

Variance-Based Sensitivity Analysis of the Biaxial Test on a Cruciform Specimen

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Keywords: Sensitivity Analysis; Sobol Indices; Biaxial Tensile Test; Cruciform Specimen.

Abstract. Parameter identification is a key aspect in the modeling of the material's mechanical behavior. The identification quality depends on the sensitivity of the test results to the values of the constitutive parameters. In this context, a variance-based sensitivity analysis is performed, in order to quantify the influence of the material parameters on the results of the biaxial test on a cruciform specimen; in particular the influence of the Swift hardening law parameters and the Hill'48 anisotropic yield criterion on the forces, the principal strains, strain path and thickness reduction. In general, it was concluded that the Swift hardening parameters influence the forces and the major principal strain value, while the anisotropy parameters have the most influence in the principal strains, strain path and thickness reduction along the surface of the cruciform specimen.

Introduction

Finite element method is a computational tool extensively used to study sheet metal forming processes. The modelling of the plastic behavior of sheet metal is a fundamental aspect to be considered in the numerical simulation of these processes. Generally, the mechanical behavior of the material is described by phenomenological models (constitutive laws). For sheet metals, it is commonly assumed that the plastic behavior is described by an orthotropic yield criterion, which represents the yield surface in a multidimensional stress space, and by hardening laws, which express the yield surface evolution during plastic deformation.

The parameters identification of the constitutive laws is traditionally performed using conventional tests, such as uniaxial tension, shear [1] and bulge tests [2]. However, the stress/strain fields generated during these tests are mainly homogeneous, not conveniently representing the more complex stress/strain states that occur during the forming processes. To overcome this problem, several authors developed mechanical tests characterized by non-homogeneous stress and strain fields [3, 4], such as the biaxial tensile test on a cruciform specimen.

The parameters identification is generally an optimization problem, where the objective is to minimize the differences between the results obtained experimentally and those predicted either analytically or numerically. The identification quality depends on the accuracy of the adopted constitutive model but also on the sensitivity of the test results to the values of the constitutive parameters [5]. In this sense, the development of the identification strategy is commonly coupled with a sensitivity analysis, in order to find which test results are more sensitive to the constitutive parameters, and *vice-versa*. Different types of sensitivity analyses can be found in literature, such as variance-based methods [6], regression [7], derivative-based methods [8] and methods based on experimental designs [9].

In this work, a variance-based sensitivity analysis is performed to evaluate the influence of the material parameters on the results of the biaxial test on a cruciform specimen. This analysis is done with the help of 1st order Sobol's indices [10], which quantify the sensitivity of each material parameter (anisotropy and hardening parameters), and total Sobol indices, which allow to evaluate

the influence of the interactions between these parameters. Initially, the sensitivity analysis will be used to quantify the influence of the material parameters on the maximum values of the output parameters (forces, strains and thickness reduction). Afterwards, sensitivity analysis is used to find the material parameters that most influence the test results in each region of the specimen. In the end, the goal of this work is to contribute to the development of future strategies, characterized by a more precise use of the results in the identification of constitutive parameters.

Numerical Model

Cruciform Specimen. The geometry of the cruciform specimen is shown in Fig. 1(a). This geometry was numerically optimized to maximize the sensitivity of the test results to the values of the constitutive parameters [3]. Due to material and geometric symmetries, only an eighth of the specimen (half the thickness) was simulated in order to reduce the computational cost. As shown in Fig. 1(b), the specimen was discretized with approximately 2 elements/mm in the gauge region and 1 element/mm in the grip region. A total of 6690 elements (8-node hexahedral solid) were used, with 2 layers of elements in thickness. During the biaxial test, the displacement on both arms of the cruciform specimen is equally imposed, until a total displacement of 3 mm is reached. The simulations were performed with the in-house code DD3IMP (Deep Drawing 3D Implicit Code) [11], on computers equipped with an Intel® Core™ i9–7900X Deca-Core processor (4.3 GHz). In average, each numerical simulation was carried out in 53 seconds.

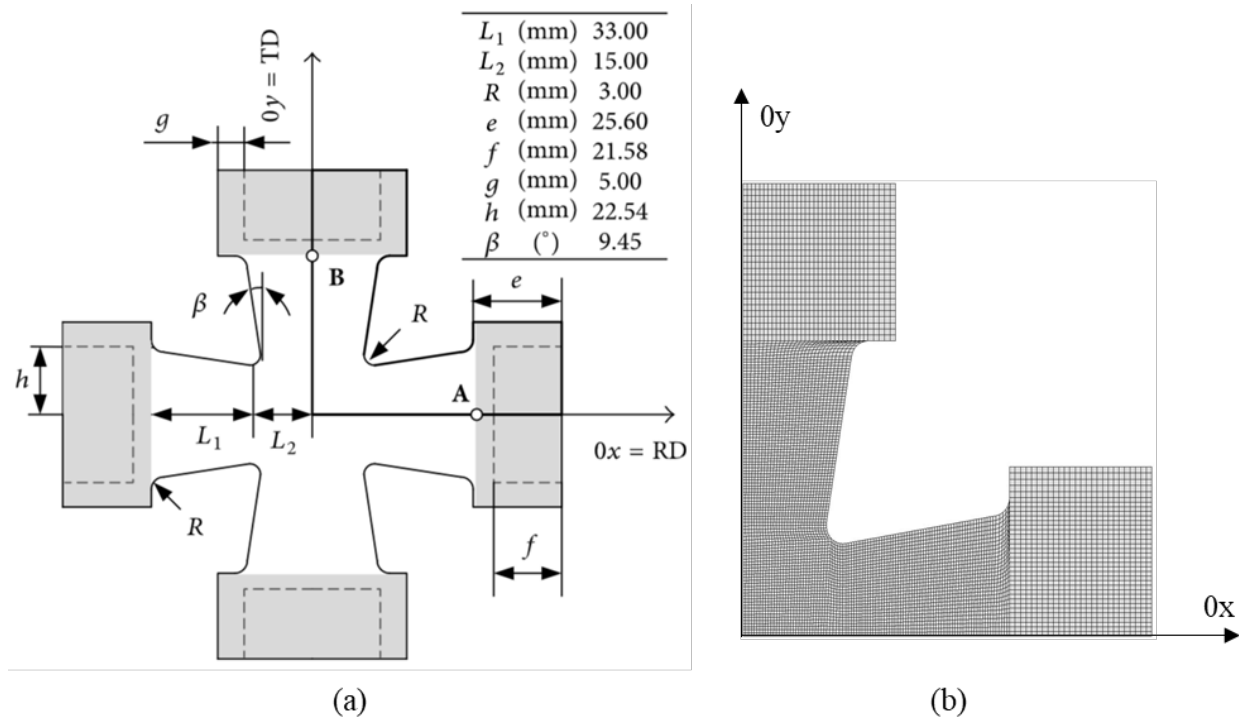


Fig. 1. Cruciform specimen: (a) geometry [3]; (b) mesh used in the numerical model.

The mechanical behavior of the metal sheet assumes that: (i) the elastic behavior is isotropic and follows the generalized Hooke's law, with a Young's modulus, $E = 210$ GPa, and the Poisson's ratio, $\nu = 0.3$; (ii) the plastic behavior is described by the orthotropic Hill'48 yield criterion and the Swift isotropic hardening law. The Hill'48 yield criterion is defined by:

$$F(\sigma_{yy} - \sigma_{zz})^2 + G(\sigma_{zz} - \sigma_{xx})^2 + H(\sigma_{xx} - \sigma_{yy})^2 + 2L\tau_{yz}^2 + 2M\tau_{xz}^2 + 2N\tau_{xy}^2 = Y^2, \quad (1)$$

where σ_{xx} , σ_{yy} , σ_{zz} , τ_{xy} , τ_{yz} and τ_{xz} are the components of the Cauchy stress tensor defined in the orthotropic coordinate system of the material (Oxyz); F , G , H , L , M and N are anisotropy parameters and Y is the yield stress. It is assumed the condition $G + H = 1$, and so the yield stress, Y , is comparable to the uniaxial tensile stress along the rolling direction of the sheet. The parameters L and

M were set to 1.5, as in von Mises. The yield stress evolution during deformation, $Y = Y(\bar{\varepsilon}^p)$, is described by the Swift hardening law:

$$Y = C(\varepsilon_0 + \bar{\varepsilon}^p)^n, \quad (2)$$

where $\bar{\varepsilon}^p$ is the equivalent plastic strain; n , C and ε_0 are hardening parameters. The initial yield stress, Y_0 , is given by $Y_0 = C(\varepsilon_0)^n$.

Input parameters and Test outputs. In the sensitivity analysis, is evaluated the sensitivity of the biaxial test results to the following material parameters (inputs): n , C and Y_0 of the Swift hardening law; and F , G and N of the Hill'48 anisotropic yield criterion. These input parameters have an average value similar to a DC06 steel, whose values are indicated in Table 1 [12]. In the sensitivity analysis it is assumed that the input parameters vary between an upper and lower limit, equal to $\pm 40\%$ of the average value (see Table 1).

Table 1. Average and variation limits of each input parameter.

	n	C [MPa]	Y_0 [MPa]	F	G	N
Upper Limit	0.363	791.45	219.97	0.3957	0.5018	1.8039
Average	0.259	562.32	157.12	0.2826	0.3584	1.2885
Lower Limit	0.155	339.19	94.27	0.1696	0.2151	0.7731

The influence of the input parameters is evaluated on the results of the biaxial test (outputs), namely, the forces along the $0x$ (F_x) and $0y$ (F_y) arms, the principal strains (ε_1 and ε_2), the strain path ($\beta = \varepsilon_2/\varepsilon_1$) and the thickness reduction (TR). All the results are numerically evaluated at the end of the test (i.e., for a displacement of 3 mm in both arms). The thickness reduction is given by:

$$TR[\%] = \frac{t_0 - t_f}{t_0} \times 100 \quad (3)$$

where, t_0 is the initial thickness and t_f is the final thickness at a given point of the specimen. The initial thickness of the specimen is 1 mm.

Sensitivity Analysis

Sobol's Indices. The sensitivity analysis allows to evaluate the influence of the material parameters on the results of the biaxial test on a cruciform specimen. The quantification of this sensitivity is performed with variance-base sensitivity indices, specifically Sobol's indices [10]. These indices are defined as follows:

$$S_i = \frac{V[E(U|X_i)]}{V(U)}, \quad (4)$$

where $V[E(U|X_i)]$, is the conditional variance of the expected value of the output U (test result), when all inputs are fixed, excluding the parameter X_i ; $V(U)$ is the unconditional variance of the output. S_i is the 1st order index and quantifies the individual influence of a given input parameter, X_i , on the output U . Furthermore, total sensitivity indices, S_i^T , can be defined by the following equation [10]:

$$S_i^T = 1 - \frac{V[E(U|X_{\sim i})]}{V(U)}, \quad (5)$$

where $V[E(U|X_{\sim i})]$, is the conditional variance of the expected value of the output U , when only the input parameter X_i is fixed. S_i^T quantifies not only the individual influence of the input parameter X_i , but also the influence of the interactions between this parameter and the remaining ones, on the output. In this sense, the difference between S_i^T and S_i quantifies the influence of the interactions, between the input parameter X_i and the remaining ones, on the output U .

Indices Computation and stabilization. Both sensitivity indices, S_i and S_i^T , are computed based on the approach proposed in [13], but using the estimators proposed in [14], which allowed to significantly improve the stabilization of the indices for smaller sample sizes. This approach was coupled with a Sobol sequence to generate the sample of input parameters [15]. The use of this sequence allowed a faster stabilization when compared to random sampling [16], and the possibility of incrementally increasing the sample size without redesign the complete sample [17].

In order to determine the required sample size (i.e., the total number of sets of input parameters), in Fig. 2 is shown the stabilization of the 1st order sensitivity indices for the principal strain ε_1 , with the increase of the sample size. Based on Fig. 2, it is safe to assume that the stabilization of the indices occurs for a sample size less than 1800. Although not shown, the same trend is observed for the other outputs. Therefore, a sample size of 1800 sets of input parameters was selected to evaluate the sensitivity indices. This implies the use of 14400 numerical simulations to compute the sensitivity indices for the 6 input parameters [13].

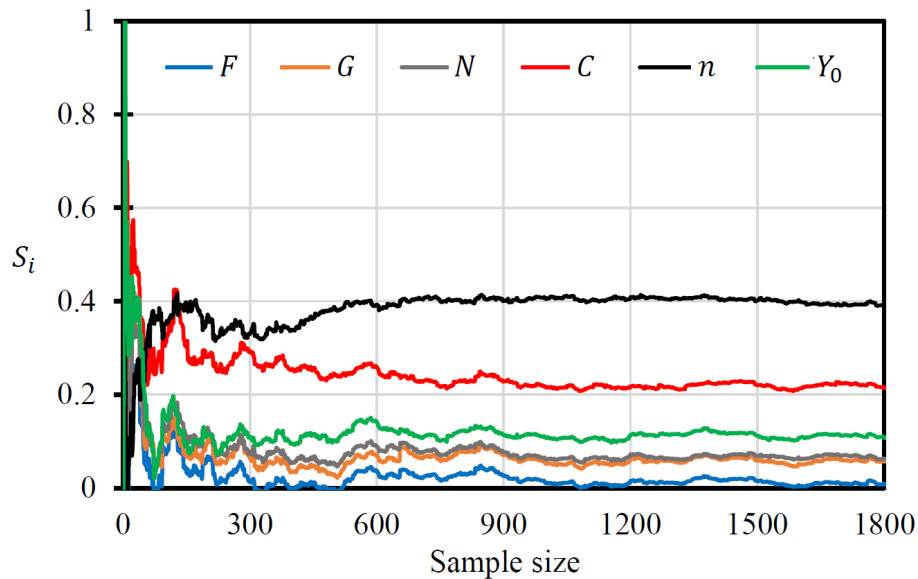


Fig. 2. Evolution of the sensitivity indices S_i for the principal strain ε_1 , with the increase in the sample size.

Sensitivity Results

Maximum Values of the Test Outputs. This section presents the results of the sensitivity indices, S_i and S_i^T , for the test outputs F_x , F_y , ε_1^{\max} , ε_2^{\max} and TR^{\max} , which are evaluated for a displacement of 3 mm in both arms. ε_1^{\max} , ε_2^{\max} and TR^{\max} are, respectively, the maximum values of ε_1 , ε_2 and TR observed at the surface of the cruciform specimen. Fig. 3 shows the sensitivity indices for the test outputs F_x , F_y , ε_1^{\max} , ε_2^{\max} and TR^{\max} . It can be concluded from Fig. 3 that:

- The force values F_x (Fig. 3(a)) and F_y (Fig. 3(b)), are mainly influenced by the hardening parameter C and, to a lesser extent, by the hardening coefficient n ;
- ε_1^{\max} (Fig. 3(c)) is mainly influenced by the hardening parameters C , n and Y_0 ;
- ε_2^{\max} (Fig. 3(d)) is mainly influenced by the anisotropy parameter G and, to a lesser degree, by the hardening parameter C ;
- TR^{\max} (Fig. 3(e)) is mainly influenced by the anisotropy parameter G and, to a lesser degree, by the anisotropy parameter N ;
- The interaction between parameters has some influence in the test results, mainly for the material parameters that have a low 1st order sensitivity index.

It should be noted that these results are only representative of the sensitivity indices computed for the cruciform regions where the maximum values of the outputs occur.

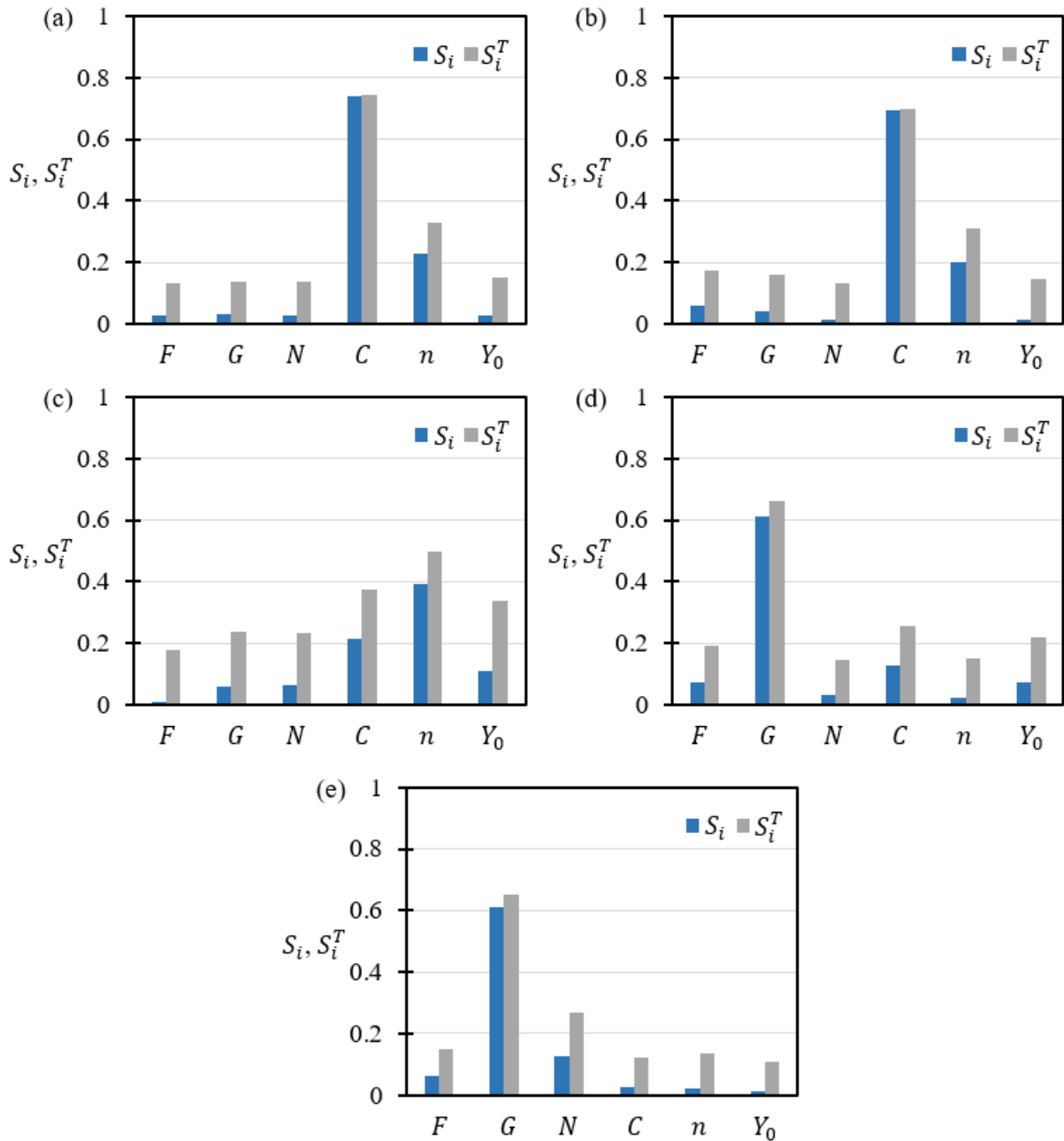


Fig. 3. Sensitivity indices of the material parameters on the test outputs: (a) F_x ; (b) F_y ; (c) ϵ_1^{\max} ; (d) ϵ_2^{\max} ; (e) TR .

Sensitivity along the Cruciform Specimen. In this section, an analysis is performed on the distributions of the sensitivity indices along the cruciform specimen, for the test outputs ϵ_1 , ϵ_2 , TR and β . These results are evaluated for a displacement of 3 mm in both arms. Fig. 4 shows the input parameters whose 1st order index is maximum at each node of the cruciform specimen. It can be concluded from this Fig. 4 that:

- G is the material parameter that most influence the results at the central region of the specimen. For the strain path, β , the parameter F has also some influence in this region;
- At the arms of the specimen, the parameters C and G have the most influence in the results. Additionally, in the arm along Oy , the parameter F also influences the strain path and the thickness reduction;
- At the curvature regions, N (associated with shear stress) is the parameter with the most influence in the principal strains and in the thickness reduction. For the thickness reduction, the parameter F has also some influence in this region. Additionally, the hardening coefficient n does also influence the major principal strain in the curvature region.

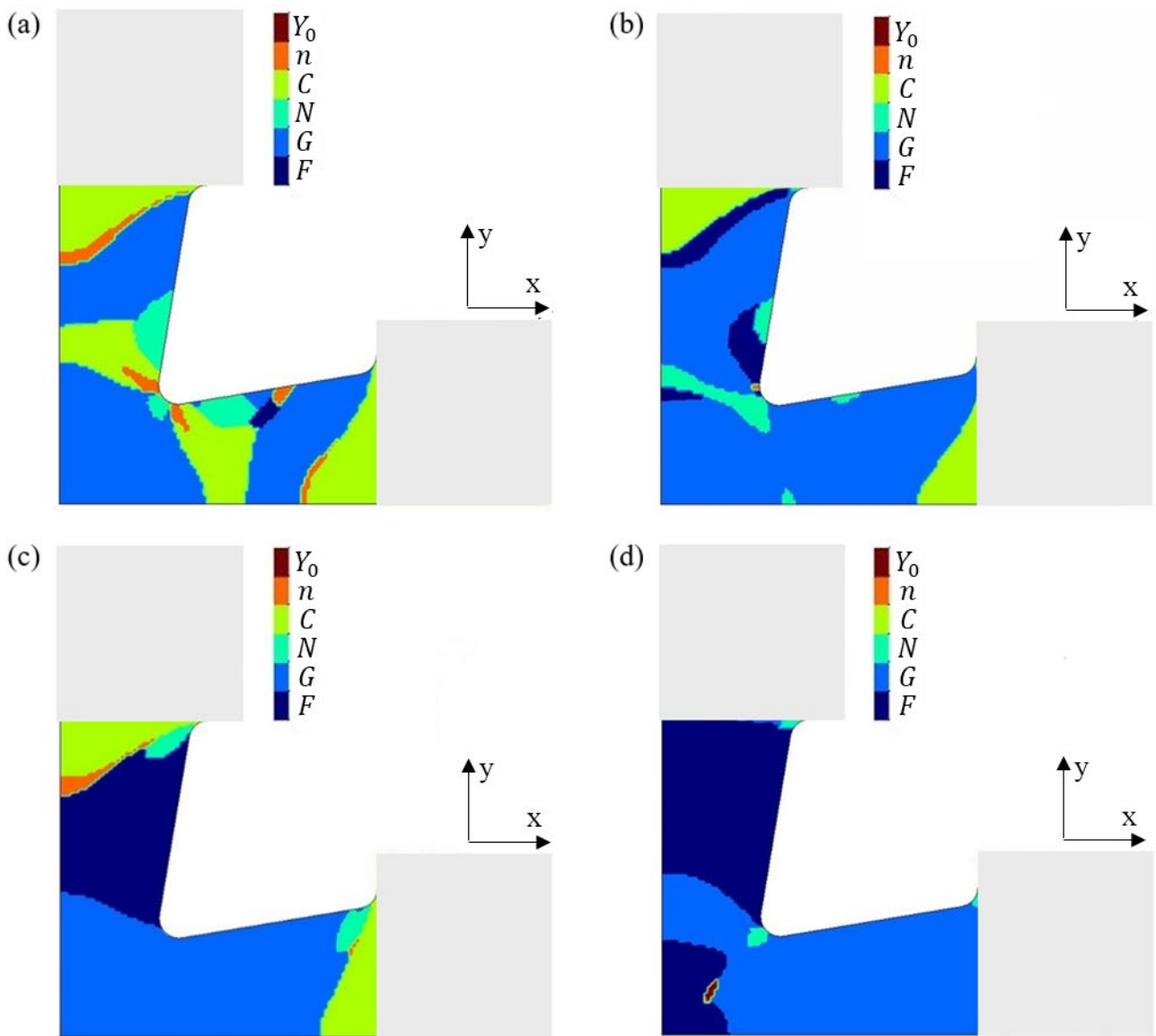


Fig. 4. Material parameter with the highest sensitivity in a given region of the specimen, for the test outputs: (a) ϵ_1 ; (b) ϵ_2 ; (c) TR ; (d) β .

Conclusion

In this work, a sensitivity analysis was performed to identify the material parameters that most influence the results of the biaxial test in a cruciform specimen. In this study, six input parameters were considered: Y_0 , C and n of the Swift hardening law; F , G and N of the Hill'48 yield criterion. The sensitivity of the input parameters was evaluated on the forces along the arms (F_x and F_y), the principal strains (ϵ_1 and ϵ_2), the strain path (β) and the thickness reduction (TR). Sobol' indices were used to quantify the sensitivity of each material parameter in the test results.

Initially, the sensitivity was studied for the regions of the specimen where the maximum values of the test results occur. It was concluded that the Swift hardening parameters have the most influence on the forces along both arms and in the major principal strain; while the parameters of the Hill'48 yield criterion have a significant influence in the minor principal strain and in the thickness reduction.

Afterwards, the input parameters that most influence the results for given region of the cruciform specimen were examined. It was concluded that, in general, the anisotropy parameters G and F have the most influence across the different regions of the specimen, with exception of the curvature region, for which the results are also affected by the parameter N . The work hardening parameters C and n have some influence in the arms of the specimen, mainly for the major principal strain value.

In future works, it is expected to use the results of the biaxial test, evaluated at different times, in order to study the evolution of the sensitivity of the parameters with the course of the test. The results of this sensitivity analysis will be used to develop an efficient identification strategy, using the biaxial test in a cruciform specimen. Additionally, in order to minimize the computational cost of the sensitivity analysis it is also planned to resort to other type of sensitivity indices and to the use of metamodeling techniques.

Acknowledgements

This research is sponsored by FEDER funds through the program COMPETE – “Programa Operacional Factores de Competitividade” and by national funds through FCT – “Fundação para a Ciência e a Tecnologia”, under the project UIDB/00285/2020. It was also supported by projects: RIFORMING (reference PTDC/EME-EME/31243/2017), co-funded by Portuguese Foundation for Science and Technology, by FEDER, through the program Portugal-2020 (PT2020), and by POCI, with reference POCI-01-0145-FEDER-031243; EZ-SHEET (reference PTDC/EME-EME/31216/2017), co-funded by Portuguese Foundation for Science and Technology, by FEDER, through the program Portugal-2020 (PT2020), and by POCI, with reference POCI-01-0145-FEDER-031216. All supports are gratefully acknowledged.

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