

# Combined Probabilistic Approach and Stress State Dependency on the Failure Modeling of HPDC Aural-2 Alloy

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**Abstract.** Both experimental method and numerical method are used to analyze the large variation in the material ductility of high pressure die casting (HPDC) Aural-2 alloy in the present work. The X-ray tomography (XRT) technique is used to characterize and reveal the significant variation of the internal porosity for the investigated material. The von Mises plasticity model in conjunction with a mixed Swift-Voce hardening law, and a stress state dependent fracture initiation criterion are used to accurately describe the deformation response of the material. Very good agreement with the experimental results is obtained in the predicted average force-displacement responses for the calibrated stress states. A probabilistic damage mechanics model is put forward to depict the apparent stochastic ductile fracture behavior over a wide range of stress states. The 5<sup>th</sup> and 95<sup>th</sup> percentiles of the fracture initiation locus are recalibrated based on the proposed probabilistic ductile fracture model, which could provide an almost perfect prediction of the maximum and minimum bounds of force-displacement curves.

## Introduction

Al-Si alloys are the most widely used foundry alloys in the automotive and aerospace industries due to a good combination of mechanical properties and castability. The high pressure-die casting (HPDC) method, which is well known for its high efficiency, enables to produce complex-shaped thin-walled castings [1]. Thus, aluminum high pressure-die castings have become essential elements in the car body design, mainly used as structural nodes and connector elements where high forces are introduced locally and various components need to be connected. However, casting defects caused by the high pressure-die casting processes always lead to a pronounced variation in the ductility and fracture properties of the material [2-4].

The factors normally governing the ductility of aluminum castings may be classified into two categories: intrinsic factor and extrinsic factor. Intrinsic factors include several microstructure characteristics, i.e., grain size of matrix, morphology and spatial distribution of eutectic Si phase etc. Gas pore, shrinkage pore, hot tear, cold shut, and inclusions are primarily involved in the extrinsic factor. The influence of the intrinsic factor, i.e., microstructure, on the ductility of aluminum castings has been investigated in our previous research [5]. The objective of the present work is to predict accurately the deformation and failure behavior of aluminum casting alloys through a numerical simulation approach considering the influence of casting defects.

It is well known that understanding void evolution is the most effective routine to predict ductile fracture of aluminum castings [6]. Ductile fracture process is microscopically characterized by nucleation, growth, and coalescence of microvoids. Damage mechanics model are divided into two groups (i.e., uncoupled and coupled) following their interaction between damage and plasticity behavior [7]. In coupled models, the effects of damage on the yield surface have been taken into account. The most classic Gurson–Tvergaard–Needleman (GTN) model in the family of coupled damage models, is based on the hypothesis that failure is activated once the  $f_c$  (the parameter of critical void volume fraction) is reached during deformation. Such model has been extended for a wide variety of applications [8]. Despite the GTN model has been widely used, it still has some disadvantages. One of the most obvious shortcomings is that there are too many parameters, which makes it ambiguous to calibrate such criteria and expensive to use in a simulation. In the alternative approach, the failure criteria in the family of uncoupled damage models are typically formulated by assuming that the failure strain is a weight function of stress state variables, typically with fewer parameters, which makes the calibration process easier. In recent developments of ductile fracture models, the influence of both stress triaxiality and Lode angle parameter has been verified in many applications. Numerous damage models have been proposed for predicting ductile fracture initiation by incorporating the stress triaxiality and Lode angle parameter in recent proposed damage models. For instance, the Bai–Wierzbicki (BW) model [9], modified Mohr–Coulomb (MMC) model [10], modified Hosford–Coulomb (MHC) model [11], and the series of ductile fracture model by Lou et al. [12] were proposed. The hybrid modified Bai–Wierzbicki (MBW) model presented by Lian et al. [13] has also been frequently used to depict damage and fracture phenomena of various types of steels under different loading conditions. Baral et al. [14] have used a new damage criterion, which is a linear combination of the Oyane and Johnson–Cook models, to predict the ductile fracture behavior of an Al–Si–Mg die-cast alloy.

In most of current studies on ductile fracture prediction, a deterministic failure criterion is typically applied, without considering the fracture uncertainties [15, 16]. This strategy is reasonable for metallic materials produced by conventional processes, where the final microstructure is homogeneous and degree of defects is relatively low. However, for materials produced from casting and additive manufacturing processes, a increased degree of microstructural heterogeneity and defects is usually observed in the final products, which leads to a more pronounced variation in the plasticity and fracture properties of such materials [17]. Therefore, it is necessary to combine the probabilistic concept with the damage mechanics models to describe the stochastic ductile fracture of some metallic alloys. In the present study, the tensile properties and ductile fracture behavior of a high-pressure die-casting Aural-2 alloy have been investigated using a probabilistic damage mechanics model. Tensile tests using different specimen geometries have been used to characterize the fracture behavior in different stress states. The isotropic von Mises plasticity model in combination with a probabilistic ductile fracture criterion is used to predict the deformation and failure response of the material. In the probabilistic ductile fracture model, the fracture criterion corresponding to different failure probabilities has been calibrated. With the developed approach, the maximum and minimum bounds of fracture displacements in different tensile tests can be predicted accurately over a wide range of stress states.

## Material Characterization and Experiments

### Material characterization

The material under investigation is Aural-2®. The AURALTHERM heat treatment was applied to all Aural-2 specimens to obtain the optimal mechanical properties, which contains a solid solution treatment at 450 °C for 1.5 hours followed by water quenching, then treated at 225 °C for 2 hours as artificial aging. Porosity characterization from X-Ray Tomography reveals an average porosity in samples gauge is in the range from 0.11%~1.04% (volume fraction), as shown in Fig. 1.

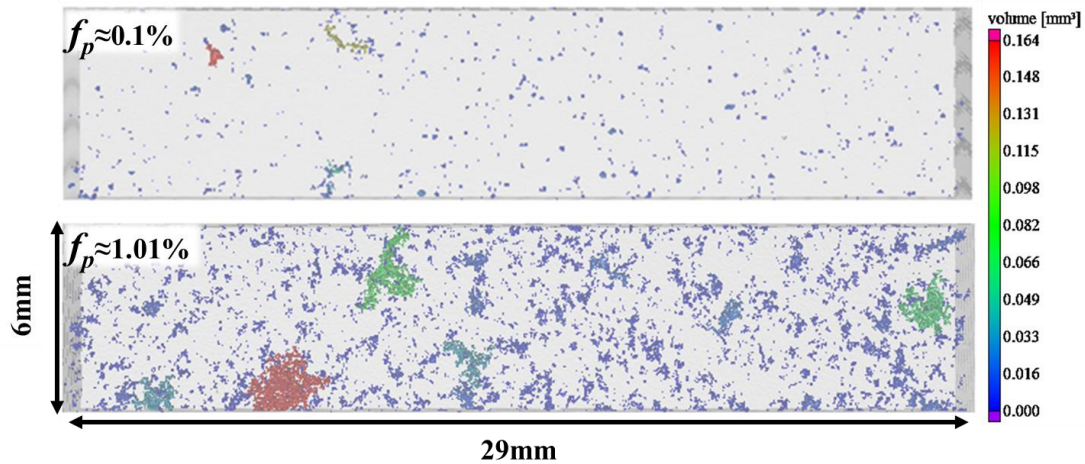


Figure 1. Stochastic distribution of the average volume fraction of porosity in HPDC Aural-2 samples.

### Mechanical tests

For the characterization of plastic flow behavior at room temperature, uniaxial tensile tests have been performed on smooth dog-bone (SDB) specimens according to DIN EN ISO 6892-1. All specimens were cut from flat samples produced via high pressure die casting process. The initial distance between the extensometer arm is 40 mm along the loading direction. The SDB tensile experiments have been conducted using a Zwick machine Z100 with a cross-head velocity of 0.2 mm/min. Six parallel tests for the SDB samples have been performed. The results of uniaxial tensile tests for all six specimens are presented in Fig. 2. The ductility of the investigated HPDC Aural-2 alloy shows a pronounced variation.

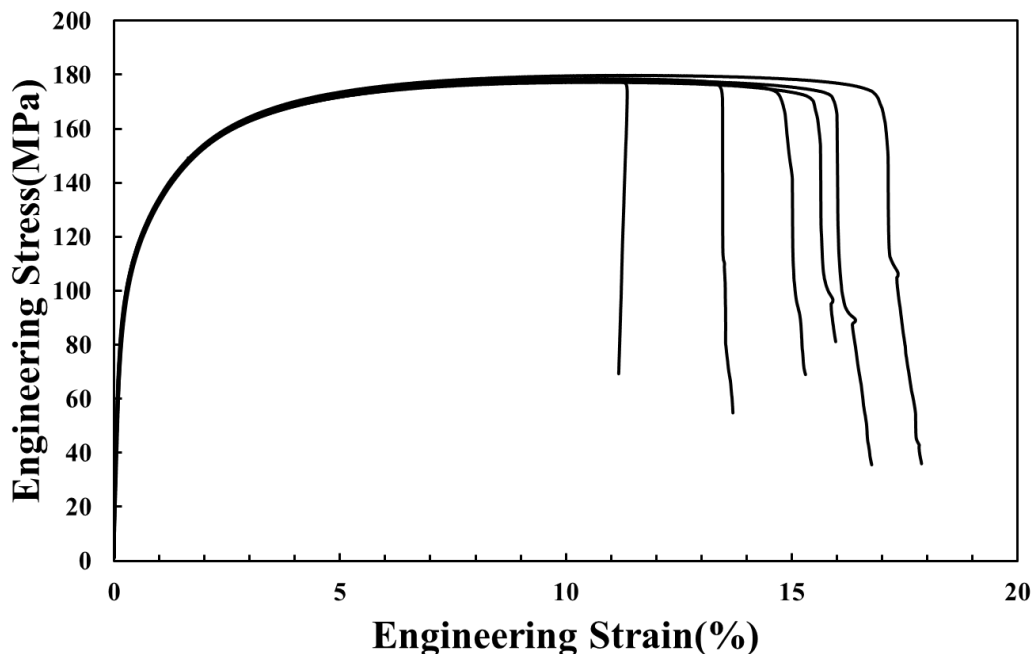


Figure 2. Experimental results of engineering stress and strain curves of the HPDC Aural-2 alloy obtained from six parallel uniaxial tensile tests.

Tensile tests on flat specimens with various geometries have been performed to analyze the effect of stress states on the ductile fracture properties of the studied material at room temperature. The dimensions of the tested samples are summarized in Fig. 3, including one shear (SH) geometry, one central hole (CH) geometry and two notched dog bone (NDB-R30, NDB-R6) geometries. The broad range of stress states between shear and plane strain tension can be covered by the designed testing program. All tensile tests have been conducted with a cross-head velocity of 0.2 mm/min to maintain

the quasi-static loading conditions. For each geometry, six repeating tests have been conducted to evaluate the scatter in the ductile fracture properties of the Aural-2 alloy. The results of fracture tests are provided in the following.

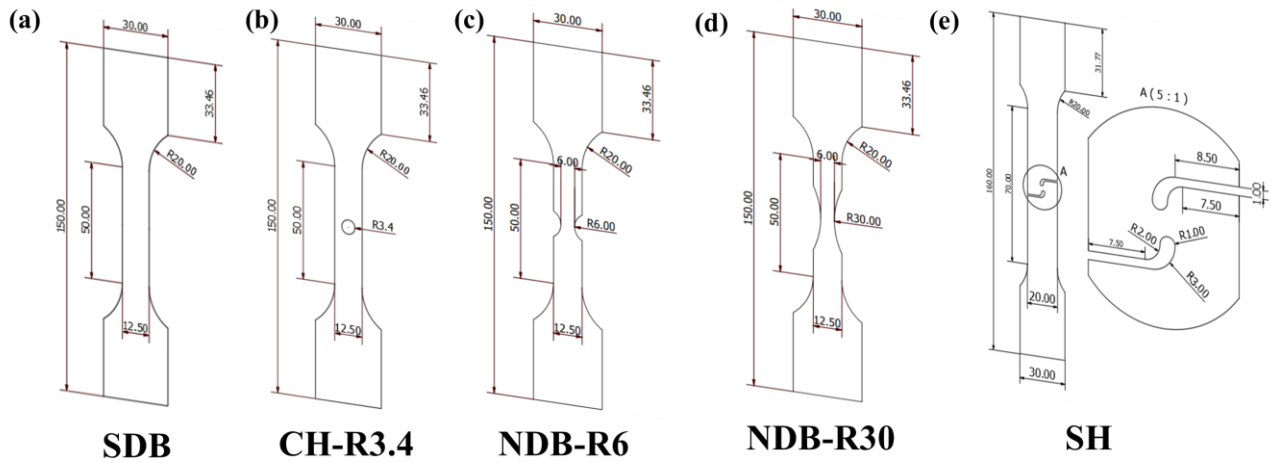


Figure 3. Overview of the geometry of plasticity and fracture specimens (all units are mm).

## Material Models

### Damage mechanics model

The modified Bai-Wierzbicki (MBW) model proposed by Lian et al. [13] has been further developed here to describe the ductile fracture behavior of a high-pressure die-casting aluminum alloy considering the influence of the stochastic distribution of the porosity. Detailed derivation and annotation of MBW model could be found in [16, 18]. The isotropic von Mises equivalent stress  $\bar{\sigma}(\boldsymbol{\sigma})$  is applied in the yield criterion  $f$  and a scalar damage variable  $D$  is used to quantify the damage effects. The flow stress  $\sigma_y(\bar{\epsilon}^p)$  is calibrated using the combined Swift-Voce strain hardening law for this material.

$$f = \bar{\sigma}(\boldsymbol{\sigma}) - (1 - D)\sigma_y(\bar{\epsilon}^p) \leq 0. \quad (1)$$

The effects of stress states on the failure properties of isotropic materials are determined by the two most widely loading indicators [18], i.e., the triaxiality  $\eta$  and the Lode angle parameter  $\bar{\theta}$ , which can be derived from three stress invariants ( $I_1, J_2, J_3$ ).

$$\eta = \frac{I_1}{3\sqrt{3}J_2} = \frac{(\sigma_1 + \sigma_2 + \sigma_3)}{3\sqrt{\frac{1}{2}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]}}. \quad (2)$$

$$\bar{\theta} = 1 - \frac{2}{\pi} \cos^{-1} \left( \frac{3\sqrt{3}}{2} \frac{J_3}{J_2^{3/2}} \right). \quad (3)$$

In the deterministic form of the ductile fracture criterion, the average stress triaxiality  $\eta_{avg}$ , average Lode angle parameter  $\bar{\theta}_{avg}$ , and an indicator  $I_{dd}$  is used to describe the ductile damage behaviour, considering the non-proportional loading effects. A ductile damage initiation locus is defined by the critical equivalent plastic strain  $\bar{\epsilon}_{di}^p$  as a function of stress state parameters. Ductile fracture occurs once the indicator  $I_{dd}$  reaches unity during plastic deformation.

$$I_{dd} = \int_0^{\bar{\epsilon}^p} \frac{d\bar{\epsilon}^p}{\bar{\epsilon}_{di}^p(\eta_{avg}, \bar{\theta}_{avg})}. \quad (4)$$

$$\bar{\epsilon}_{di}^p(\eta_{avg}, \bar{\theta}_{avg}) = [D_1 \exp^{-D_2 \eta_{avg}} - D_3 \exp^{-D_4 \eta_{avg}}] \bar{\theta}_{avg}^2 + D_3 \exp^{-D_4 \eta_{avg}}. \quad (5)$$

### Probabilistic failure criterion

To consider the variation of ductile fracture properties due to random distribution of porosity in HPDC Aural-2 material, the normal distribution function is selected to describe the stochastic characteristics of the ductile initiation strain. The cumulative probability concept is incorporated into the ductile failure criterion. As ductile failure is more likely to be triggered with increasing plastic strain, the cumulative failure probability can be expressed by the cumulative distribution probability ( $P_f$ ) of failure strain  $\bar{\varepsilon}_f^*$ . The normal cumulative distribution function of failure strain is described as:

$$P_f = P_f(\bar{\varepsilon}_f \leq \bar{\varepsilon}_f^*) = \frac{1}{2} \left[ 1 + \operatorname{erf} \left( \frac{\bar{\varepsilon}_f^* - \bar{\varepsilon}_f^m}{\sqrt{2} \cdot \Delta} \right) \right]. \quad (6)$$

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt. \quad (7)$$

$\operatorname{erf}(x)$  is the Gaussian error function in the normal distribution;  $\bar{\varepsilon}_f^m$  is the mean failure strain of each stress states. The ductile failure strain  $\bar{\varepsilon}_f^*$  corresponding to a specific failure probability  $P_f$  can be explicitly determined as:

$$\bar{\varepsilon}_f^* = \bar{\varepsilon}_f^m + \sqrt{2} \cdot \Delta \cdot [\operatorname{erf}^{-1}(2P_f - 1)]. \quad (8)$$

For the application of the probabilistic damage and failure criteria, the mean values of damage initiation strain  $\bar{\varepsilon}_{di}^{p,m}(\eta_{avg}, \bar{\theta}_{avg})$  need to be determined first based on experimental results. Noted here that  $m$  in  $\bar{\varepsilon}_{di}^{p,m}$  represent the mean strain for each stress states. Then the damage initiation strain  $\bar{\varepsilon}_{di}^p(\eta_{avg}, \bar{\theta}_{avg}, P_f)$  at any specific failure probabilities can be explicitly calculated. The calibration and validation of this model are elaborated by the numerical prediction of ductile fracture properties of the HPDC Aural-2 alloy.

### Numerical Simulations

#### Calibration of model parameters

A fine mesh of  $0.1 \times 0.1 \times 0.1 \text{ mm}^3$  has been created in the critical region of different tensile geometries. Solid elements with quadratic reduced integration from the ABAQUS/Explicit software have been used in these finite element models. The combined Swift-Voce hardening law is used to describe the plasticity of the material.  $A$ ,  $\varepsilon_0$  and  $n$  are material parameters of the Swift hardening law and  $k_0$ ,  $Q$  and  $\beta$  are coefficients of the Voce equation. The corresponding  $\alpha$  coefficient, as summarized in Table 1, has been inversely calibrated based on the uniaxial tensile stress and strain curves and displacement results obtained from tensile tests of different sample geometries.

Table 1 Calibrated hardening parameters.

Material	$\sigma = \alpha \cdot A \cdot (\bar{\varepsilon}^p + \varepsilon_0)^n + (1 - \alpha) \cdot [k_0 - (Q \exp^{-\beta \cdot \bar{\varepsilon}^p})]$						
	$A$	$\varepsilon_0$	$n$	$\alpha$	$k_0$	$Q$	$\beta$
Aural-2	200	0.01	0.25	0.54	300.00	137.40	51.99

Based on the finite element simulation results using the calibrated plasticity model, the evolution of local stress states (stress triaxiality and Lode angle parameter) with equivalent plastic strain (PEEQ) in the critical element of different specimens has been collected, as demonstrated in Fig. 4 for four geometries.

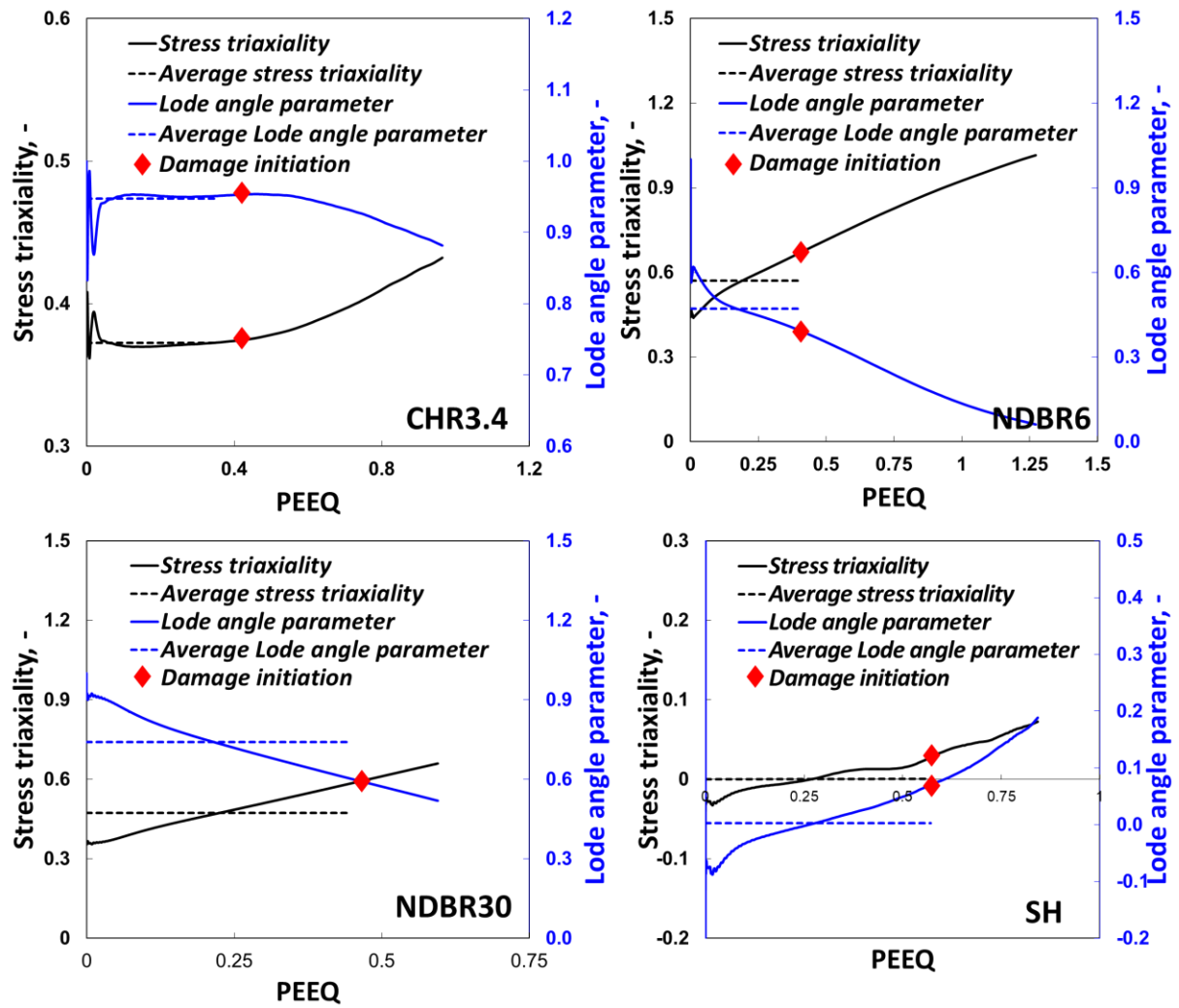


Figure 4. Evolution of the stress states in the critical element of the CH, NDB-R30, NDB-R6, and SH samples.

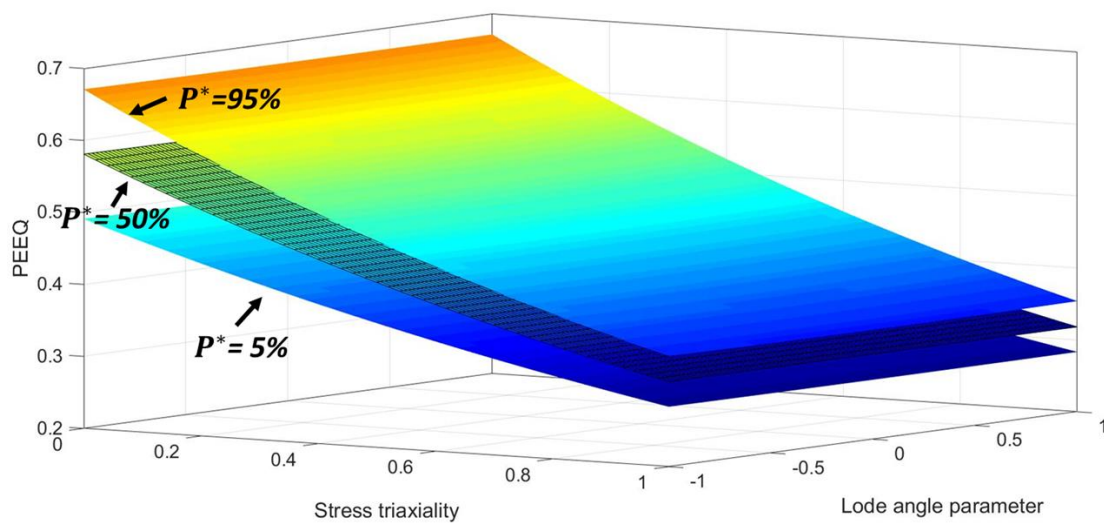


Figure 5. Calibrated ductile fracture criteria corresponding to different failure probabilities of the HPDC Aural-2 alloy.

Given the failure strain and stress state variables collected from tensile tests using different sample geometries, the normal distribution parameters of failure strain for each geometry have been calibrated. Based on the calibrated distribution function, the failure strain corresponding to a specific failure probability  $P_f$  can be determined explicitly for each loading condition, according to Eq. 8. In



this study, the failure strain at three different failure probabilities ( $P_f = 5\%$ ,  $50\%$  and  $95\%$ ) has been determined for the CH, NDB-R30, NDB-R6, and SH samples. A optimization algorithm is applied to determine the fracture locus parameters for each failure probability. The determined fracture locus corresponding to three failure probabilities are shown in Fig. 5. It is noticed that the influence of Lode angle parameter on ductile fracture is not very pronounced for the investigated HPDC Aural-2 alloy.

### Validation of probabilistic failure criterion

For the validation of the probabilistic damage mechanics model, numerical simulations have been carried out using the determined failure criteria at two different failure probabilities ( $P_f = 5\%$  and  $95\%$ ). The predicted fracture displacements precisely cover the range of experimental results. From the force and displacement results shown in Fig. 6, it is concluded that the distribution of fracture displacement and force can be accurately predicted using the probabilistic damage mechanics model.

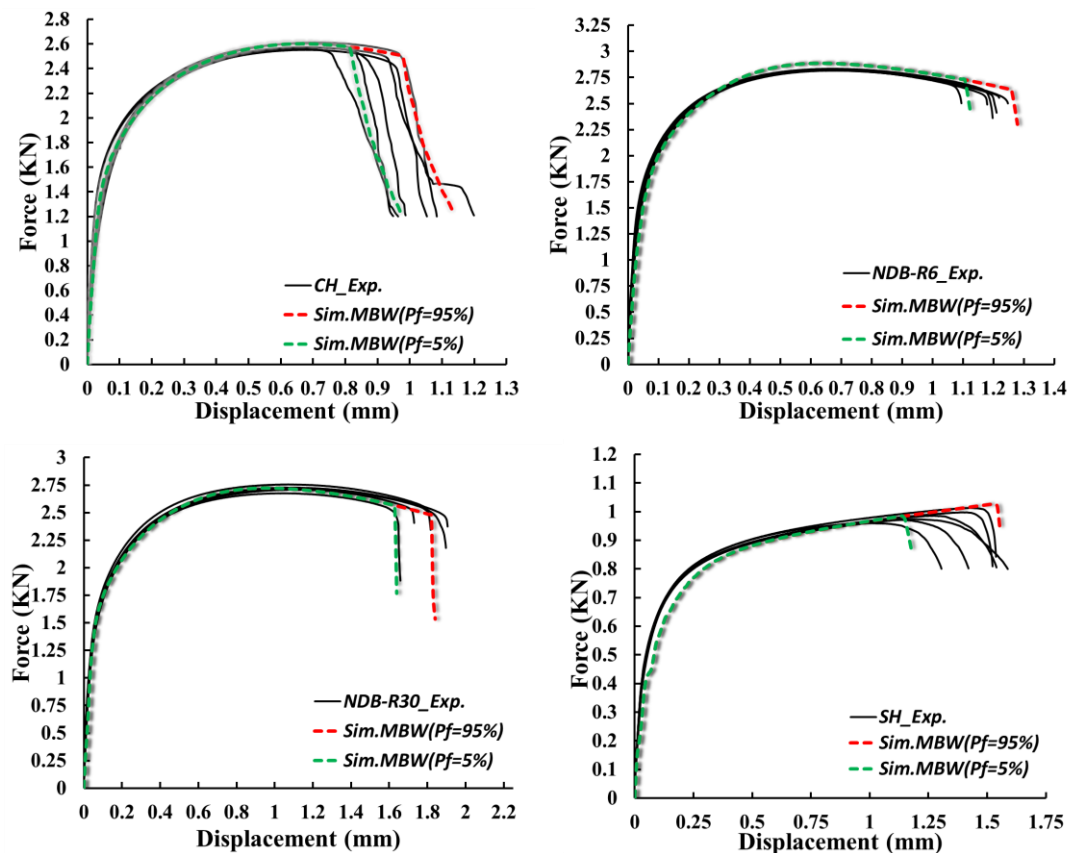


Figure 6. Prediction of the stochastic ductile fracture behavior of the HPDC Aural-2 alloy using a probabilistic damage mechanics model.

### Summary

The large variation in the material ductility of HPDC Aural-2 alloys was experimentally and numerically analyzed in the present work. The key findings are summarized as follows:

- Non-uniform distribution of porosity was found in gauge sections of tensile specimens, ranging from 0.1% to 1.01% (volume fraction) determined via the x-ray tomography technique. A pronounced scatter exists in the ductility of the material.
- The numerical investigation on the ductile fracture behavior has revealed a pronounced stress triaxiality dependence of fracture initiation strain. However, the effect of the Lode angle on the ductile fracture behavior is not significant in this material.
- A probabilistic damage mechanics model is proposed to describe the apparent stochastic ductile fracture behavior over a wide range of stress states considering different failure probabilities. The statistical function can be replaced according to the material.

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