

A New Ex-Situ Method for Real Contact Area Determination for Sheet Metal Forming

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Abstract. Experimental determination of the real contact area is essential for the development and validation of advanced friction models in sheet metal forming. However, existing experimental approaches are limited by the need for transparent counterfaces or third agents in the interface, high sensitivity to relocation errors, or unreliable assumptions about the contact shape. This study proposes a novel ex-situ method to determine the real contact area using topographical information from the original and deformed sheet surfaces. The approach identifies a minimum contact area with high confidence and reconstructs from it the full contact area. It provides the real contact area ratio, the contact topography and height distribution. The method is evaluated using results from normal load simulations based on the Pullen and Williamson model as the reference and is compared with other ex-situ methods from literature. Results demonstrate that the proposed method is robust against realistic relocation errors and yields more accurate contact area values than existing approaches. The method offers a reliable experimental tool for tribological analysis and friction modelling in sheet metal forming.

Introduction

Sheet metal forming is a complex manufacturing process governed by a wide range of material, geometrical, and process parameters. Among these, tribological phenomena play a decisive role in controlling material flow, forming forces, tool life, and the final surface quality [1]. Consequently, accurate numerical simulation of sheet metal forming requires advanced friction models that can reliably capture the underlying contact mechanics. The interacting surfaces in forming operations are inherently rough, exhibiting asperities and valleys across a broad range of length scales [2]. When a sheet comes into contact with a tool, contact occurs only at discrete asperity junctions, which occupy a small fraction of the nominal contact area, as originally demonstrated by Bowden and Tabor [3]. This real contact area fundamentally governs frictional behaviour, making its accurate determination crucial for advanced friction model development and validation in sheet metal forming.

The first step is to predict the real contact area under normal loading conditions. To this end, numerous models have been developed which can be broadly categorized into asperity-based, fractal, and continuum approaches [4]. While the theoretical understanding of rough surface contact is well established, systematic experimental validation remains limited [5]. The primary challenge lies in the difficulty of directly measuring the real contact area in experiments [6]. To address this limitation, several experimental techniques have been proposed, which are classified as in-situ and ex-situ methods [7].

In-situ methods aim to measure the real contact area during loading, either indirectly or directly. Indirect methods infer the real contact area from ultrasonic signals [8] or electrical [9] and thermal resistance measurements [10]. However, these approaches require extensive calibration and are

limited in spatial resolution, restricting their practical applicability [7]. Direct in-situ techniques rely on optical access to the contact interface using a transparent counterface. Methods such as optical microscopy with CCD cameras [11], confocal microscopy [12], and optical interferometry [13] have been employed. Their main limitation is the requirement for at least one optically transparent and smooth surface, which restricts their applicability to rough–flat contacts and limits the range of materials and surface topographies that can be investigated.

Ex-situ methods estimate the real contact area after unloading. Some methods introduce a third medium at the interface, such as thin metallic films [14] or liquids [15], to reveal contact regions. However, the presence of an interfacial agent can alter the contact mechanics and compromise the validity of the results. Other approaches rely on surface topography measurements performed before and after contact. Relocation profilometry techniques compare height maps acquired at the same surface location before and after testing [10]. Although conceptually simple, these methods require extremely accurate surface relocation, particularly when deformations are small. Alternative strategies analyse changes in the height distribution functions of the surfaces and define a transition height above which all points are assumed to be in contact [16], [17]. A key limitation of these approaches is the implicit assumption that contact occurs on a flat plane, which is generally not valid for rough surface contacts.

Despite significant progress, there is a need for experimental methodologies that can reliably determine the real contact area without requiring transparent counterfaces, interfacial agents, or restrictive assumptions regarding the contact surface shape. The objective of this study is to address this gap by proposing a novel ex-situ method that exploits topographical information from the original and deformed sheet surfaces to identify contact points with high confidence and reconstruct the real contact area from them. The proposed approach is robust against relocation errors and provides not only the real contact area ratio, but also the associated contact topography and height distribution, thereby offering deeper insight into the contact interface. This will enable an accurate experimental determination of the real contact area for advancing the understanding of tribological systems and validating advanced friction models in sheet metal forming applications.

Review of Current Methods

In this section, ex-situ methods reported in the literature that use topographical information from the contacting surfaces are reviewed.

Transition height methods. Transition height methodologies estimate the real contact area by comparing the height distribution functions of the original and deformed surfaces. The method assumes that asperities deform plastically once they come into contact with the rigid plane as purely elastic deformation of asperities on metallic surfaces occurs only at extremely small surface compression heights. They also assume the contact between a deformable rough surface and an ideal perfectly flat rigid plane. Hence, when a rough surface is plastically deformed against a perfectly rigid flat plane, the distribution of surface heights shifts toward lower values, producing a sharp cut-off peak. The position of this transition height (h_t) defines the extent of the real contact area (Fig. 1a).

However, in practice the height distribution of the deformed surface does not exhibit a sharp transition but instead displays a gradual slope. This deviation arises from elastic recovery, small misalignments, and the roughness of the rigid counterpart. The absence of a distinct transition point complicates the determination of h_t , which strongly influences the resulting contact area estimation.

Two main approaches have been reported to set h_t . Westeneng [18], defined the transition height at the local peak of the distribution function (h_{tMax}) (see Fig. 1b). In contrast, Kucharski and Starzynski [17] proposed that asperities in contact correspond to points lying outside the original height distribution, establishing the transition height at the deviation between the original and deformed functions (h_{tDif}) (see Fig. 1c). The methods are named transition height peak method and transition height difference method respectively for the rest of the document. In both approaches, the real contact area is estimated from the material ratio corresponding to the h_t (see Fig. 1b and c)

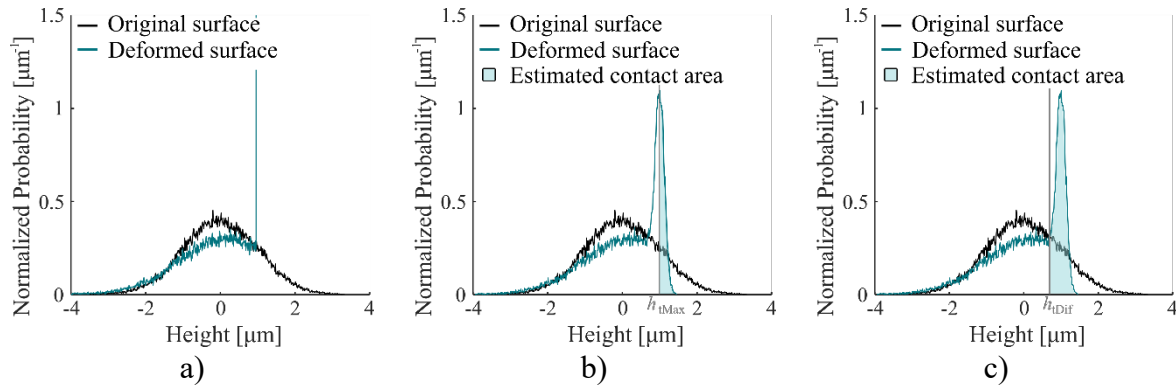


Fig. 1 a) Height distribution function of a sheet deformed against an ideally flat rigid plane. Height distribution function of sheet deformed against a tool with an $R_a = 0.1 \mu\text{m}$ and contact area determined with b) Transition height peak method and c) Transition height difference method.

Relocation methods. The relocation height method uses the original and deformed 3D surface topography to determine the real contact area. When two surfaces come into contact and deform plastically the height at the contact points is reduced. Hence, if measurements of the same location are acquired before and after deformation (see Fig. 2a and b respectively), the undeformed topography can be subtracted from the deformed topography. Ideally, a positive difference indicates a reduction in height, confirming that the point was in contact (Fig. 2c). Therefore, the real contact area ratio, the distribution of contact patches, defined as continuous surface regions where neighbouring asperities are in contact with the counterface, and the height distribution function of the contact area can be determined.

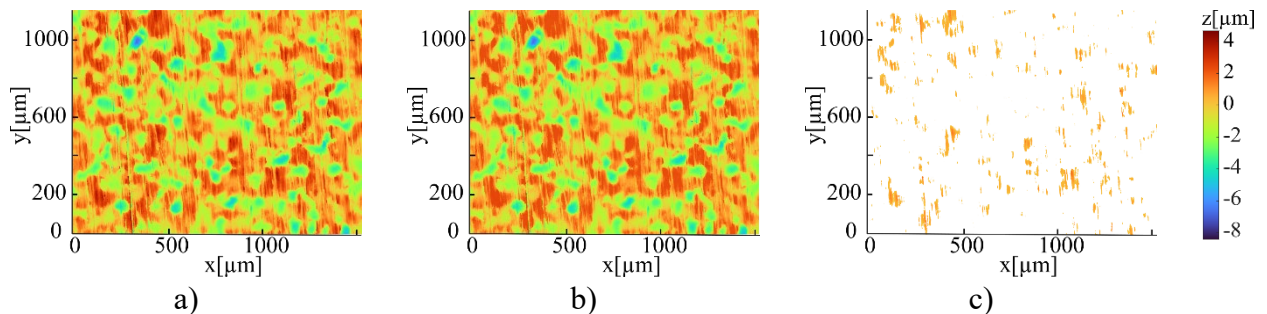


Fig. 2 a) Original sheet surface. b) Deformed sheet surface. c) Plastic deformation of the contact points

Accurate relocation is therefore essential; otherwise, observed changes in the profile could be mistakenly attributed to misalignment rather than plastic deformation. This is particularly critical in small deformation scenarios, where the noise introduced by the relocation procedure and measurement process may not be possible to filter out.

Novel method

When a sheet is compressed against a tool, sheet surface asperities flatten and the overall height range decreases compared with the original surface (see Fig. 3a). Height levels present in the original topography but absent after deformation indicate asperities that have undergone plastic deformation. It should be noted that the mean plane of the surfaces is taken as the reference plane for the height levels. Consequently, these locations must have been in direct contact with the tool and constitute the minimum contact area. The key idea of this method is to use the information from the minimum contact area to estimate the real contact area.

The determination of the minimum contact area is performed through the following steps. First, the minimum contact points are identified by determining the height corresponding to 99.7% of the material ratio curve of the deformed surface, denoted as h_{CMAX} (see Fig. 3a). The 99.7% threshold is chosen based on the three-sigma rule to exclude extreme outliers. The original surface is then thresholded at this level to extract the spatial distribution of the minimum contact points (see Fig. 3b).

This is converted into a mask (Mask A_{Min}), (see Fig. 3c). Finally, Mask A_{Min} is applied to the deformed surface obtaining the minimum contact area surface and height distribution of it (see Fig. 3d and e respectively).

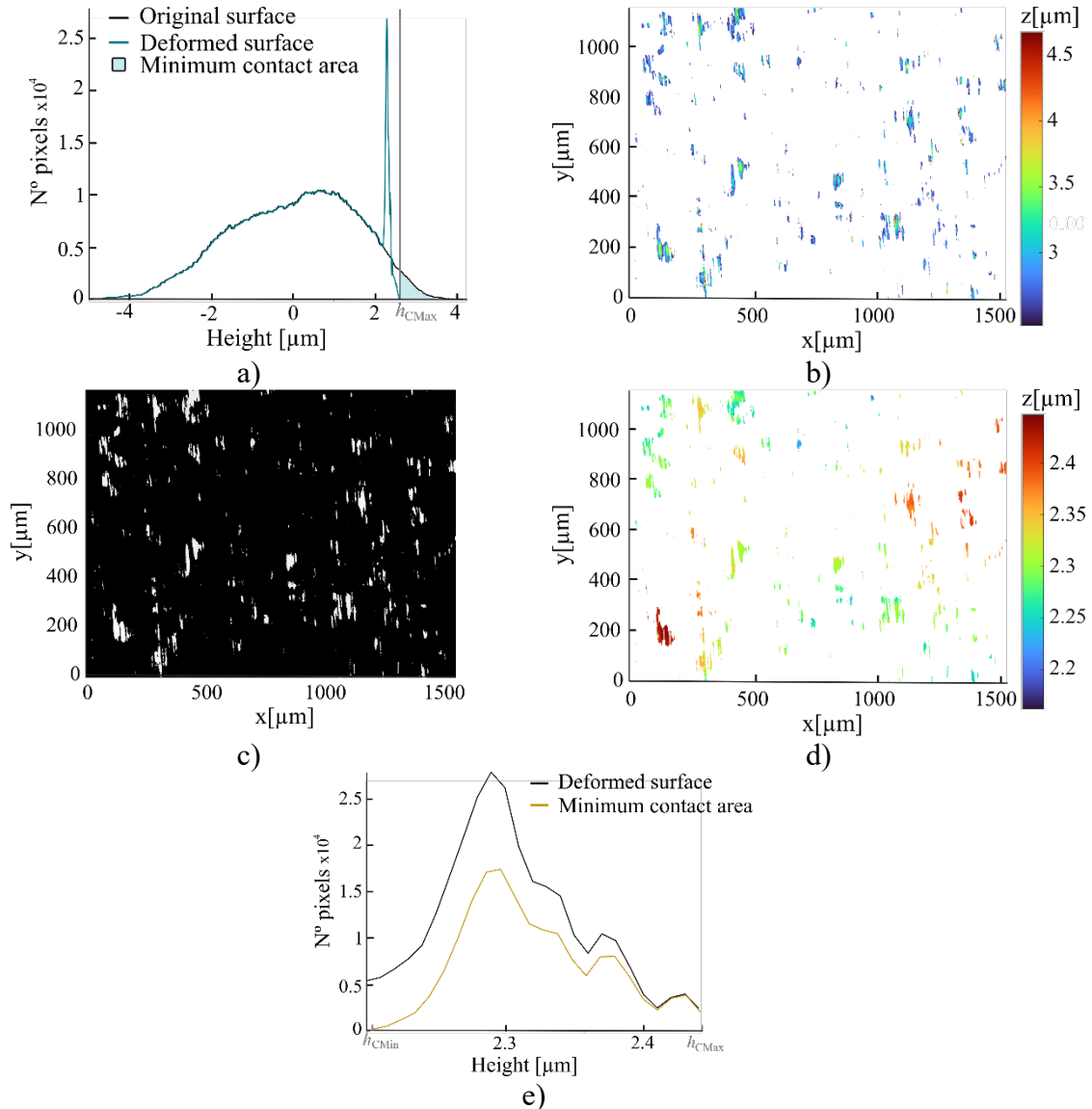


Fig. 3 a) Height distribution function analysis to determine A_{Min} . b) Topography of the minimum contact area in the original sheet surface. c) Mask of the minimum contact area (Mask A_{Min}). d) Topography of the minimum contact area in the deformed sheet surface. e) Height distribution function of the minimum contact area

The minimum contact area represents a conservative lower bound of the real contact area. Based on the identified minimum contact area, the real contact area is reconstructed under the following physically grounded assumptions: (i) At least one point has been detected in each contact patch of the real contact area. (ii) The remaining contact points are located in the vicinity of these detected points. (iii) All contact points have heights within the height range defined by the minimum contact points on both the original and deformed surfaces. (iv) The smallest height difference between the original surface and the minimum contact surface represents the minimum plastic deformation (δ_{Min}) experienced by the asperities.

The reconstruction proceeds as follows. First, the lowest height of the determined minimum contact area (Fig. 3e), h_{CMin} , is obtained from the 99.7% material ratio curve. δ_{Min} is determined by subtracting the identified contact points of the original sheet topography and the minimum contact surface (surfaces shown in Fig. 3b and d respectively) and determining the value of 99.7% of the material ratio curve.

The perimeter of Mask A_{Min} (Fig. 4a) is then iteratively expanded. At each iteration, newly included points are retained only if their height on the deformed surface exceeds h_{CMin} . This procedure is repeated until no additional points are identified, resulting in Mask A_{Max} (see Fig. 4b).

This mask is then filtered in two consecutive consistency checks to ensure that the reconstructed real contact area reflects physically admissible contact conditions and is robust against measurement noise, surface relocation errors, and purely morphological artefacts. First, Mask A_{Max} is filtered by applying it to the original surface and removing all the points that have a height smaller than h_{CMin} , ensuring geometric compatibility with pre-existing asperities yielding Mask Filtered (see Fig. 4c). Mask Filtered is further refined applying it to the surface obtained from subtracting the original topography from the deformed one. Points with deformation values smaller than δ_{Min} are removed, resulting in the final mask Mask A_{Real} (see Fig. 4d).

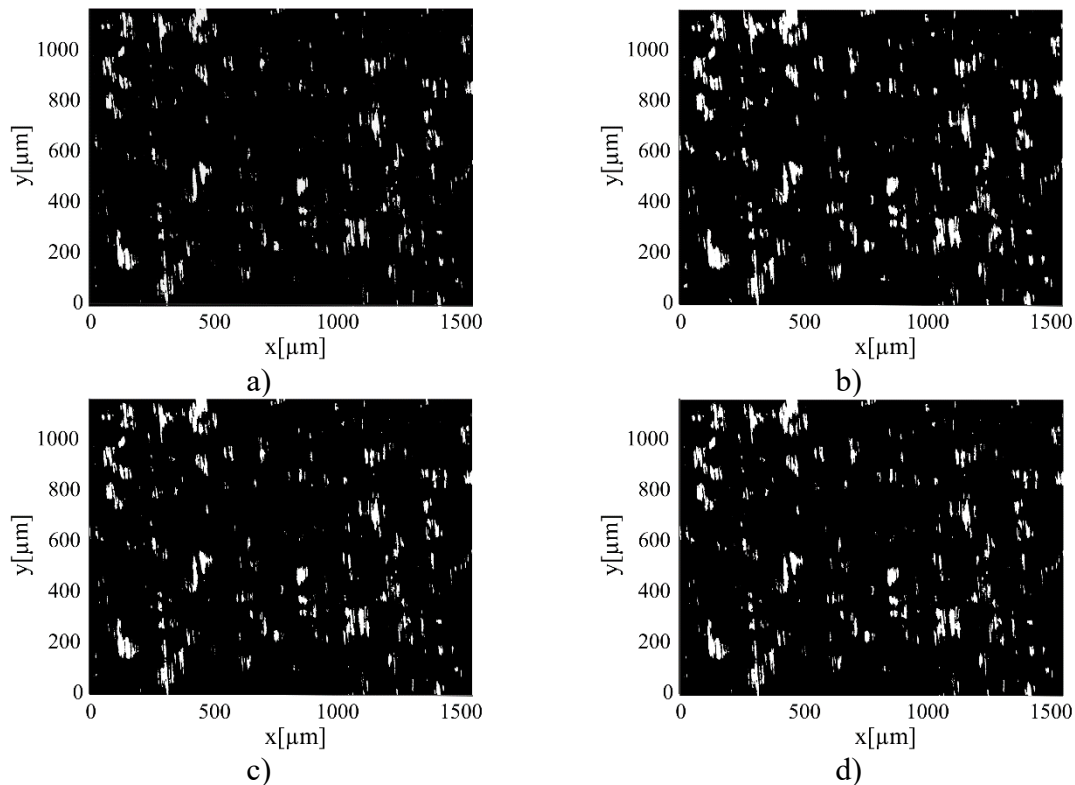


Fig. 4 Real contact area determination procedure through masking a) Mask A_{Min} . b) Mask A_{Max} . c) Mask Filtered d) Mask A_{Real} .

Mask A_{Real} is finally applied to the deformed sheet surface obtaining the real contact area ratio (RCA), the topography (see Fig. 5a), and height distribution function (see Fig. 5b) of the real contact area.

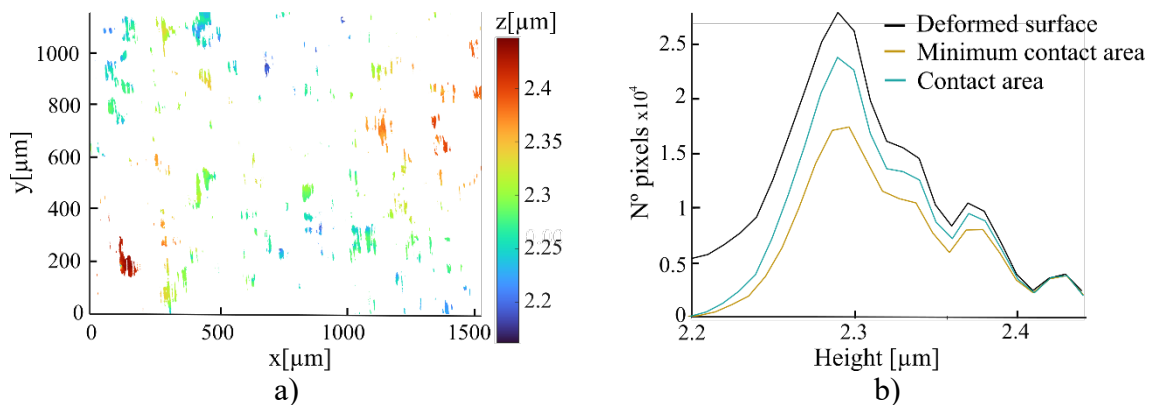


Fig. 5 Real contact area a) Topography, b) Height distribution function

Material and Methods

To assess the capability of the proposed method to determine the real contact area ratio (RCA), as well as, the corresponding contact topography and height distribution, normal load simulations were performed. The contact area calculated in the simulation is considered as the true contact area for validation (A_{True}). It needs to be highlighted that no experimental normal-loading tests were performed because the real contact area cannot be directly measured and depends on the chosen method, preventing meaningful validation. Instead, simulations were used because they provide a known contact area under ideal conditions, allowing the accuracy of different estimation methods to be compared. The goal of this work is to assess the validity of the methods to estimate the real contact area not to validate the accuracy of the simulation itself.

The Pullen and Williamson model with volume conservation was adopted [19], based on the following assumptions: (i) the tool is considered rigid; (ii) plastic material behaviour of the sheet; (iii) the asperity persistence effect is ignored; (iv) material displaced from contacting asperities is uniformly redistributed over the non-contacting regions; and (v) contact is frictionless.

To account for the tool and sheet surfaces, the equivalent surface method is employed [20]. Hence, the surfaces of the tool (Z_{Tool}) and sheet (Z_{Sheet}) are combined,

$$Z_{\text{eq}} = Z_{\text{Sheet}} + Z_{\text{Tool}} \quad (1)$$

where Z_{Tool} and Z_{Sheet} are matrices containing the Z -coordinates of the tool and sheet surfaces at each (x, y) position.

The Z_{eq} is conceptualized as a collection of discrete bars, with areas equal to the topography resolution (see Fig. 6). Z_{eq} becomes into contact against a rigid plane.

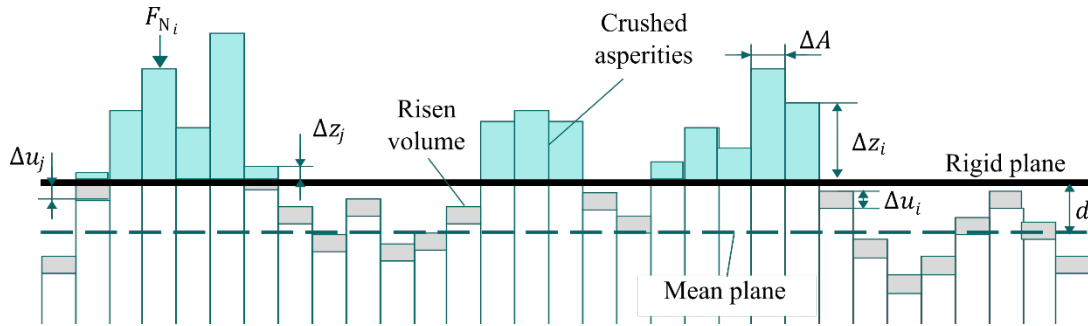


Fig. 6 The equivalent surface crushed by a rigid flat plane. Adapted from [21].

The asperities are categorized into three distinct groups based on their interaction with rigid plane: (i) the bars that are in contact (N), (ii) bars that will become in contact due to the risen volume (N^*) and (iii) the bars that will never come into contact (N^{**}). As the volume is conserved, the volume of the crushed asperities must be equal to the volume of the material redistributed into the non-contacting areas as stated in Eq (2)

$$\sum_{i=1}^N \Delta z_i \Delta A + \sum_{j=1}^{N^*} \Delta z_j \Delta A = \sum_{j=1}^{N^*} \Delta u_j \Delta A + \sum_{i=1}^{N^{**}} \Delta u_i \Delta A \quad (2)$$

where Δz is the crush height and Δu is the risen volume height (see Fig. 6) and ΔA the area of each bar. In order to determine A_{real} , Pullen and Williamson derived Eq (3)

$$A_{\text{real}} = \frac{F_N}{P_{\text{nom}} + H} \quad (3)$$

where P_{nom} is the nominal pressure and H the material hardness. Hence, for each distance between surface d , resultant P_{nom} , deformed sheet and contact area topography and the RCA, can be calculated by $\frac{A_{\text{real}}}{A_{\text{nom}}}$.

Simulations were carried out using the material properties of DC01 steel. Uniaxial tensile tests determined a yield strength of 209 MPa. Additionally, the material hardness was measured through a Vickers hardness test with a 30 kgf load (HV30), resulting in a hardness value of 1066 MPa. Nominal pressures ranging from 10 MPa to 60 MPa in steps of 10 MPa were simulated. A total of 6 simulations were performed. The same polished die topography was employed for all the simulations while different surface topography measurements extracted from the same sheet material were employed for each pressure range. From the simulation, the RCA and deformed sheet topographies were obtained. The deformed topographies were then analysed using the proposed method and benchmarked against the state-of-the-art real contact area estimation methods reviewed in Section 2, using the simulation results as reference.

Sheet and die surface topographies were measured using a Sensofar SNEOX optical profilometer in interferometric mode with a 20× EPI objective (spatial sampling = 645 nm) to feed the simulation with real topographies. Post-processing of the acquired data was carried out with SensoMap Premium 7.4 metrology software. The surface form was removed using a first-order polynomial fitting (F-operator), while noise was suppressed with a Gaussian filter ($\lambda_s = 1.3 \mu\text{m}$, S-filter).

The tool surface was a polished die with $R_a = 0.01 \mu\text{m}$ (see Fig. 7a and b) and the DC01 sheet surface was sanded to obtain a Gaussian height distribution with $R_a = 1.6 \mu\text{m}$ (see Fig. 7c and d)

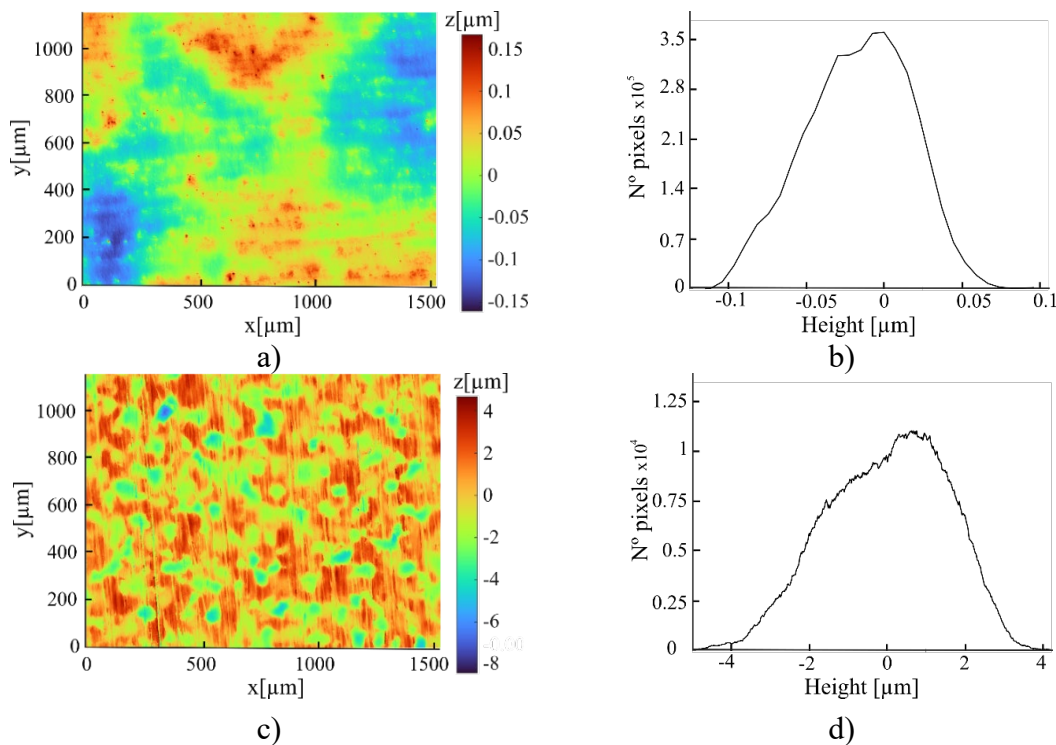


Fig. 7 Tool's a) Topography and b) Height distribution function. Sheet's c) Topography and d) Height distribution function.

As the methods require topographical information from the same locations on the original and deformed sheet surfaces, measurements must be performed both before and after the normal loading.

In practice, exact repositioning is not possible, leading to a relocation error that can affect the accuracy of the real contact area determination. Hence, this error was quantified experimentally.

To ensure accurate relocation, each sheet was marked at its centre with a femtosecond-laser pattern that served as a reference. The sheet was placed and removed three times, with a surface measurement acquired after each placement. The automatic surface relocation algorithm available in SensoMap Premium 7.4 was then applied to reduce the relocation error between measurements. Differences between all measurement pairs (1–2, 1–3, and 2–3) were computed. The relocation noise was defined as the largest standard deviation of these difference maps, which correspond to $\sigma = 0.584 \mu\text{m}$.

To incorporate relocation error into the simulations, the original sheet topography was shifted in the x-y directions until the standard deviation of the difference maps between the original and shifted surfaces matched the measured relocation noise. The same shift was then applied to the deformed surface, corresponding to displacements of approximately $6.5 \mu\text{m}$ in the x-direction and $7 \mu\text{m}$ in the y-direction.

Results and Discussion

Fig. 8a shows the RCA obtained for each simulated contact pressure using the methods reviewed in Section 2 (transition height difference, transition height peak and relocation methods) and the proposed method (New method), both under perfect relocation conditions (ideal) and with relocation noise introduced (real noise). The figure also includes the minimum contact area identified by the proposed method. Fig. 8b presents the error between the true contact area from the simulations and the values estimated by the different methods following Eq (4)

$$Error = A_{True} - RCA. \quad (4)$$

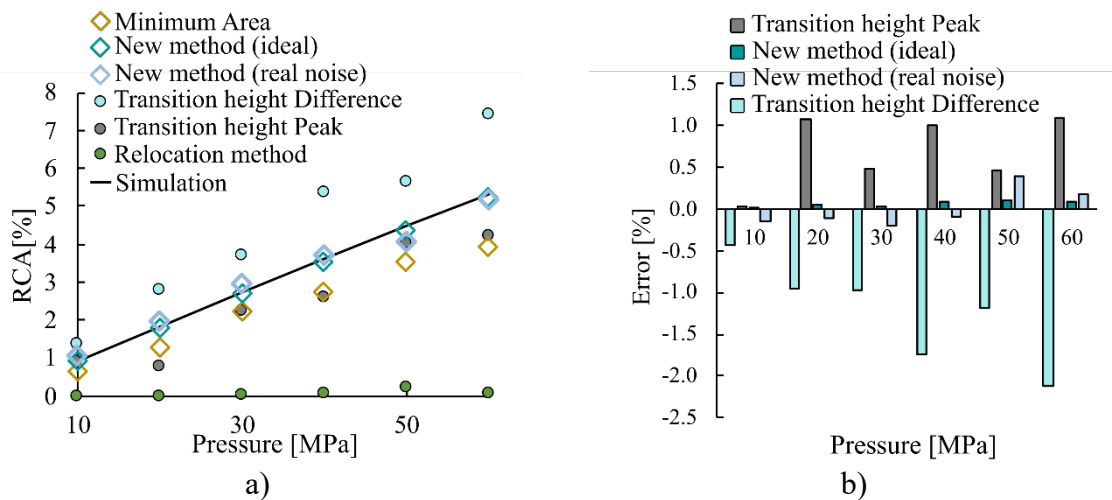


Fig. 8 a) Real contact area ratio results. b) Errors between the simulated true contact area and the real contact area estimated.

As observed, the relocation-based method yields RCA values close to zero. This occurs because only deformations exceeding the relocation noise can be detected, while the actual asperity deformations remain below this threshold as observed in Fig. 9. Consequently, the relocation method is not applicable under the considered conditions.

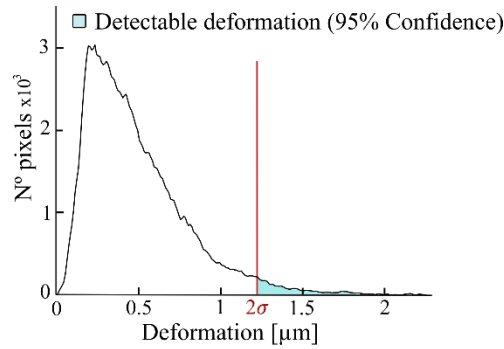


Fig. 9 Height distribution of plastic deformations in the DC01 sheet simulated at 60 MPa.

The transition height methods do not exhibit a consistent linear relationship with contact pressure, indicating a strong sensitivity to the resulting height distribution function. Even under ideal conditions, both approaches show significant errors, with maximum deviations from the true contact area of 2.11 % and 1.1 % for transition height difference method and transition height peak method respectively. The transition height difference method systematically overestimates the real contact area whereas the transition height peak method tends to underestimate it. These trends arise from the common assumption that contact occurs above a single plane, whereas in reality contact occurs at multiple heights.

As shown in Fig. 10a, the transition height difference method captures all contact points but also includes original surface points that were already above h_{tDif} . In contrast, the transition height peak method may locally overestimate the contact area for the same reason, while simultaneously neglecting contact points located below h_{tMax} , which normally leads to an underestimation (see Fig. 10b). In some cases, these opposing effects may compensate, resulting in apparently accurate RCA values, as observed for the 10 MPa condition. However, the corresponding contact topography and height distribution are not physically representative. In conclusion, although truncation height methods are simple to apply when a clear h_t can be identified, they suffer from fundamental theoretical limitations that compromise their reliability. Moreover, contacts involving non-Gaussian surface topographies may not exhibit a distinct local maximum in the height distribution, further limiting the applicability of these methods across different forming scenarios.

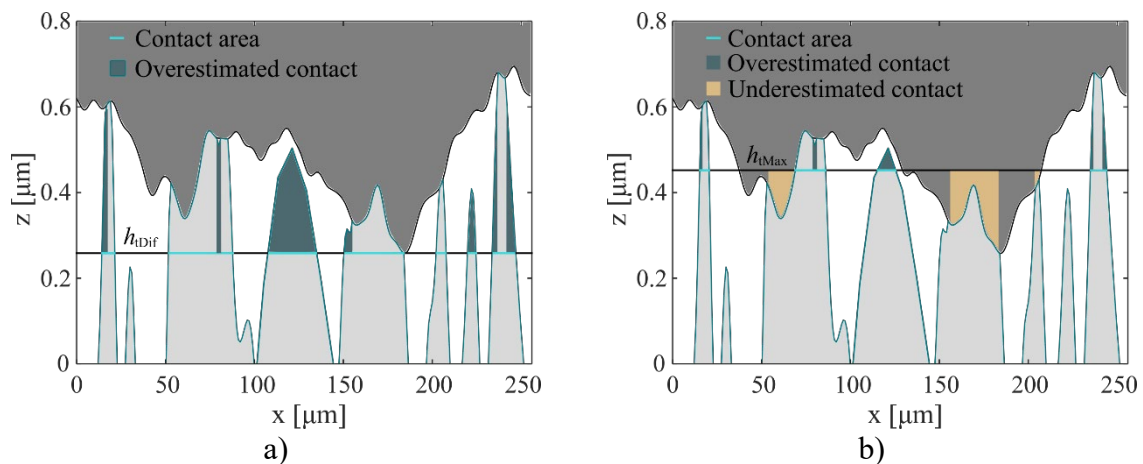


Fig. 10 2D profile of the contact between two rough surfaces. Real contact area estimation errors with a) Transition height difference method and b) Transition height peak method.

The minimum contact area identified by the proposed method follows the same trend as the true contact area and is insensitive to relocation noise, as it is determined from a threshold applied to the original surface at h_{CMax} .

Under ideal relocation conditions, the proposed method slightly underestimates the real contact area. This discrepancy arises from the assumption that at least one pixel is detected in each contact patch, which is not always satisfied, as shown in Fig. 11. The smallest contact patches are occasionally

not detected, and their surrounding regions are therefore not explored. However, the maximum difference between the estimated and true contact areas is approximately 0.1%, indicating that the contribution of these undetected patches is negligible and likely within experimental variability. Consequently, the method is validated under ideal conditions.

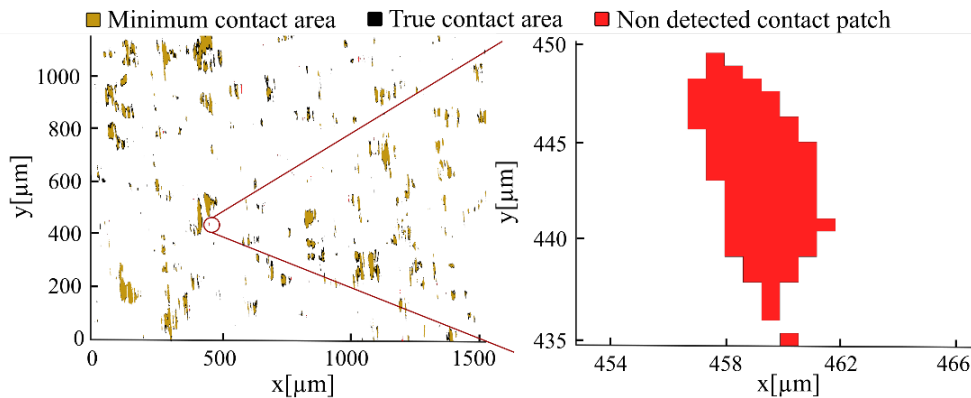


Fig. 11 Comparison between the true contact area, minimum contact area detected and non-detected contact patches of a DC01 sheet simulated at 60 MPa. Detailed view of the size of a non-detected contact patch.

When relocation error is introduced, the proposed method exhibits a similar overall trend to the ideal case. Although the estimation error increases slightly, the method remains more accurate than the approaches reported in the literature. The maximum deviation from the true contact area is of 0.4%. The method tends to overestimate the real contact area at pressures up to 40 MPa and to underestimate it at higher pressures.

Overall, the results demonstrate that the proposed method provides a robust and reliable determination of the real contact area, even in the presence of realistic relocation errors and provides reliable characterization of the contact topography and the height distribution function.

Conclusions

This study presented and validated a novel ex-situ methodology for determining the real contact area by employing the topographical information from the original and deformed sheet surfaces. The method is based on the identification of a minimum contact area that can be established with high confidence and on the subsequent reconstruction of the full real contact area using physically grounded assumptions. A systematic comparison with ex-situ methods found in literature was conducted. The following was concluded:

- Relocation-based methods were found to be unsuitable for the studied conditions, as the asperity deformations were of the same order of magnitude as the relocation noise.
- Transition height methods lead to an overestimation or underestimation of the contact area depending on where the transition height was established. The common assumption of contact occurring at flat plane convert the methods in unreliable.
- The proposed method provides a way of determining a minimum contact area regardless of the contact conditions
- The proposed method provided accurate and robust estimates of the real contact area over a wide range of contact pressures. Under ideal relocation conditions, the maximum deviation from the simulated real contact area was below 0.1%,
- When realistic relocation errors were introduced, the method preserved the overall trend and remained more accurate than existing techniques, demonstrating a high tolerance to measurement uncertainty.

Beyond estimating the real contact area ratio, the proposed method also provides the contact topography and the height distribution function of the real contact points, offering deeper insight into

the tribological behavior of the system. These outputs are essential for validating normal-loading models used in advanced friction formulations. The method thus constitutes a robust and valuable experimental tool for advancing tribological modelling and improving the predictive accuracy of numerical simulations in sheet metal forming, as it enables the experimental determination of the real contact area under normal loading conditions. Notably, the method requires both pre- and post-deformation surface measurements, meaning it relies on more experimental information than transition-height-based approaches.

Future work will assess the applicability of the method to cases involving combined normal pressure and in-plane deformation; and the increase on real contact area due to sliding which are important aspects of advanced friction modelling in sheet metal forming. Additionally, the applicability of the method to contacts involving non-Gaussian surface topographies or surfaces with comparable roughness levels (R_a of the same order of magnitude) will be evaluated.

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