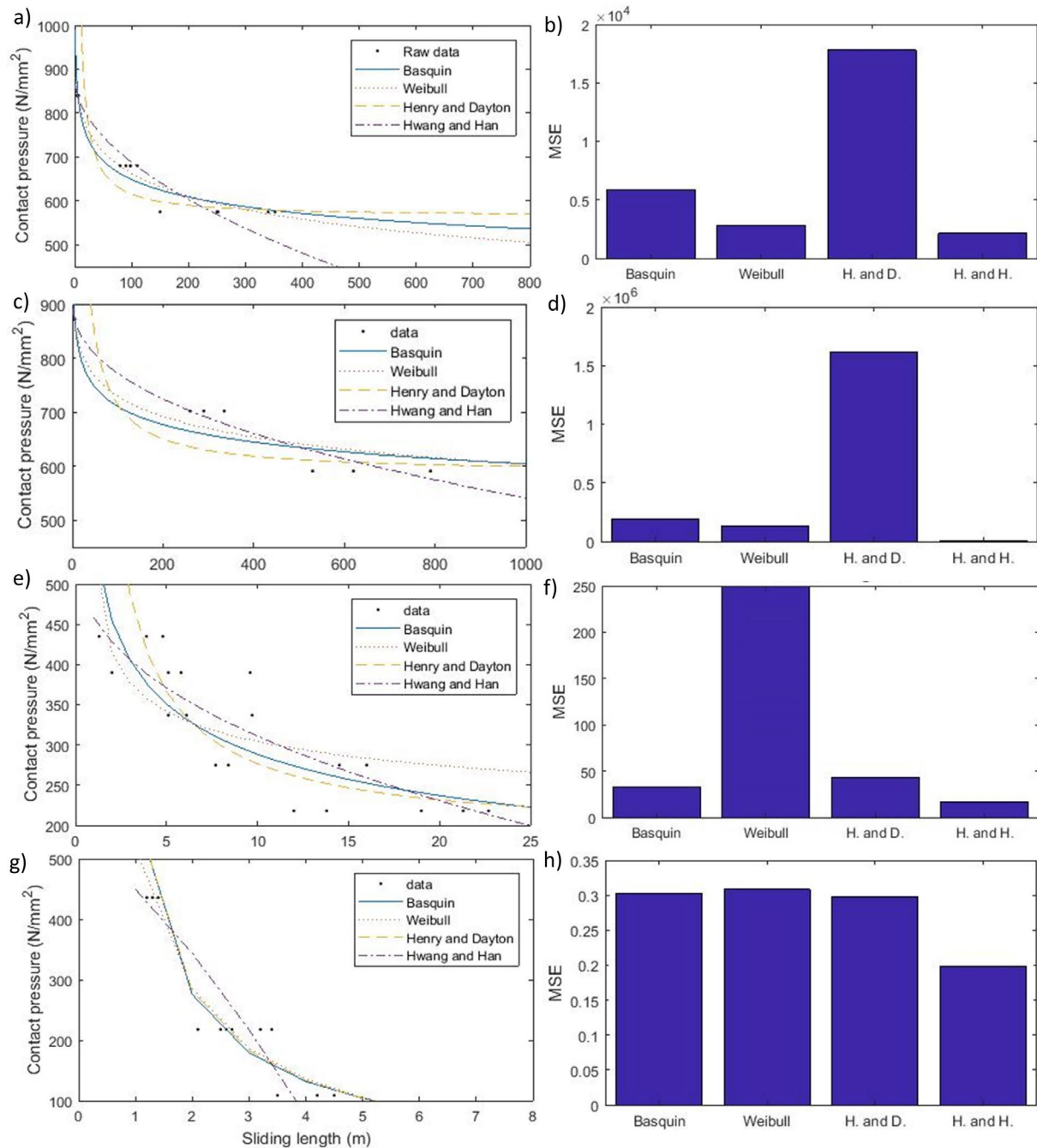

Figure 1 The results of curve fitting and its goodness of fitting for different fitting models
a) b) tribological system No.1; c) d) tribological system No.2; e) f) tribological system No.3;
g) h) tribological system No.4 (Abbreviation: H. and D. – Henry and Dayton; H. and H. – Hwang and Han)

Regarding the calculated MSEs, it can be seen that the Hwang and Han model has the least MSE in comparison to the other three models for all four tribological systems. In addition, the tribological systems tested belong to both high and low wear resistance tribological systems. From the results, it can be seen that the Hwang and Han model has a general validity for estimating the average tool life span of tribological systems, regardless of the wear resistance of the tribological system. Since the validity of the models can be approved under different experimental conditions for different tribological systems, the robustness of the models can be approved [33].

3.2 Quantile calculation for uncertainty estimation

The results in section 3.1 show that the median estimation is suitable for tribological systems with low variation of the life span at both higher and lower load levels. However, in practice, the variation of the life spans is load-dependent [3]. This shows the need for uncertainty estimation for lower tribological loads.

In section 2.2, the equation for quantile estimation is described. The difference between the median and the quantile estimation is the conversion of the fitting parameter according to equation (5).

The parameter $\Phi^{-1}(q)$ can be obtained from the „Standard normal table" according to the targeted confidence grade of the estimation. For example, a 95 % confidence level corresponds to the 5 % quantile of the normal distribution. This means that 95 % of the obtained life span data is located on the right side of the estimated curve with a 95 % confidence and the probability of error is 5 %.

In this section, the curves with confidence levels 95 %, 80 %, 70 % and the median (50 %) are plotted for all four tested tribological systems according to the three models with the best goodness of fit (see Figure 1). The results are illustrated in Figure 2.

After the derivation of the quantile, it is possible to determine the uncertainties of the estimation, especially for decreasing load levels. From the results it can be seen that most failures can be pre-estimated through the application of the curve with a confidence level 95 %, with only minor exceptions for several test points, which are located on the left side of the estimated curve (see Figure 2a and d).

The main difference between these models is the service lifespan at lower load levels. For example, for tribological system No.1, the lowest load level of the test data is 580 MPa, but the estimation for a lower load level like 500 MPa reveals large deviations when different fitting models are applied (with 95 % confidence 230 m is estimated by the model from Hwang and Han, while 800 m is estimated by Basquin's model). In practice, the effectiveness of the fitting for lower load levels also determines its applicability. Therefore, it is necessary to validate the estimation by additional tests for the life spans at lower load levels.
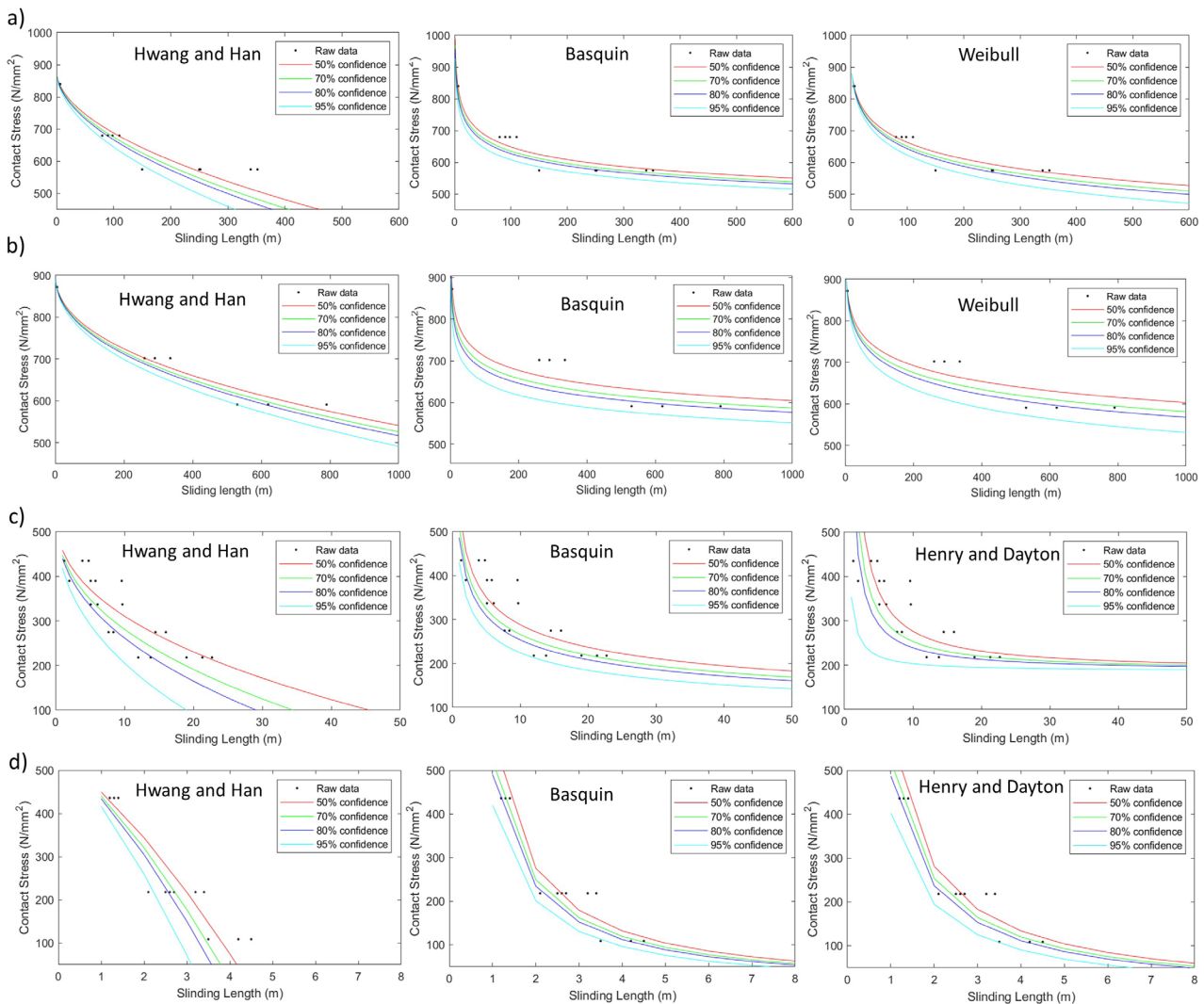
Figure 2 Quantile of different confidence levels of the three best models with goodness of fitting for a) tribological system No.1; b) tribological system No.2; c) tribological system No.3; d) tribological system No.4

3.3 Validation of the models at lower load levels

For examining the fitting models at lower load levels as well as the load levels other than those tested, several tests are deployed for validation. In this section, the tribological system No. 3 is used, since the data volume and the load levels are higher than for the other tribological systems. For characterizing the life span at lower load levels, wear tests are deployed twice at the load 163 N/mm^2 (30% to p_{uT}), which is about 75 % compared to the lowest load level of the fitting data. For further validation, three tests with randomly selected contact stress are deployed. The test matrix and the resulting life spans are illustrated in Table 5.

Table 5 Test matrix for fitting model validation

Test No.	Contact stress (N/mm^2)	Repetition	Life span (m)	Lubricant
1	163	4	15.1, 16.3, 33,42.6	PL61 of Zeller Gmelin Amount 1g/m^2
2	247	1	12.5	
3	312	2	8.9, 11.5	

The life spans of the test matrix are illustrated as scattered points in the diagram of estimated WRC in Figure 3. The life spans of Test No. 2 and 3 with intermediate load levels according to Table 5 are

located to the right side of the curve with 80 % confidence level for all the three models. The four life spans for the validation under the lowest contact stress (Test No.3) show a greater variation, which corresponds to the former research of the authors that the variation of the life span increases with decreasing contact stress.

From the validated results of Figure 3b and c, it can be seen that the estimated life spans of the model „Basquin" and „Henry and Dayton" for the lower load levels are extremely overestimated even for the estimation with 95 % confidence. In comparison, the estimation by the the model "Hwang and Han" shows much better performance in estimating the life span under the lower load levels, since all data points are located on the right side of the WRC with 95 % confidence.
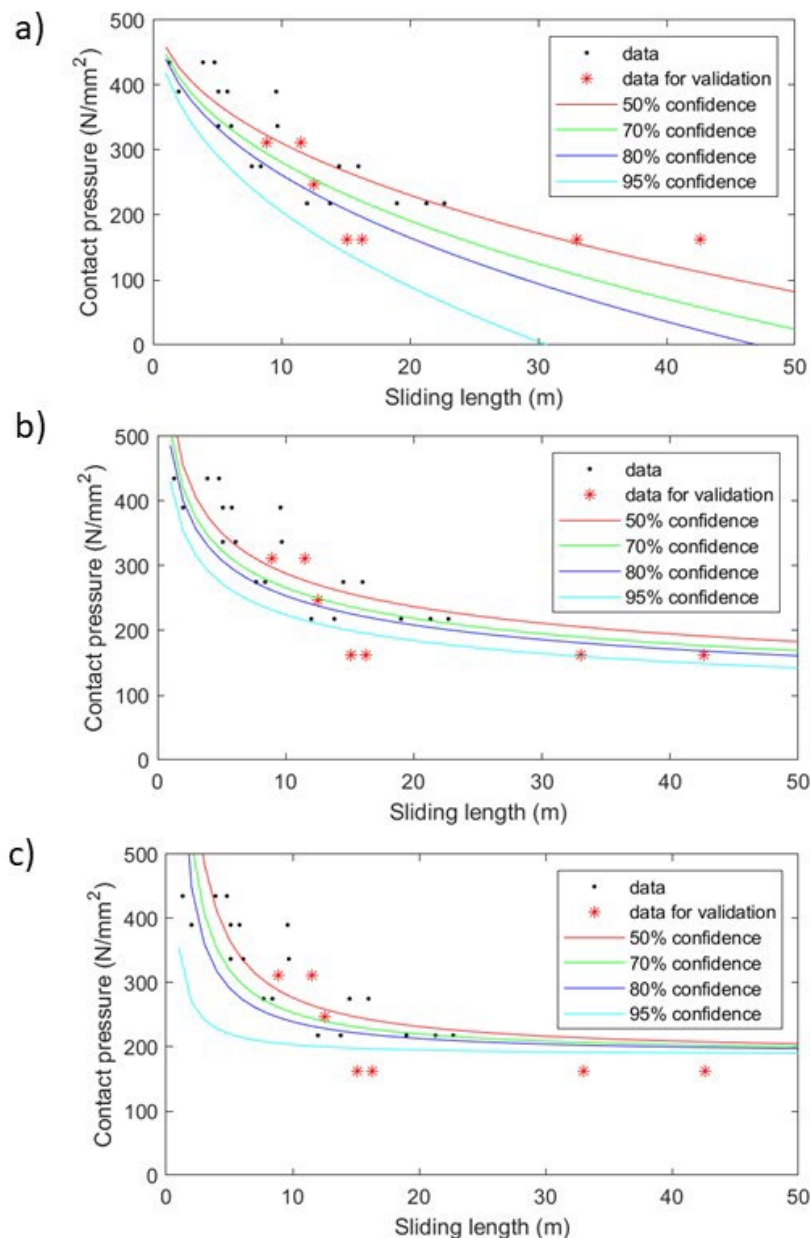

Figure 3 Results of validation of three different models a) Hwang and Han; b) Basquin; c) Henry and Dayton

Summarizing the results of sections 3.2 and 3.3, it can be seen that the estimation with the "Hwang and Han" model not only has the highest accuracy of curve fitting compared to the other fitting models, but also a better performance in estimating the service life for even lower load levels.

4.   Evaluation of the Models in Different Circumstances of Application

In the former sections, the applicability of the models was evaluated merely by calculating the MSE of all data. However, it is necessary to consider the models' performances in industrial applications. In industrial applications, the load levels vary in different sheet metal forming processes. Therefore, the goodness of fit should be evaluated for both higher and lower load levels.

In this section, tribological system No. 3 is used as an example to show how the required parameters are obtained. First, the MSEs are calculated separately for the highest load level (435 N/mm^2) and the lowest load levels (218 N/mm^2) of the tested data (see Figure 2b).

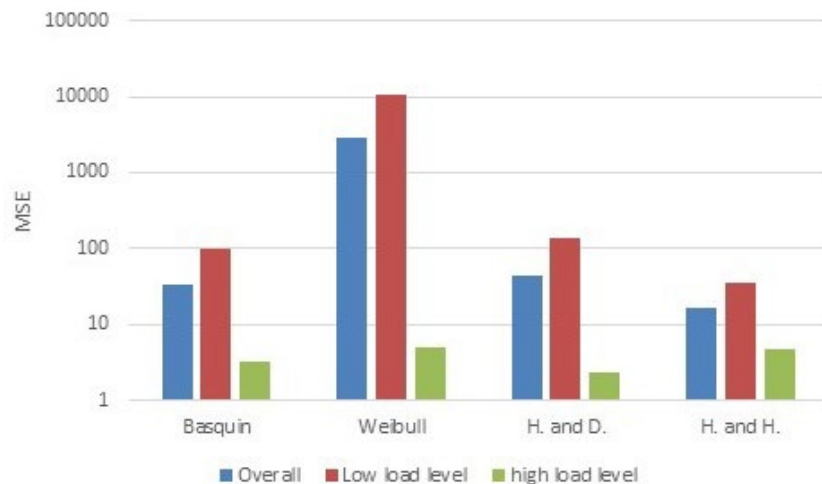The results of the parameter calculation are shown in Figure 4.

Figure 4 MSE for different load levels (Tribological system 3)

Combining the goodness of fit at both low and high load level, it can be seen that the fitting model "Hwang and Han" shows the best agreement in fitting the life spans at both high and low load levels (Figure 4).

5.   Interpretation of the Results

In the former sections, it is found that the model "Hwang and Han" is suitable for the life span estimation of uncoated tools. This model was originally derived for the fatigue strength estimation of composite materials with glass fiber [19]. It seems that the fatigue strength of composite material and the wear resistance of the uncoated tools are two different kind of problems. However, it is well worth considering why the "Hwang and Han" model can be used to estimate the wear resistance of the uncoated tools. In this chapter, the similarities of both concepts are discussed for interpretation of the obtained results above.

Firstly, regarding the damage development, both the fatigue damage [34] of the composite material and the wear development shows three phases (see Figure 5a and e). In the first phase, the running-in phase, the degradation of the composite material, which is characterized by the "fatigue modulus" [19], shows a regressive development until saturation. After the saturation point, it experiences a long-term "stationary phase" until an abrupt "failure" [34]. Comparatively, wear development shows a similar trend according to Habig [35], who proposed the typical wear development curve for metals. In former researches of the authors, the obtained wear development shows a similar development with three phases [3,9,10,31]. Therefore, the damage development describing fatigue strength and wear is the first similarity.

Secondly, in terms of the damage mechanisms of each phase, several similarities can also be obtained. In the first phase of the fatigue damage of composite material, cracks are initiated on the whole matrix of the composite material (see Figure 5b) [34], while tiny adhesion marks emerge on the tool surface, which is shown in the tribological system No. 3 (see Table 3 and Figure 5f). These cracks within the composite materials, as well as tiny adhesion marks, grow in the second phase, the stationary phase of the damage development (see Figure 5g). According to the damage development of material under

cyclic load, the length of crack can not only be extended, but also be suppressed [36]. This leads to a relative slow development in the stationary phase. Similarly, in the wear development, the roughness asperities of the adhesion marks can grow but can also be flattened, which also provokes a lower gradient. However, the overall size of wear marks tends to increase instead of being flattened and the wear marks merge gradually to form a severe wear mark (see Figure 5h), which cannot be flattened and therefore is irreversible. After the formation of the merged wear mark, the "failure" of the tool surface occurs. In comparison, in the development of fatigue failure of composite materials, the crack propagates to a certain extent and leads to a fracture of the material (see Figure 5c and d).
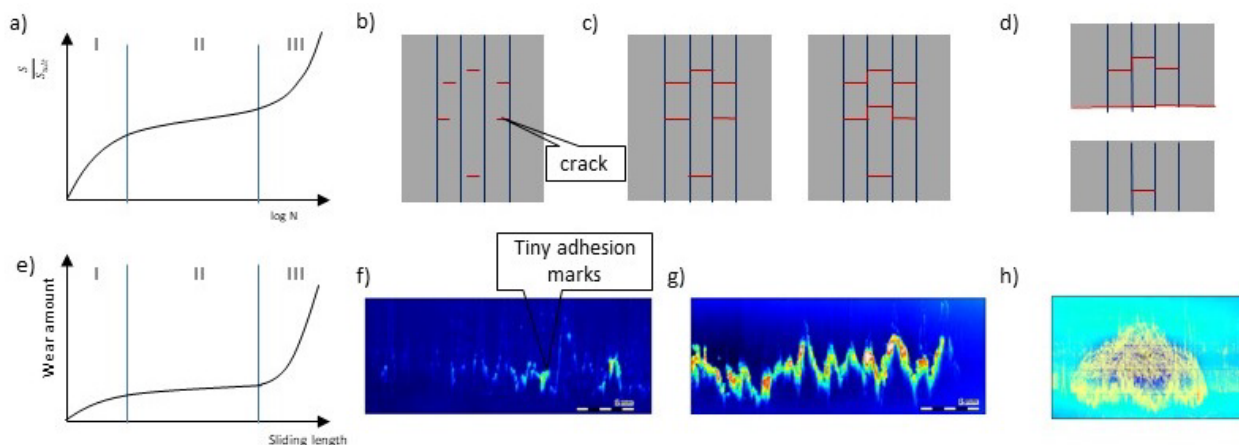

Figure 5 a) Fatigue development of composite material and its failure mechanism at b) running-in, c) steady-state and d) fracture phase [37]; e) typical wear development according to Habig [35] and the wear mechanism at f) running-in, g) steady-state and h) severe wear phase

Another question is, why the fitting model for composite materials such as "Hwang and Han" [19] instead of those for single models (such as "Basquin" [15]) is more suitable to the life span estimation of the uncoated tools investigated in this paper. This can be explained by the number of cracks in the initiating phase of failure development between composite and single material. Within the composite material, a larger number of cracks initiates during the running-in phase [37], while the fatigue damage of metal starts from a single crack initiated within the material [36]. Similar to the composite material, the tiny wear marks initiated during the "running-in" phase occur at several loci, which is also similar to the crack initiation within the composite material.

After analyzing the failure development and failure mechanism of composite fatigue failure and wear in composite materials, several similarities have been observed. These evidences support the applicability of the model "Hwang and Han" for the life span estimation of uncoated tool wear from physical similarities.

6. Discussion of Minimal Number of Tests for Wear Estimation

As mentioned in section 1, the other objective of estimating wear by the derivation of WRC is that the estimation should be satisfactory with a limited number of wear tests. Therefore, in this section, the influence of the number of tests and the selected load levels on the goodness of curve fitting will be discussed.

As an example, the tribological system No. 3 with 19 data points at five load levels is used. Firstly, it will be examined, if it is possible to estimate the tool lives for lower load levels through the test data of higher load levels. Therefore, the 19 data points are split into two parts, the data for fitting and the data for validation. The test matrix is shown in Table 6.

Table 6 1st Attempts for influence for load levels on fitting quality

Attempts	Load levels for fitting	Load level(s) for validation
1	1,2	3,4,5
2	1,2,3	4,5
3	1,2,3,4	5

The models with the best performance "Hwang and Han" with 50 % confidence are used for this investigation. Three test are made. In the first experiment, the results with the highest loading level 1 and 2 serve as the fitting data, and the fitting parameters are defined according to the fitting model in equation (4).

The estimated life spans for load levels 3, 4 and 5 are calculated and summarized in Table 7. The same procedure is repeated for the 2nd and 3rd experiment according to Table 6. In Table 7, the estimated load levels at the whole load levels with 19 data points are also included as a reference. The relative errors of attempts 1 - 3 with respect to the reference (attempt 4) are also listed in Table 7 in brackets.

Table 7 Estimated life span and its relative error with higher load levels as reference to the estimated life span with whole load levels

Attempts	Estimated life span (m) and its (relative errors)		
	Load level 3	Load level 4	Load level 5
1	205.7 (2610%)	2197 (15604%)	12593 (57219%)
2	-	58.04 (315%)	137.3 (525%)
3	-	-	31.24 (42.2%)
4 (reference)	7.59	13.99	21.97

The results in Table 7 indicate that the errors for test 1 and 2, which use 2 and 3 higher load levels for life span estimation, are high. In test 1, for example, the estimation of load level 3 has already an error with factor 26, while the estimation for load level 5 has an error with factor 572. For the 3rd experiment, the error is decreasing to an error of 42 %. Nevertheless, it is still unacceptably high and the estimated life spans are overestimated with all the experiments. From these results, it can be seen that the estimation for lower load levels through fitting data from higher load levels is of poor quality and the estimation of life span is overestimated. Therefore, these experiments are not suitable for WRC estimation for all load levels in practical application.

In the following, three more tests are deployed. In these tests, the number of load levels corresponds to the test 1 - 3 (see Table 6). The only difference is that the data of load level 5, the lowest load level of the test data, always serve as fitting data. The attempts 5-7 are summarized in Table 8.

Table 8 2nd Attempts for influence for load levels on fitting quality

Attempts	Load levels for fitting	Load level(s) for validation
5	1,5	2,3,4
6	1,2,5	3,4
7	1,2,3,5	4

Similar to test 1 - 3, the model „Hwang and Han" with 50 % confidence is used for curve fitting. The estimated life span and its relative errors are demonstrated in Table 9.

Table 9 Estimated life span and its relative error with higher load levels as reference to the estimated life span with whole load levels

Attempts	Estimated life span (m) and its (relative errors)		
	Load level 2	Load level 3	Load level 4
5	3.91 (23.5%)	8.82 (16.2%)	18.26 (30.5%)
6	-	8.42 (10.9%)	15.27 (9.1%)
7	-	-	14.89 (6.4%)
4 (reference)	3.82	7.59	13.99

From the results, it can be seen that the estimation quality of experiments 5 - 7 has improved dramatically compared to experiments 1 - 3 after the consideration of load level 5 for curve fitting. Even the lowest number of load levels for fitting data (test 5) offers a better quality of fitting than the fitting data of four load levels (test 3) without the participation of load level 5. When the number of load levels for curve fitting increases to three (test 6), the relative error decreases to around 10 %. Therefore, it can be concluded that the pre-requisite of a high-quality fitting for WRC is the consideration of the wear data of both high and low load levels. By using the pre-selected models in Table 2, it is hard to obtain a satisfactory estimation of the whole load levels by using only the first two load levels with the shortest test duration of the strip drawing test. The wear data of lower load level is needed for a more accurate estimation.

7. Conclusions

In this paper, the process for estimation of wear resistance characteristic curves (WRC) is described. Four basic models for S-N curve estimation are used in this approach. Not only the median but also the curves with different quantiles are calculated. Of all the models, the model „Hwang and Han" is most suitable for the WRC estimation for different kinds of tribological systems with different wear resistance through the analysis of the mean square errors (MSEs). Regarding the fitting functionality for higher and lower load levels separately, the fitting through model „Hwang and Han" also has a good fitting accuracy at both higher and lower load levels. From the results it can be seen that the model for fatigue life estimation for composite material "Hwang and Han" serves for the life span analysis of uncoated tools of forming processes. The similarities in the mechanisms and development between composite fatigue life and tool wear life are also discussed in this paper to support this finding.

Since wear tests are very time consuming, this paper also discusses the possibility of estimating life span at lower load levels using data at higher load levels with short-term tests. It is concluded that the accuracy of fit in predicting wear behavior at lower load levels based on data from only the higher load levels is unsatisfactory. Thus, this strategy for life span estimation is not applicable. For a satisfactory estimation of the WRC, the prediction of wear data for higher and lower life spans is the pre-requisite.

The approach described in this paper offers a new aspect for life span estimation in sheet metal forming processes. With empirical data on the life span of the actual forming process corresponding to the lower load level, and the wear data in model tests at higher load levels such as strip drawing tests, it is possible to estimate the life spans and its uncertainties by applying estimation models like „Hwang and Han" for all load levels. As a result, through the estimation process of the WRC, the tool maintenance can be optimized.

Appendix – Nomenclature

σ_{max}	Stress amplitude (N/mm^2)
N_f	Fatigue load cycles (-)
α, β and γ	Fitting parameters (-)
σ_∞	Fatigue limit (N/mm^2)
σ_{uT}	Tensile strength of the material (N/mm^2)
L	Sliding length until wear occurrence (m)
p	Contact stress (N/mm^2)
p_{uT}	Ultimate contact stress (N/mm^2)
$\hat{\alpha}, \hat{\beta}, \hat{a}$ and $\hat{b}$	Fitting parameters for estimated curves (-)
ε	Random error (-)
l	Likelihood function (-)
σ	Standard deviation (-)
$\widehat{a_q}$	Fitting parameters for quantile calculation (-)
q	Quantile (-)
MSE	Mean square error (-)

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