

Parameter Identification and Modeling of Hardening to Preserve Identical Predictions under Monotonic Loading

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Abstract. Advanced hardening models accurately describe the transient plastic behavior (reyielding, stagnation, resumption...) after various strain-path changes (reverse, orthogonal...). However, a common drawback of these models is that they usually predict monotonic loading with lower accuracy than the regular isotropic hardening models. Consequently, the finite element predictions using these models may sometimes lose in accuracy, in spite of their tremendous theoretical superiority. This drawback has been eliminated in the literature for the Chaboche isotropic-kinematic hardening model. In this work, a generic approach is proposed for advanced hardening models. Arbitrary models could be successfully compensated to preserve rigorously identical predictions under monotonic loading. A physically-based model involving a 4th order tensor and two 2nd order tensors was used for the demonstration. The parameter identification procedure was greatly simplified by rigorously decoupling the identification of isotropic hardening parameters from the other parameters.

Introduction

The hardening model is one of the important ingredients for a predictive numerical simulation of sheet metal forming processes. Isotropic hardening models have been used since the 1950's and are still widely used in industry nowadays. The shortcomings of isotropic hardening models are known and more advanced models were proposed. Kinematic hardening, classically described by a “backstress” tensorial variable, has allowed for the description of the early re-yielding after strain-path change, subsequent increased hardening rate and eventual resumption of the hardening curve. More complex models were further proposed to describe other transient phenomena like hardening stagnation, cross-hardening etc. [1][2][3][4].

One of the limitations for such advanced models to be adopted in practice is related to the fact that the assets of the already known isotropic hardening models must be abandoned when adopting a new model. Indeed, abundant know-how was built with the simpler models, and robust material parameters are available for them from material providers or from formerly validated identification campaigns. Such models can be used for predictive simulations in early stages of process development for example. At later steps of process development, more accurate material models may be needed: for example, kinematic hardening is often required to better predict springback or failure. However, when changing the model, say, by adding a backstress variable, one cannot use any longer the existing parameters of the isotropic hardening. The available data is discarded, and the new model's material parameters must be identified from scratch. Another consequence is that all the simulations will be affected at some degree when adopting a new model, including those where strain-path change does not play any significant role – only because the new model may describe monotonic loading in a slightly different way. A similar inconsistency concerns monotonic model predictions which are identical in rolling direction, but different under different material directions; this problem

was treated and solved in [5]. This is a significant drawback for industrial application, where back-compatibility is very important.

Thus the scientific question to solve is the following: *starting from a satisfactory isotropic hardening, how to add kinematic / physically-based hardening features without affecting the predictions under monotonic loading?* The proposed solution is to add a compensation term to the original isotropic hardening every time a new feature is included in the model. These compensation terms are scalar. The algebraic expression is available in the literature [6][7] to rigorously compensate for the addition of Frederick-Armstrong kinematic hardening terms. In this work, rate (rather than algebraic) equations of the compensation terms are adopted. This allows for an extension of the approach to virtually any hardening model.

A dual phase DP600 sheet steel is used for the illustration. Monotonic (tensile and simple shear) and sequential (reverse shear) tests were performed using the facilities of the University of Aveiro.

The paper is organized as follows: the reference models used in the paper are briefly recalled first, and their parameters identified in the classical manner. Then, the proposed self-compensation method is proposed and applied to the Teodosiu-Hu model. The parameter identification results illustrate the potential benefits.

Reference Models Used in the Study

Modeling framework: Von Mises associated plasticity is considered. Usually, the size of the yield surface Y is decomposed into an initial yield stress Y_0 and an isotropic hardening variable R ,

$$Y = Y_0 + R. \quad (1)$$

The evolution of R is often given by an algebraic equation (power law, saturating law, etc). In a more generic approach, it can be given by the rate equation

$$\dot{R} = H_R \cdot \dot{\lambda}, \text{ formally leading to } \dot{Y} = H_Y \cdot \dot{\lambda}, \quad (2)$$

where H_R is an expression depending on R and potentially on other internal variables. Similarly, kinematic hardening may also be described by a rate equation of the form

$$\dot{X} = H_X \cdot \dot{\lambda}. \quad (3)$$

The classical yield condition writes $\bar{\sigma}(\sigma' - X) - Y \leq 0$, implying that the predicted stress is a result of the combination of Y and X .

Isotropic hardening: As an example, the Swift-Voce (SV) isotropic hardening model is adopted here, which is a combination of a power law and a saturating law:

$$\dot{R}_v = H_{R_v} \cdot \dot{\lambda} = C_R \cdot (R_{sat} - R_v) \cdot \dot{\lambda} \quad ; \quad \dot{R}_s = H_{R_s} \cdot \dot{\lambda} = n \cdot K^{1/n} (K \varepsilon_0^n + R_s)^{n-1/n} \cdot \dot{\lambda}, \quad (4)$$

where C_R and R_{sat} are the parameters of Voce isotropic hardening and K , ε_0 , n are the parameters of Swift isotropic hardening. The resulting model is given by

$$\dot{Y} = \dot{R} = \dot{R}_v + \dot{R}_s, \text{ with } Y_0 = K \varepsilon_0^n \text{ and } H_Y = H_{R_v} + H_{R_s}. \quad (5)$$

The SV equation has been identified by many authors as a very adequate model to describe large strain hardening behavior of sheet metals, particularly steels. One monotonic mechanical test is sufficient to completely identify the parameters of isotropic hardening models. The simple shear experiments were used here for the identification of the SV model. As shown in Fig. 1, excellent predictions are provided by this model; the identification could be performed for shear strains up to

0.5. Of course, the reverse shear experiments were not used for the identification and their specific features are not described by the model. Kinematic hardening or more physically-based models must be adopted.

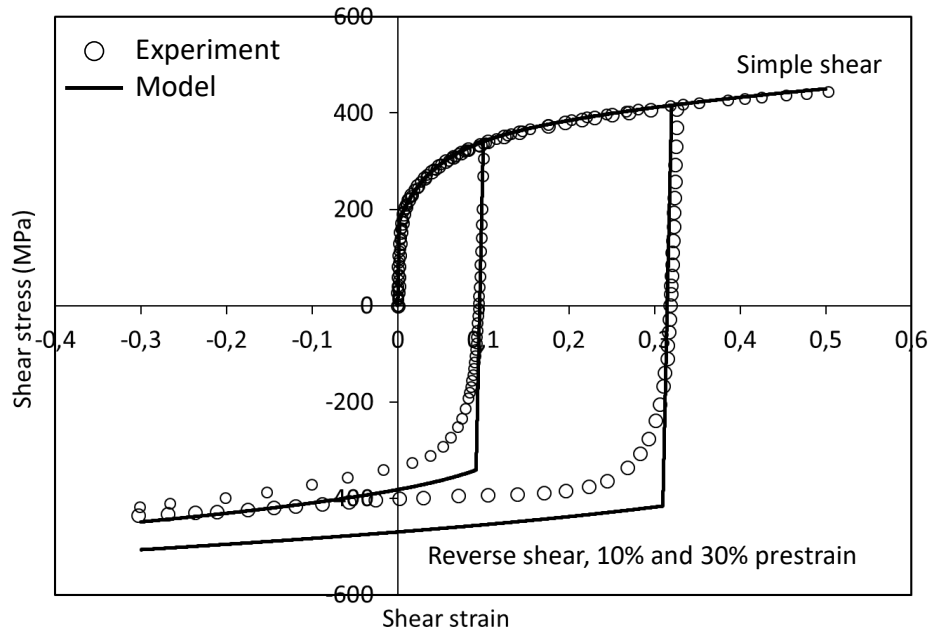


Fig. 1. Stress-strain curves for DP600: experimental (symbols) and predicted with the Swift-Voce model (solid line). Only the monotonic curve has served for the parameter identification; two reverse shear curves are provided for comparison only.

The Teodosiu-Hu microstructure-based hardening model: Several models are available in the literature for the prediction of various strain-path change hardening phenomena: Bauschinger effect, hardening stagnation and resumption, cross-hardening, etc [1-4]. Here, the Teodosiu-Hu (TH) model is adopted [4][8]. In this model, the yield surface size is calculated as

$$Y = Y_0 + R + f|\mathbf{S}|, \quad (6)$$

where R describes the contribution of the randomly distributed dislocations to the isotropic hardening. The term $f|\mathbf{S}|$ represents the contribution of plane persistent dislocation structures (PPDS) to isotropic hardening, where \mathbf{S} is a fourth-order tensor describing the directional strength of the PPDS and f is a material parameter. The kinematic hardening evolution law, described by the back-stress variable, is a classical Frederick-Armstrong equation

$$\dot{\mathbf{X}}_i = \mathbf{H}_{Xi} \cdot \dot{\lambda} = C_{Xi} \cdot (X_{sat\ i} \cdot \mathbf{n} - \mathbf{X}_i) \cdot \dot{\lambda}, \quad (7)$$

where index “ i ” simply indicates that several backstress tensors can be used (two were used in this work). Nevertheless, the saturation value X_{sat} is no longer a material parameter in Teodosiu-Hu’s model, but a function of the internal state variable \mathbf{S} . The variable \mathbf{S} is therefore decomposed into two parts: S_D (scalar) representing the strength associated with the currently active slip systems, and \mathbf{S}_L (fourth-order tensor), associated with the latent part of the PPDS. The decomposition of \mathbf{S} writes

$$S_D = \mathbf{N} : \mathbf{S} : \mathbf{N}, \mathbf{S}_L = \mathbf{S} - S_D \mathbf{N} \otimes \mathbf{N}, \quad (8)$$

where $\mathbf{N} = \dot{\boldsymbol{\epsilon}}^p / |\dot{\boldsymbol{\epsilon}}^p|$ represents the plastic strain rate direction. The evolution laws of S_D and \mathbf{S}_L are given by

$$\dot{S}_D = H_{SD} \cdot \dot{\lambda} = C_{SD} [g(S_{sat} - S_D) - hS_D] \cdot \dot{\lambda}, \quad \dot{S}_L = H_{SL} \cdot \dot{\lambda} = -C_{SL} \left(\frac{|S_L|}{S_{sat}} \right)^{n_L} S_L \cdot \dot{\lambda}, \quad (9)$$

where S_{sat} , C_{SD} , C_{SL} and n_L are material parameters. Finally, the norm $|S| = \sqrt{|S_L|^2 + S_D^2}$ is governed by the rate equation

$$|\dot{S}| = H_{|S|} \cdot \dot{\lambda}, \quad (10)$$

Thus, the scalar function H_Y in Eq. (2) is deduced for this model as

$$H_Y = H_R + f H_{|S|}. \quad (11)$$

It appears that formally this advanced model reduces to the generic form assumed in Eqs (1)-(3) [9], while it involves additional tensorial and scalar state variables along with the rate and algebraic equations describing their evolution. The complete description of the Teodosiu model can be found in [4].

The 16 parameters of the TH model were identified using the monotonic and reverse shear tests. Fig. 2 shows the excellent predictive ability of the model for the transient hardening behavior. In turn, one may note that the predictions under monotonic loading were slightly modified, although the SV equation is still part of the model. The aim of this work is to eliminate this drawback.

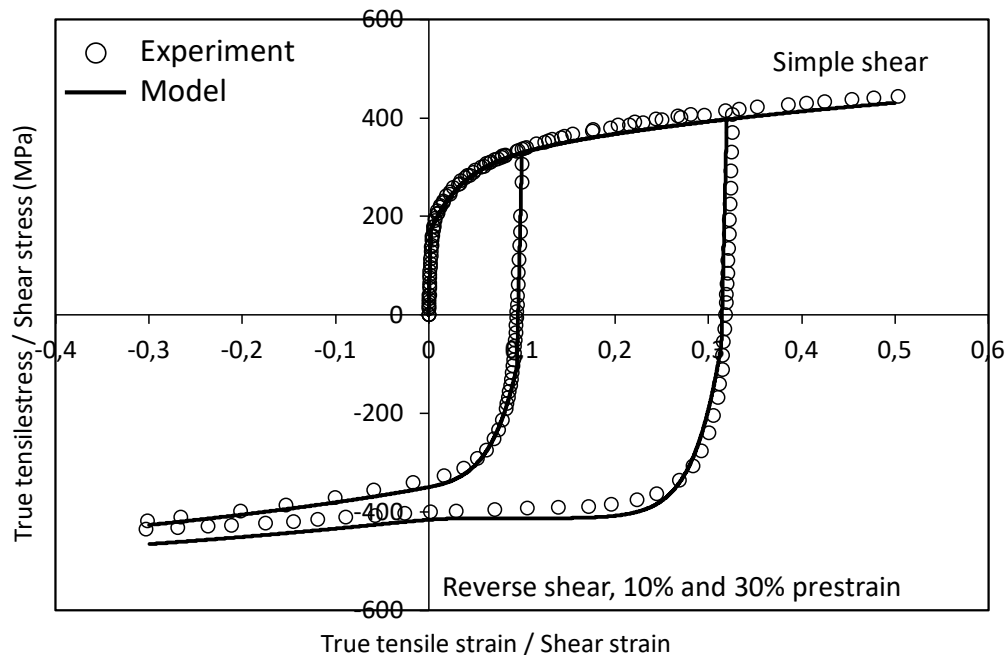


Fig. 2. Result of the parameter identification of the Teodosiu-Hu model. Both the monotonic and the reverse shear curves were used for the parameter identification, and all parameters were identified simultaneously.

Isotropic Hardening Self-Compensation and Simplified Parameter Identification

The basic idea is to use the isotropic hardening model to directly model the σ_T stress under a reference monotonic test (say, uniaxial tension). The size of the yield surface Y would be a consequence of this isotropic hardening and the (scalar) effect of the kinematic hardening under monotonic loading:

$$Y = \sigma_T - \sigma^*. \quad (12)$$

The scalar term σ^* is used to “self-compensate” the contribution of any other model components under monotonic loading. Its algebraic expression was proposed in the literature [6] for the particular case of Armstrong-Frederick kinematic hardening, $\sigma^* = X^*$ with

$$X^* = X_{sat} \cdot (1 - \exp(-C_X \cdot \bar{\epsilon}^p)), \quad (13)$$

where $\bar{\epsilon}^p$ is the equivalent plastic strain. Nevertheless, such an explicit expression is impossible (or very difficult) to derive for more complex models.

The proposed approach simply consists in using the rate form of Eq. (12), and calculate the rate of σ^* instead of σ^* itself. Notoriously, for the Armstrong-Frederick model the result is

$$\dot{\sigma}^* = \dot{X}^* = H_X^* \cdot \dot{\lambda}, \quad H_X^* = C_X(X_{sat} - X^*). \quad (14)$$

For the TH model, the expression further expands as

$$\sigma^* = X^* + f S_D^*, \quad (15)$$

or, in rate form,

$$H_Y = H_{\sigma_T} - H_{\sigma}^*, H_{\sigma}^* = H_X^* + f H_{S_D}^*, \quad (16)$$

where S_D^* is a scalar determined by

$$\dot{S}_D^* = H_{S_D}^* \cdot \dot{\lambda} = C_{SD}[g^*(S_{sat} - S_D^*) - h^* S_D^*] \cdot \dot{\lambda}. \quad (17)$$

It is obvious that the contributions of the scalar functions g , h etc. can be directly deduced from the base equations of the TH model. Thus, deriving the rate of σ^* appears as a trivial extension of any arbitrary hardening model, while its analytical integration under an algebraic form may be difficult or impossible. The full development of the corresponding equations for the TH model can be found in [10].

The parameter identification procedure is both simplified and clarified by the self-compensation of isotropic hardening. Several experiments were required for the parameter identification: i) monotonic simple shear tests (and, optionally, monotonic tensile tests) and ii) two-sequence reverse shear tests, with different levels of shear pre-strains. The parameters were identified by minimizing the objective function given by the following expression

$$\sum_{i=1}^N \frac{1}{M_i} \sum_{j=1}^{M_j} \left(\frac{\sigma_{i,j}^{exp} - \sigma_{i,j}^{sim}}{\sigma_{i,j}^{exp}} \right)^2 + \left(\frac{h_{i,j}^{exp} - h_{i,j}^{sim}}{h_{i,j}^{exp}} \right)^2, \quad (18)$$

combining the gap between simulation (sim) and experiments (exp) in terms of both stress values σ and hardening slopes h . N designates the number of tests and M_i the number of experimental points on curve i . For the parameter identification of the original TH model, $N=3$ (one monotonic and two reverse shear tests) or more. As a consequence of the self-compensation, the isotropic hardening parameters can now be identified once for all using the available monotonic test(s), and they are no longer changed (five parameters for the SV model). In a second step, all the remaining parameters of the hardening model (kinematic hardening, cross-hardening etc.) are identified with respect to the sequential tests. Here, the reverse shear tests were used to identify the eleven remaining parameters. Fig. 3 illustrates the differences between the parameter identification procedures required for the compensated and non-compensated models, respectively. On the one hand, the sequential parameter identification of independent groups of parameters allows for simplified parameter identification procedures. On the other hand, it allows for the simplified identification of various models: here, the

same isotropic hardening parameters can be reused without change in combination to arbitrary, more complex, hardening models. This is similar to the classical reuse of anisotropy parameters whatever the hardening model. The final set of parameter values is the following: $C_r=45$; $R_{sat}=137$ MPa; $K=664$ MPa; $n=0.32$; $\varepsilon_o=0.004$ for the isotropic hardening and $C_{x1,2}=90/6$; $X_{sat1,2}=199/125$ MPa; $X_o=167$ MPa; $S_{Sat}=566$ MPa; $C_{SD}=2,85$; $C_p=0,7$; $n_p=890$; $f=r=0,8$.

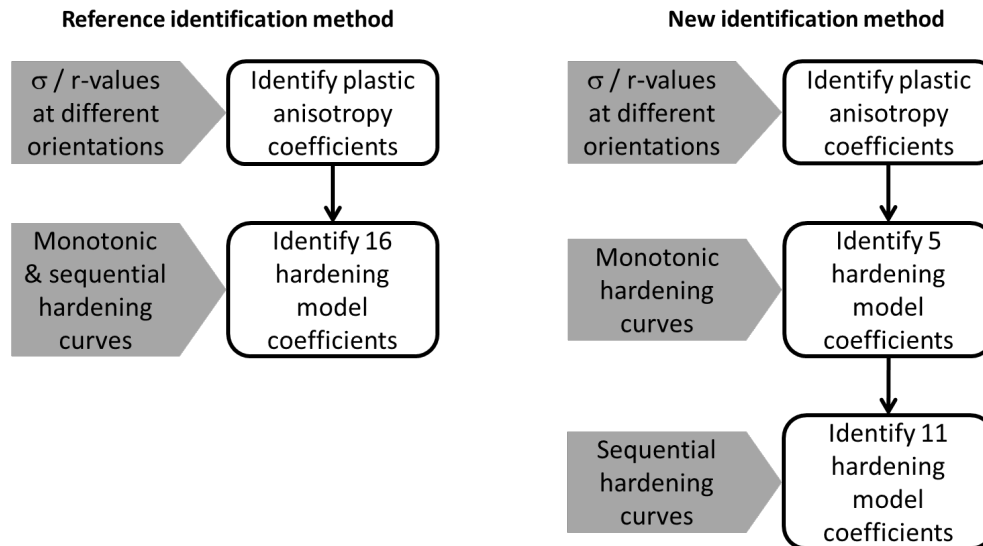


Fig. 3. Schematic description of the parameter identification procedure for the reference models (left) and the new parameter identification method for the self-compensated models (right).

Fig. 4 shows the result of the parameter identification for the self-compensated TH model. Compared to Fig. 2, a similar accuracy is observed with respect to the two reverse shear experiments. However, the monotonic shear test is described identically as in Fig. 1. This not only improves the overall accuracy; it improves the *consistency* when the respective prediction of isotropic and “advanced” hardening models are compared. Indeed, a majority of industrial processes involve mainly monotonic loading paths, and the differences between various models come more from their parameter identification procedure rather than from the supposed mathematical superiority. With the proposed approach, consistently identical results can be obtained with all models as long as the loading modes are monotonic. If significant differences appear, this clearly indicates that the material model really has an incidence on the results, and should be further considered with care.

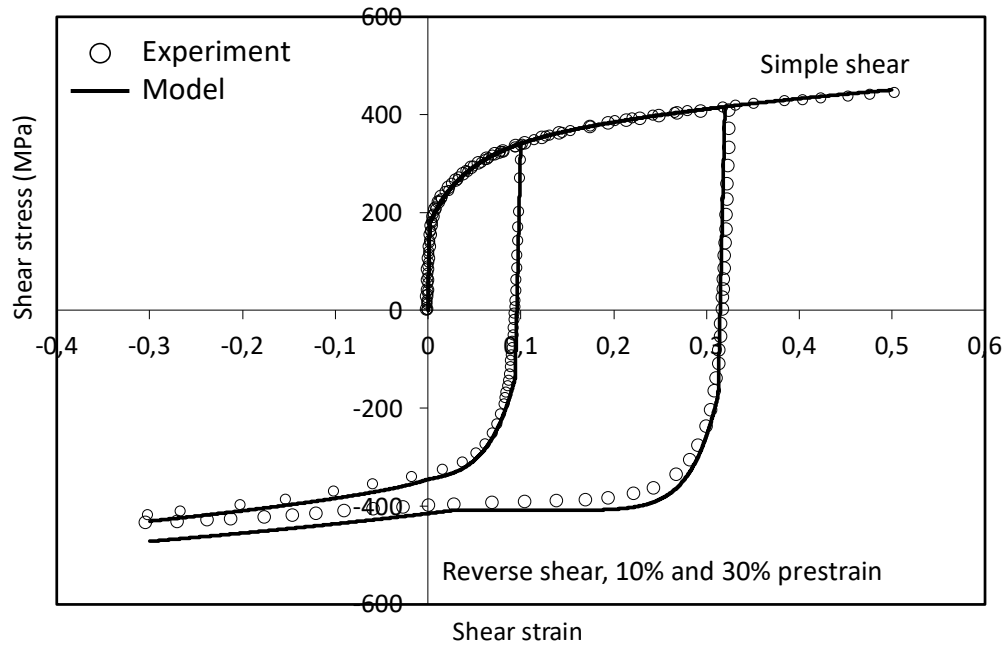


Fig. 4. Result of the parameter identification of the self-compensated Teodosiu model. The isotropic hardening part of the model is identical to Fig. 1; the reverse shear experiments were used to identify only the remaining of the model parameters.

Conclusions

A modeling approach was proposed in order to decouple the parameter identification of the isotropic hardening component of an arbitrary, more complex, hardening model. A rate equation is used to describe the evolution of the newly introduced scalar variable σ^* , which allows the extension of this approach to a wide family of hardening models. The microstructure-based Teodosiu-Hu model was used for the demonstration. The benefits of this method are:

- Available values for the parameters of isotropic hardening for the material at hand can be used in the framework of a more complex model, without change. Alternatively, the parameters of the isotropic hardening part of the model can be identified independently, using monotonic tests.
- The number of parameters to be identified using the strain-path change experiments is accordingly reduced.
- The predictions of the complete model on one hand, and of its isotropic hardening part on the other hand, are rigorously identical under any monotonic loading conditions.
- The proposed approach can be generalized to anisotropic materials, rate-dependent materials etc.

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