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Study of Direct Metal Extrusion by the Upper-Bound and Finite Volume Methods

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Abstract. The velocity fields of axisymmetric direct extrusion of metals was analysed by the upper-bound method and compared with the results from the finite-volume method, FVM. The upper-bound technique proposed by Avitzur and by Zhao et al. together with the streamline functions were employed to calculate the analytical velocity fields, which consider the friction at die wall. Moreover, the components of strain-rate are also presented. Additionally, the axisymmetric extrusion process was modelled by the FVM method to calculate the velocity fields and compared with the Avitzur's and by Zhao's solutions. The FVM velocity fields were calculated by using the Eulerian approach of fixed grid, the governing equations of metal plastic flow and conservation laws discretized by the FVM and the Explicit MacCormack method in structured and collocated mesh were also employed. Friction at die wall was modelled by the friction factor model, using the tangential shear stress boundary conditions. The examined material experimental parameters were obtained from the Al 6351 aluminium alloy in the direct extrusion process at 450º C. Velocity fields of the longitudinal and radial velocity distributions by the upper-bound and FVM methods are presented and compared. Good agreement is shown between the radial velocity component V_r from the Avitzur´s and FVM results, but poor for the longitudinal velocity V_z. From the analysis of velocity fields, the most severe condition of wear on the inner wall of the die and material surface damage occurs in the area near the exit corner of the die. However, the predicted location of the severe wear region in the die wall by the FVM method is located prior to the point predicted by the Avitzur model.

Introduction

The extrusion process of metals is a thermo-mechanical metalworking operation largely applied in the manufacturing of steel and aluminium tubes, bars and aluminium profiles. Thus, it is an important metalworking and industrial process for manufacturing structural parts. Extrusion process of metals is carried out at warm and hot temperatures, consequently, process modelling has been investigated by two independent approaches: the mechanics of metal flow and the metallurgical evolution of microstructural features such as grains and porosities.

Historically, the Upper-bound, the Slip-line fields and the Finite Element Methods have been applied with relative success for decades to calculate loads, stress, strain, strain-rate and velocity fields in the mechanics analysis of metal extrusion [1,2,3]. Experimental visioplasticity techniques have been also employed to physically simulate and visualize the details of metal flow with model materials such as plasticine, lead, aluminium and the gridded split billet and the strip-pattern techniques. Nevertheless, recently in the academy, Bressan et al. [4] have also investigated the use of the Finite Volume Method, FVM, to analyse metal flow in extrusion: literature suggests that metal flow in extrusion can be analysed as a *viscous fluid*, therefore, modelled by the plastic flow formulation [4, 5]. Applications of FVM to solid mechanics was also recently reviewed by Cardiff and Demirdžić [6]. Thus, plastic flow of metal in extrusion and drawing can be modelled as the *flow of an incompressible non-linear viscous fluid*. This hypothesis can be assumed because metal extrusion is a metal flows without volume change.

The *MacCormack Numerical Method* is traditionally used to solve the governing differential equations in fluid dynamics applications, particularly in the complex aerodynamics situations. This method has the advantage to smooth the pressure pick discontinuities produced by the pressure shock waves. Additionally, it is a numerical method of second order accuracy in time and space. Thus, the MacCormack method is commonly used to model compressible and incompressible fluid flow, using the FVM to solve the governing differential equations [7].

Present work compares the solutions of FVM numerical scheme proposed by Bressan et al. [4] with the upper-bound method (model proposed by Avitzur [8] and the model proposed by Zhao et al. [9]) for calculating the velocity fields of metal flow in the extrusion process, in steady state conditions. In the Finite Volume Method, the governing equations were discretized using the Explicit MacCormack Method in structured and collocated mesh.

Axisymmetric Extrusion Analysis by the Upper-Bound Method and the Streamlines

The Upper-bound technique is an analytical method to calculate metal forming operation loads and power. The method assumes a kinematically admissible velocity fields and the existence of surfaces of velocity discontinuities. The kinematics of plastic deformation or kinematically admissible velocity field of the metal forming process should be obtained from the die geometry and velocity components which satisfy the boundary conditions and volume constancy. Flow streamlines, on the other hand, are defined as the lines that describe the path followed by a particle during plastic flow. Thus, the velocity field components are derived from the streamfunction which is assumed to describing the flow and generally no friction is assumed at the die.

According to the Avitzur´s extrusion model [8] in the (z,r)-axis for a spherical velocity field, the velocity components, the surfaces of velocity discontinuities and streamlines in the deforming region II of the axisymmetric direct extrusion process, as seen in Fig.1, are calculated by the equations,

$$V_z = V_0\left(\frac{\rho_0}{\rho}\right)^2 \cos^2\varphi \quad ; \quad V_r = \frac{V_0}{2}\left(\frac{\rho_0}{\rho}\right)^2 \operatorname{sen}(2\varphi) \qquad \text{(1a,b)}$$

where V_z and V_r are the velocity components, V_o is the die entry velocity, R_o is the entry radius of the die, r is the current radius in the deforming region II, φ is the streamline angle in the region II and α is the die semi-angle. Avitzur´s streamlines drawing is depicted in Fig.2a.

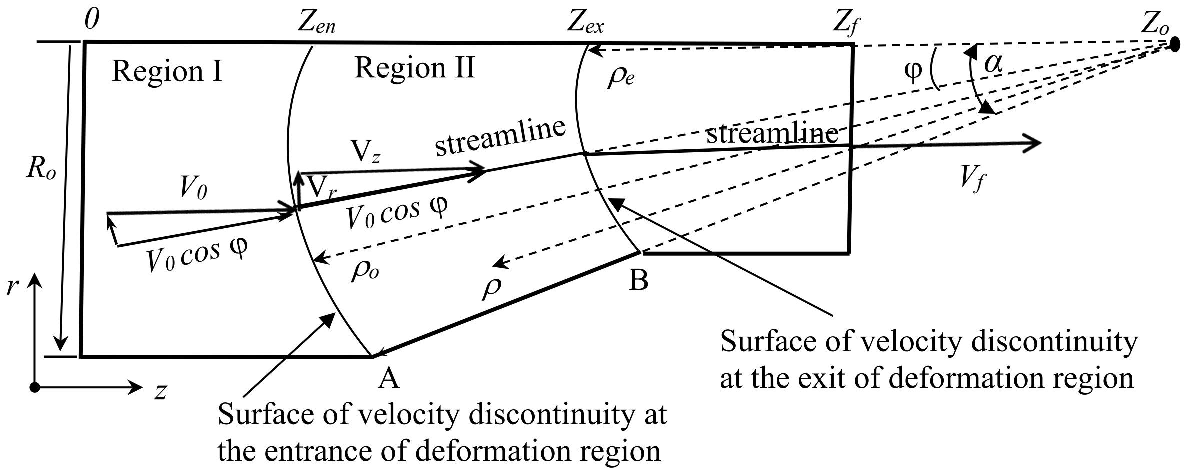

Fig. 1. Avitzur´s model: schematic drawing of the geometry of axisymmetric extrusion process with the entrance velocity V_o, velocity components and streamlines.

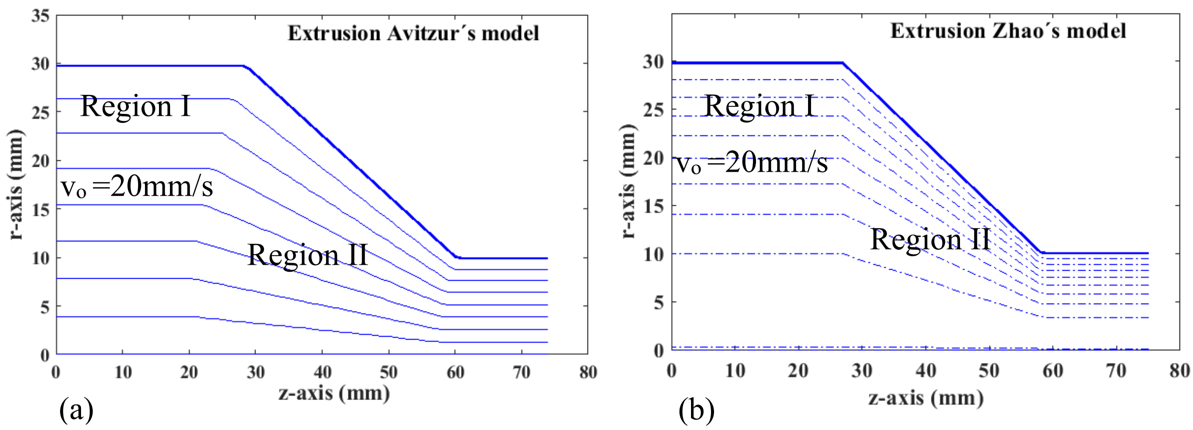

Fig. 2. Extrusion streamlines, die semi-angle α=32° : a) Avitzur´s model and b) Zhao´s model.

An alternative streamfunction and the correspondent velocities components, assuming no friction at die, for the axisymmetric direct extrusion process was proposed by Zhao et al. [9].

In cylindrical coordinates system (r,θ,z), the governing equation for an incompressible flow or the plastic flow continuity governing equation is given by,

$$\vec{\nabla}\cdot\vec{V} = \frac{1}{r}\frac{\partial(rV_r)}{\partial r} + \frac{1}{r}\frac{\partial V_\theta}{\partial \theta} + \frac{\partial V_z}{\partial z} = 0 \qquad (2)$$

where V_r , V_θ and V_z are the velocity components. For an axisymmetric extrusion process without rotation, the velocity component V_θ= 0, therefore, the governing Eq. 2 is reduced to,

$$\frac{1}{r}\frac{\partial(rV_r)}{\partial r} + \frac{\partial V_z}{\partial z} = 0 \qquad (3)$$

Assuming the definition of Stokes´ streamfunction Ψ(r,z), which satisfy Eq.3, the velocity components V_r , V_z and the resultant velocity V are calculated by,

$$V_r = -\frac{1}{r}\frac{\partial \Psi}{\partial z} \quad ; \quad V_z = \frac{1}{r}\frac{\partial \Psi}{\partial r} \quad ; \quad V = \sqrt{V_r^2 + V_z^2} \qquad (4a,b,c)$$

After the integration of Eq.4, defining Ψ(r,z)= 0 at the axisymmetric extrusion axis and Ψ(r,z)= C_{ab} , constant at the die boundary AB, the Zhao´s streamline function is obtained as,

$$\Psi(r,z) = \frac{R_o^2 V_o}{2}\left(\frac{r^2}{z^2 \tan^2\alpha} - 1\right) + C_{ab} \qquad (5)$$

And the Zhao´s velocities components in the region II for the axisymmetric extrusion are [9],

$$V_r = \frac{c\,r}{\pi z^3 \tan^2\alpha} \quad ; \quad V_z = \frac{c}{\pi z^2 \tan^2\alpha} \qquad (6a,b)$$

where the volume constant, $c = \pi R_o^2 V_o$. Zhao´s streamlines drawing is shown in Fig.2b.

The strain-rate components are calculated from the following equations, $\dot{\varepsilon}_{z\theta} = \dot{\varepsilon}_{r\theta} = 0$,

$$\dot{\varepsilon}_z = \frac{\partial V_z}{\partial z} = \frac{2c}{\pi z^3 \tan^2 \alpha} \quad ; \quad \dot{\varepsilon}_r = \dot{\varepsilon}_\theta = \frac{V_r}{r} + \frac{1}{r}\frac{\partial V_\theta}{\partial \theta} = \frac{c}{\pi z^3 \tan^2 \alpha} \quad ;$$

$$\dot{\varepsilon}_{zr} = \frac{1}{2}\left(\frac{\partial V_z}{\partial r} + \frac{\partial V_r}{\partial z}\right) = \frac{3cr}{2\pi z^4 \tan^2 \alpha} \qquad \text{(7a,b,c)}$$

Axisymmetric Extrusion Modelling by the Finite Volume Method

The FVM method or control volume method is a numerical method to calculate all metal forming process variables. The deforming material domain of the analysed process is divided into small volume units, named control volumes, which constitute the grid or mesh.

The differential equations of the governing conservation laws (of mass, momentum and energy) in cylindrical coordinates system (r,θ,z) for the axisymmetric extrusion case, for $\partial F_\theta / \partial \theta = 0$, can be merged into a compact matrix structure in the form of [4]:

$$\frac{\partial \boldsymbol{Q}}{\partial t} + \frac{1}{r}\frac{\partial (r\boldsymbol{F}_r)}{\partial r} + \frac{\partial \boldsymbol{F}_z}{\partial z} = \boldsymbol{S} \qquad (8)$$

where t is time, and $\boldsymbol{Q}$, $\boldsymbol{F}_r$, F_z and $\boldsymbol{S}$ are flow vectors which assume the following format in the Euler approach of fixed mesh in space:

$$\mathbf{Q} = \begin{Bmatrix} \rho \\ \rho v_r \\ \rho v_z \\ \rho c T \end{Bmatrix} ; \quad \mathbf{F_r} = \begin{Bmatrix} \rho v_r \\ \rho v_r^2 - \sigma_{rr} \\ \rho v_r v_z - \sigma_{rz} \\ \rho c T v_r + \dot{q}_r \end{Bmatrix} ; \quad \mathbf{F_z} = \begin{Bmatrix} \rho v_z \\ \rho v_z v_r - \sigma_{rz} \\ \rho v_z^2 - \sigma_{zz} \\ \rho c T v_z + \dot{q}_z \end{Bmatrix} ; \quad \mathbf{S} = \begin{Bmatrix} 0 \\ -\frac{\sigma_{\theta\theta}}{r} \\ 0 \\ \bar{\sigma}.\dot{\bar{\varepsilon}} \end{Bmatrix} \qquad \text{(9a,b,c,d)}$$

where ρ is material density, T temperature, $\dot{q}$ the rate of heat transfer, $\sigma_{rr}, \sigma_{\theta\theta}, \sigma_{zz}, \sigma_{rz}$ the stress components, $\bar{\sigma}$ and $\dot{\bar{\varepsilon}}$ the equivalent strain-rate. The field variables to be determined are in the flow vector $\boldsymbol{Q}$. the equivalent stress

Integrating Eq.8 over the control volume, employing Gauss' theorem, the following differential equation is obtained:

$$\frac{\partial \mathbf{Q}_{mn}}{\partial t} = -\frac{1}{V_{mn}}\left\{\frac{1}{r}\left[(r\boldsymbol{F}_r \cdot \mathbf{s})_{m,n-\frac{1}{2}} + (r\boldsymbol{F}_r \cdot \mathbf{s})_{m,n+\frac{1}{2}}\right] + \left[(\boldsymbol{F}_z \cdot \mathbf{s})_{m-\frac{1}{2},n} + (\boldsymbol{F}_z \cdot \mathbf{s})_{m+\frac{1}{2},n}\right]\right\} + \mathbf{S}_{mn} \qquad (10)$$

where $\boldsymbol{s}$ is the outward vector of the surface and V_{mn} is the control volume area. Therefore, the conservation laws are applied to each control volume.

Eq.10 was solved numerically, employing the MacCormack numerical strategy [7], which comprises the predictor and corrector steps in the numerical convergence process. Thus, the MacCormack converged current step is calculated by the average between the predictor and corrector steps, $Q_{mn}^{t+1} = (\underline{Q_{mn}^{t+1}} + \underline{\underline{Q_{mn}^{t+1}}})/2$, t is the current time and Δt is a virtual time step. Therefore, the numerical MacCormack method is a pseudo-transient calculation process, where Δt is a virtual time increment to obtain the final converged solution. The required time step to attain numerical stability in the numerical convergence process was $\Delta t = 10^{-18}$ s.

Present metal plasticity constitutive equation for the axisymmetric plastic flow in extrusion considered the material to be incompressible, isotropic and rigid-viscous-plastic. The relation between the stress component σ_{ij} and strain rate tensor $\dot{\varepsilon}_{ij}$ was written in the following form $\sigma_{ij} = -\sigma_m + 2\eta\dot{\varepsilon}_{ij}$, where σ_m is hydrostatic pressure calculated by $\sigma_m = (\sigma_{rr} + \sigma_{\theta\theta} + \sigma_{zz})/3$. Metal plastic flow equivalent viscosity η was calculated from the *associated plastic flow potential* $\left(\dot{\varepsilon}_{ij} = \dot{\lambda}s_{ij}\right)$, thus, $\eta = 1/2\dot{\lambda} = \bar{\sigma}/3\dot{\bar{\varepsilon}}$. The equivalent strain rate was defined as $\dot{\bar{\varepsilon}} = \sqrt{(2/3)\dot{\varepsilon}_{ij}\dot{\varepsilon}_{ij}}$ and the equivalent stress was $\bar{\sigma} = \sqrt{(3/2)s_{ij}s_{ij}}$, where s_{ij} is the deviatoric stress component.

The boundary conditions applied in the present direct extrusion analysis [4], at the solid wall interface between material and die, considered friction factor, $\tau_{nt} = mk$, where τ_{nt} is the tangential friction stress, k the material yield shear stress and *m* the friction factor. Assuming isotropic material, von Mises yield criteria and the shear strain rate $\dot{\gamma}_{nt}$, the tangential friction shear stress is: $\tau_{nt} = (2\bar{\sigma}/3\dot{\bar{\varepsilon}})\dot{\gamma}_{nt}$.

Material and Experimental Procedure

Comparison of results of direct extrusion of Al 6351 aluminium alloy by the analytical solutions and the FVM method developed by Bressan et al. [4] are presented and discussed in the next session. The Al 6351 billets were extruded in controlled conditions of 450 ºC and the extrusion speed of 10 mm/s (details are reported in [10]) and were simulated by FVM method.

The experimental average parameters of Al 6351 alloy, the extrusion process and simulation parameters used in the FVM simulations are seen in Table 1. The tools were modeled as rigid solid. The viscoplastic friction model for sliding without lubrication and no-stick conditions was adopted with the friction factor m equal to 0.5. Flow stress of Al 6351 alloy were obtained from tensile tests at 450ºC and strain rate of 0.001 and 0.1/s. The curves were fairly linear with almost constant yield stress which can be represented by a rigid-perfectly-plastic material. Thus, the material flow stress behavior was modeled as rigid-strain rate sensitive material with the hardening law: $\bar{\sigma} = K\dot{\bar{\varepsilon}}^M$.

Table 1. Parameters used in the analysis of Al 6351 alloy direct axisymmetric extrusion.

Parameters	value	Parameters	value
Density (ρ)	2710 [kg.m^{-3}]	Quantity of control volumes	1110
Yield stress ($\sigma_Y = 2k$)	255 [MPa]	Time step (Δt)	10^{-18} [s]
Die entry radius (R_o)	30 [mm]	Die semi-angle (α)	32.3°
Area reduction (Ar = 9)	89 %	Friction factor parameter (m)	0.5
Die exit radius (R_{ex})	10 [mm]	Material extrusion temperature (T)	450 [ºC]
Inlet extrusion velocity (V_o)	20 [mm/s]	Material strain hardening law: rigid perfect-plastic, strain rate sensitivity	K=255 [MPa] M = 0.10

Results and Discussions

The geometry of the fixed mesh with 1110 volumes employed in the present FVM numerical simulations and the Avitzur´s model of the axisymmetric plastic flow of direct hot extrusion of Al 6351 aluminium alloy is shown in Fig.3. The required time step to attain numerical stability in the numerical convergence process was $\Delta t = 10^{-18}$ s, and the number of iterations loops was approximately 30.000 loops.

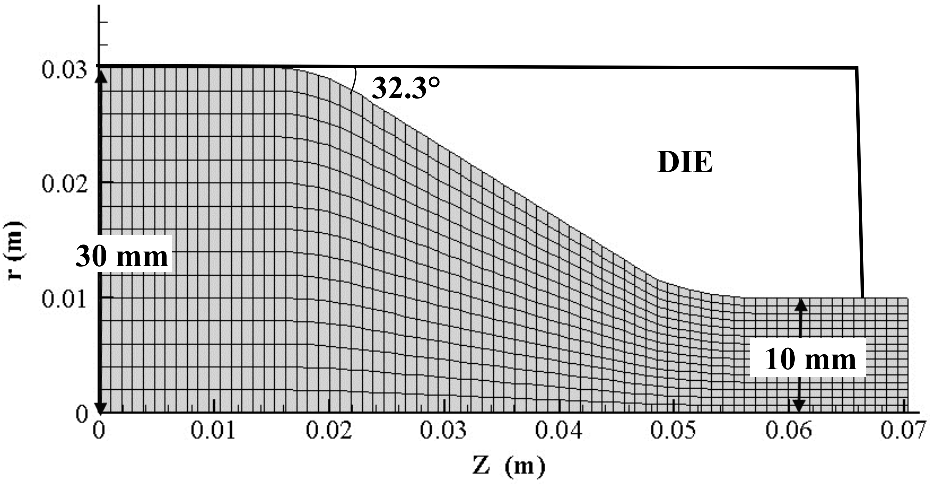

Fig. 3. Geometry of *fixed mesh* in space used in the present FVM numerical simulations of the axisymmetric direct extrusion of Al 6351 aluminium billets, using 1110 control volumes.

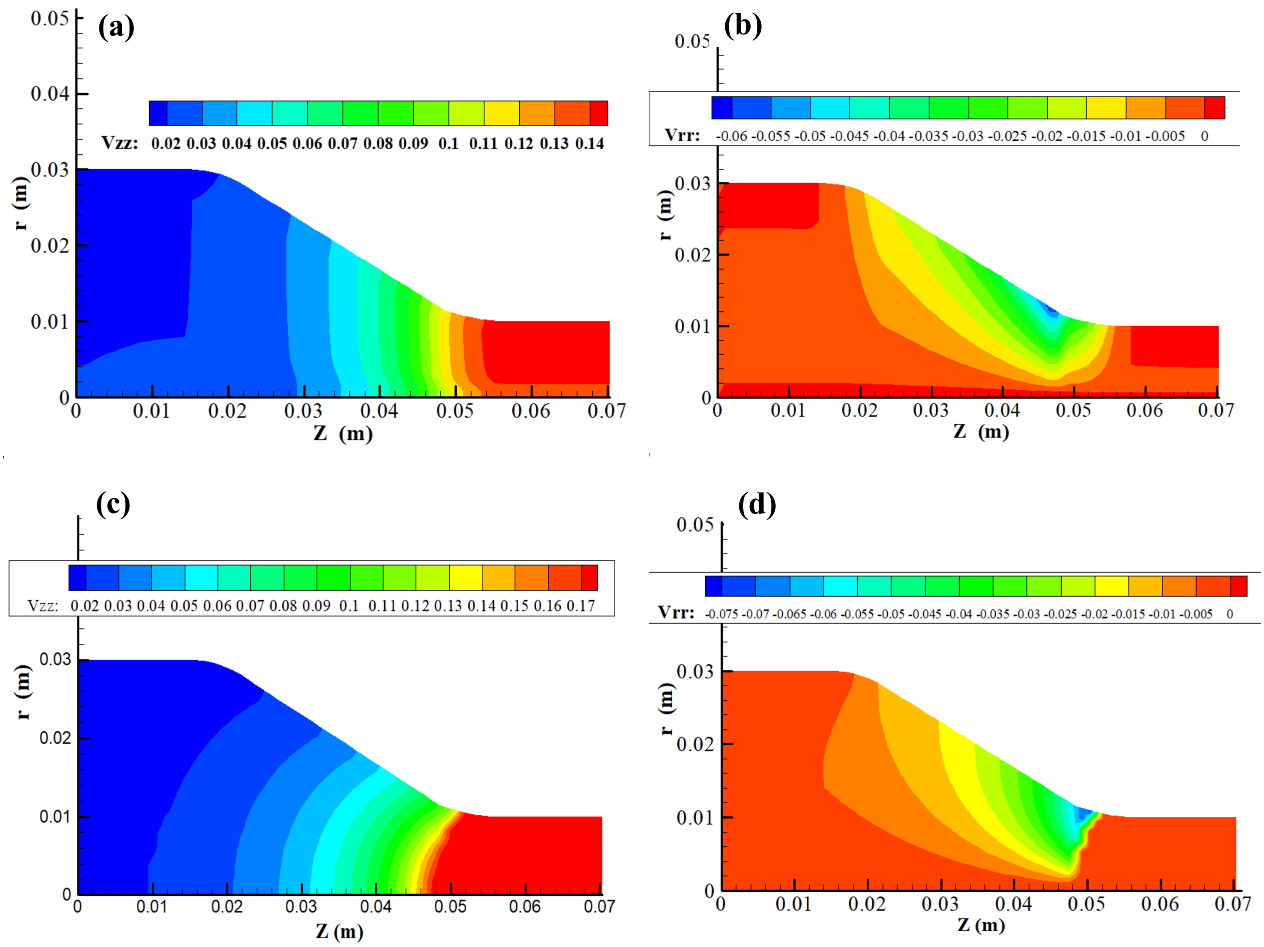

Fig. 4. Comparison of the velocity field results by the FVM numerical simulations and the Avitzur's model: (a) axial component Vz and (b) radial component Vr by the FVM simulations. (c) axial component Vz and (d) radial component Vr by the Avitzur's model. Friction factor m=0.5.

The comparisons of the velocity field results by the FVM numerical simulations and the Avitzur's analytical model are depicted in Fig.4. In the FVM approach, the friction factor at the die wall, $\tau_{nt} = mk$, was assumed equal to m=0.5. However, in the Avitzur's analytical velocity field model seen in Eq.1a and Eq.1b no friction was considered. Therefore, the Avitzur's field of the axial velocity component Vz at the die deformation region is quite different from the FVM model. Nevertheless, the Avitzur's field of the radial velocity component Vr is very similar to the FVM model.

In Fig.5, assuming friction factor m=0 in the FVM numerical simulations, the FVM field of the axial velocity component Vz has no change, except near the die wall. Thus, the difference of the fields, distribution of velocity bands of the axial velocity Vz is quite large compared with Avitzur.

Comparisons of the velocity component Vr field results by the FVM simulations and the Zhao's model is shown in Fig.6. The maximum radial velocity attained in extrusion at die exit corner was: Vr = -0.06 m/s (FVM approach) and -0.12 m/s (Zhao´s approach).

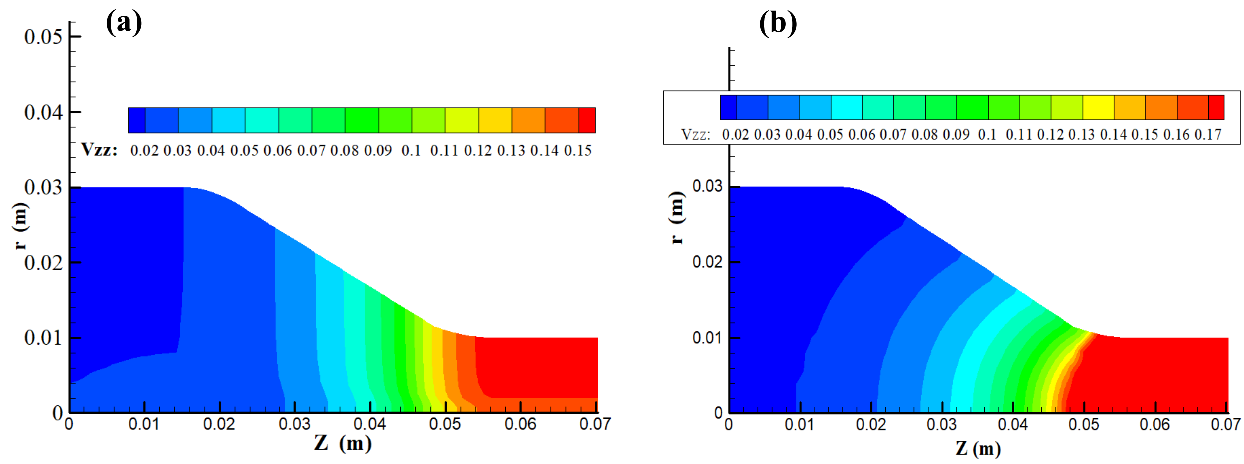

Fig. 5. Comparison of the velocity Vz field results by the (a) FVM numerical simulations and the (b) Avitzur's model. Assuming friction factor m at die wall equal to zero, m=0.

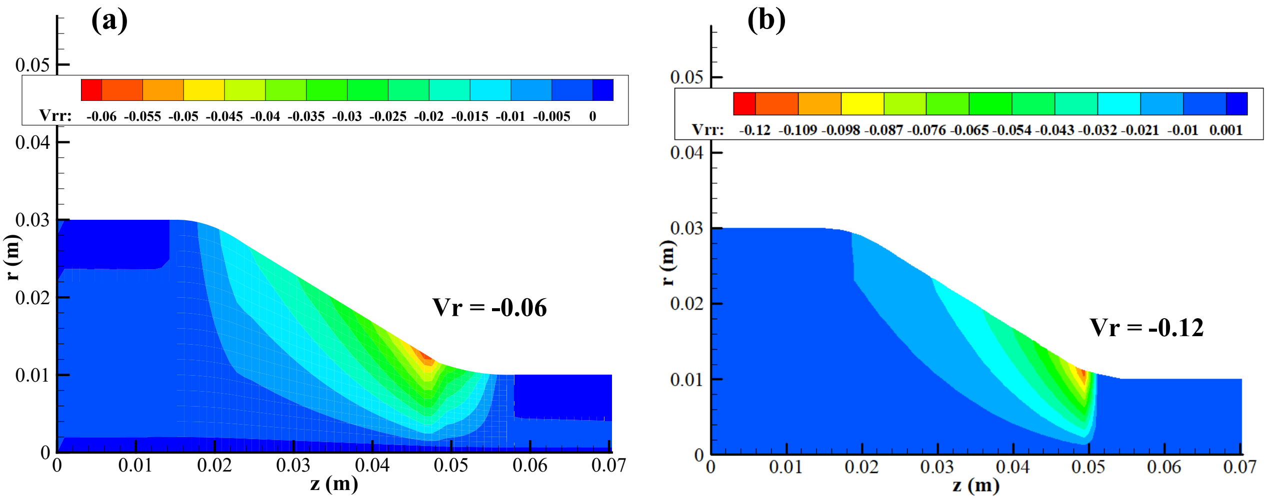

Fig. 6. Comparison of the velocity Vr field results by the (a) FVM numerical simulations and the (b) Zhao's model. Assuming friction factor m at die wall equal to zero, m=0.

Conclusions

Based on the analysis of present analytical models of axisymmetric extrusion proposed by Avitzur and by Zhao, and the FVM numerical simulation results for the velocity fields of direct hot extrusion of Al 6351 aluminium alloy billet, the following conclusions can be summarized,

a) In the analytical models proposed by Avitzur and the Zhao for the field of velocity components, no friction between the die wall and the extruded material is present in the equations.
b) The distribution of the radial component of velocity Vr from the Avitzur's and the FVM's models are quite similar.
c) However, the difference of the fields, distribution of velocity bands, for the longitudinal component of velocity Vz from the Avitzur's and the FVM's models is quite large.
d) Assuming friction zero, m=0, in the FVM numerical simulation, the difference of the longitudinal component of velocity Vz with the Avitzur' solution is still great.
e) The distribution and values of the longitudinal velocity Vz of FVM simulations are similar to Zhao´s approach, but had poor correlation with the radial velocity Vr.
f) The maximum radial velocity attained in extrusion at die exit corner was: Vr = -0.06 m/s (FVM approach),-0.07 m/s (Avitzur´s approach) and -0.12 m/s (Zhao´s approach).
g) Therefore, the region of most severe condition of wear on the inner wall of the die and material surface damage occurs in the area near the exit corner of the die. However, the predicted location of the severe wear region in the die wall by the FVM method is located prior to the point predicted by the Avitzur model.

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