Materials Science Forum
ISSN: 1662-9752, Vol. 681, pp 165-170
doi:10.4028/www.scientific.net/MSF.681.165

Online: 2011-03-28

Residual Stress Measurement in Coated Plates Using Layer Growing/ Removing Methods: 100th Anniversary of the Publication of Stoney's Paper "The Tension of Metallic Films Deposited by Electrolysis"

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Keywords: residual stress, experimental techniques, coated plates, layer growing/removing method, computer program, Stoney's equations.

Abstract. In connection with the 100th anniversary of Stoney's equations, some historical remarks are made with respect to the development of these equations. As an example of the extension of Stoney's equations, a unique algorithm of the layer growing/removing methods is presented for determination of residual stresses in isotropic inhomogeneous coated plates. Using a computer program based on this algorithm, residual stresses are computed in the galvanic steel coating on the copper plate substrate.

Introduction

The layer growing method (non-destructive method) and the layer removing method (destructive method) are used for experimental determination of residual stresses in coated plates. Elaboration of the theory of the layer growing method started 100 years ago with a seminal paper by Stoney [1] and was evolved in studies by Brenner and Senderoff [2], by Kõo [3], by Davidenkov [4], by Birger and Kozlov [5], by Doi et al. [6] as well as by other authors. The theory of the layer removing method was further developed in papers by Stäblein [7], Treuting and Read [8] and Moore and Evans [9], who treated homogeneous plates. Unique consideration of the layer growing and layer removing methods started with paper [10] presenting the general principles of this approach.

In paper [11] a common algorithm of the layer growing and layer removing methods is presented for determination of biaxial residual stresses in a free rectangular orthotropic inhomogeneous elastic plate whose elastic parameters depend on its thickness coordinate continuously or piecewise. This algorithm enables to calculate residual stresses from the curvature or strain measured on the stationary surface of the plate (substrate), as well as from initial stresses measured on the moving surface by the X-ray diffraction technique. In this study a special algorithm, following from the general algorithm for an isotropic inhomogeneous coated plate, is considered and applied for determination of residual stresses in a galvanic steel coating from the curvatures of a copper substrate measured during the growing process of coating.

Some Historical Remarks Relating to Stoney's Equations

In 1909 G. Gerald Stoney published his famous paper "The Tension of Metallic Films deposited by Electrolysis" [1]. In this paper he derived three equations for experimental determination of residual stresses originating from a galvanic coating applied onto one side of a steel strip from the curvature of the strip measured after deposition.

The premier equation, known usually as Stoney's formula, follows from the equilibrium conditions of the strip substrate with a very thin coating (film) and reads as:

$$\bar{\sigma} = \frac{Eh_1^2}{6h_2 r}, \tag{1}$$

where $\bar{\sigma}$ is residual (initial) stress in the coating, E is the modulus of elasticity (Young's modulus) of the substrate, h_1 is the thickness of the substrate, h_2 is the thickness of the coating and r is the radius of the curvature of the initially flat substrate after deposition of the coating.

The second equation, which represents Stoney's formula for a thick coating ($h_2 \approx h_1$) in a differential form and follows from Eq. 1, is:

$$\bar{\sigma} = \frac{E\left(h_1 + y\right)^2}{6}\frac{\mathrm{d}æ}{\mathrm{d}y}, \tag{2}$$

where y is the current thickness of the coating and $æ = 1/r$ is the curvature of the substrate.

The third equation, resulting from differential Eq. 2 after integration provided that stress $\bar{\sigma}$ remains constant with thickness, is

$$\bar{\sigma} = \frac{E\left(h_1 + h_2\right)h_1}{6h_2}æ. \tag{3}$$

In 1949 Brenner and Senderoff [2] extended Stoney's equations for the case of thick coatings of strip substrates with various boundary conditions and with the modulus of elasticity differing from that of the coating. In particular, they considered the substrate with slipping ends and developed a theory which allows determination of residual stress σ in the coating as the sum of initial (instantaneous) stress $\bar{\sigma}$ and additional stress σ^*:

$$\sigma = \bar{\sigma} + \sigma^*. \tag{4}$$

It is worth mentioning that Stoney, Brenner and Senderoff treated the coating process as a consecutive application of elementary layers equidistant from the substrate surface, i.e. they used a model that is known as the model of continuous growth in layers [10].

The above analyses were based on the same assumption, i.e. the state of stress of the coated substrate is uniaxial. In 1959 Kõo [3] and in 1960 Davidenkov [4] pointed out that the stress state of a substrate with coating is biaxial but not uniaxial as was assumed previously; they showed that the difference can be taken into account by multiplying the stress values, obtained from Eqs. 1 - 3, by factor $1/(1-\mu)$, where μ is the Poisson's ratio of the substrate material.

Thus, today Stoney's Eqs. 1 and 2 are generally used in the form:

$$\bar{\sigma} = \frac{E}{1-\mu}\frac{h_1^2}{6h_2}æ; \qquad \bar{\sigma} = \frac{E}{1-\mu}\frac{\left(h_1 + y\right)^2}{6}\frac{\mathrm{d}æ}{\mathrm{d}y}, \tag{5}$$

where factor $E/(1-\mu)$ is known as the biaxial modulus of the substrate material.

It is possible to express curvature $æ$ by means of strain ε measured by a strain gauge on the free stationary surface of the substrate. Then, instead of Eqs. 5, one has [10, 12]

$$\bar{\sigma} = \frac{E}{1-\mu}\frac{h_1}{2h_2}\varepsilon; \qquad \bar{\sigma} = \frac{E}{1-\mu}\frac{h_1 + y}{2}\frac{\mathrm{d}\varepsilon}{\mathrm{d}y}. \tag{6}$$

Today, Stoney's Eqs. 5 and their extensions are frequently used for relating the substrate's curvature to coating stress. In particular, the first equation of Eqs. 5 serves as the basis for standard [13].

An Algorithm of Layer Growing/Removing Methods for Plates

Following algorithm [11], consider a thin layer growing on one face of a free rectangular plate, whose elastic parameters depend on its thickness coordinate continuously or piecewise (Fig. 1).

Let the initial thickness of the plate be z_1, variable thickness h and final thickness z_2. Rectangular coordinates x, y and z are used, where the free stationary surface of the plate is taken as the reference surface (x, y) and coordinate z is perpendicular to the stationary surface. It is assumed that axes x

and y are the principal axes of the state of residual stresses depending on coordinate z only. It is also assumed that the edges of the plate are parallel to axes x and y.

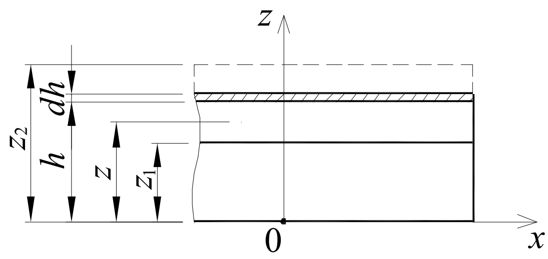

Figure 1. Layer-growing on the upper face of a rectangular plate

According to the general algorithm, residual stresses in layer z of the coating can be calculated as the sum of initial and additional stresses:

$$\{\sigma\} = \{\bar{\sigma}\} + \{\sigma^*\}, \tag{7}$$

where $\{\sigma\} = \begin{bmatrix}\sigma_x & \sigma_y\end{bmatrix}^{\mathrm{T}}$, $\{\bar{\sigma}\} = \begin{bmatrix}\bar{\sigma}_x & \bar{\sigma}_y\end{bmatrix}^{\mathrm{T}}$, $\{\sigma^*\} = \begin{bmatrix}\sigma_x^* & \sigma_y^*\end{bmatrix}^{\mathrm{T}}$ are the vectors of residual stresses, initial stresses and additional stresses, respectively.

Initial stresses $\bar{\sigma}_x = \bar{\sigma}_x(h)$, $\bar{\sigma}_y = \bar{\sigma}_y(h)$ in differential surface layer dh can be expressed by strains $\varepsilon_x = \varepsilon_x(h)$, $\varepsilon_y = \varepsilon_y(h)$, and curvatures $æ_x = æ_x(h)$, $æ_y = æ_y(h)$ measured on the stationary surface ($z = 0$) as follows:

$$\{\bar{\sigma}\} = [B]\left\{\frac{\mathrm{d}\tilde{\varepsilon}}{\mathrm{d}h}\right\} - [C]\left\{\frac{\mathrm{d}\tilde{æ}}{\mathrm{d}h}\right\}; \qquad h\{\bar{\sigma}\} = [C]\left\{\frac{\mathrm{d}\tilde{\varepsilon}}{\mathrm{d}h}\right\} - [D]\left\{\frac{\mathrm{d}\tilde{æ}}{\mathrm{d}h}\right\}, \tag{8}$$

where

$$\left\{\frac{\mathrm{d}\tilde{\varepsilon}}{\mathrm{d}h}\right\} = \begin{bmatrix}\frac{\mathrm{d}\tilde{\varepsilon}_x}{\mathrm{d}h} & \frac{\mathrm{d}\tilde{\varepsilon}_y}{\mathrm{d}h}\end{bmatrix}^{\mathrm{T}} \text{ and } \left\{\frac{\mathrm{d}\tilde{æ}}{\mathrm{d}h}\right\} = \begin{bmatrix}\frac{\mathrm{d}\tilde{æ}_x}{\mathrm{d}h} & \frac{\mathrm{d}\tilde{æ}_y}{\mathrm{d}h}\end{bmatrix}^{\mathrm{T}}$$

are the vectors of the derivatives of strain changes $\tilde{\varepsilon}_x = \varepsilon_x(z_2) - \varepsilon_x(h)$, $\tilde{\varepsilon}_y = \varepsilon_y(z_2) - \varepsilon_y(h)$ and curvature changes $\tilde{æ}_x = æ_x(z_2) - æ_x(h)$, $\tilde{æ}_y = æ_y(z_2) - æ_y(h)$, respectively. [$B$], [$C$] and [$D$] are the matrices of the elastic parameters given by

$$[B] = \begin{bmatrix}B & B_\mu \\ B_\mu & B\end{bmatrix}, \ [C] = \begin{bmatrix}C & C_\mu \\ C_\mu & C\end{bmatrix}, \ [D] = \begin{bmatrix}D & D_\mu \\ D_\mu & D\end{bmatrix} \tag{9}$$

and

$$\begin{bmatrix}B & B_\mu \\ C & C_\mu \\ D & D_\mu\end{bmatrix} = \int_0^h \left[E^0(z)\right] \begin{Bmatrix}1 \\ z \\ z^2\end{Bmatrix} \mathrm{d}z \tag{10}$$

with

$$\left[E^0\right] = \begin{bmatrix}E^0 & E_\mu^0\end{bmatrix} \text{ and } E^0 = \frac{E}{1-\mu^2}, \ E_\mu^0 = \mu E^0, \tag{11}$$

where $E = E(z)$ denotes the modulus of elasticity, and $\mu = \mu(z)$ denotes Poisson's ratio.

The expression for computing additional stresses in the coating $(z_1 \le z \le z_2)$ is

$$\{\sigma^*\} = \left[E^*\right] \int_z^{z_2} \left[-\left\{\frac{\mathrm{d}\tilde{\varepsilon}}{\mathrm{d}h}\right\} \quad z\left\{\frac{\mathrm{d}\tilde{æ}}{\mathrm{d}h}\right\}\right] \mathrm{d}h, \tag{12}$$

where

$$\left[E^*\right] = E^0(z)\begin{bmatrix}1 & \mu(z) \\ \mu(z) & 1\end{bmatrix}. \tag{13}$$

For computing residual stresses in the substrate $(0 \le z \le z_1)$, the lower limit z of Eq. 12 should be replaced by z_1.

If measurement of strains or curvatures is not performed during coating growth then, using the removing procedure, it should be assumed in the above algorithm that $\varepsilon_x(z_2)=\varepsilon_y(z_2)=$ $=æ_x(z_2)=æ_y(z_2)=0$.

Eqs. 7 - 13 form a common algorithm of the layer growing/removing methods for isotropic inhomogeneous plates, allowing calculation of residual stresses at growing/removing on one face of the plate:

(1) From curvatures and strains measured on the free stationary surface (z = 0) depending on thickness h. In this case initial stresses are computed by using Eqs. 8. From Eq. 12 the expression for computing additional stresses is

$$\{\sigma^*\}=\left[E^*\right]\left[\{\tilde{\varepsilon}\} \quad -z\{\tilde{æ}\}\right]. \tag{14}$$

(2) From measured strains or curvatures only. In this case the unmeasured deformation parameter is computed from the equation

$$\left[[C]-h[B]\right]\left\{\frac{\mathrm{d}\tilde{\varepsilon}}{\mathrm{d}h}\right\}=\left[[D]-h[C]\right]\left\{\frac{\mathrm{d}\tilde{æ}}{\mathrm{d}h}\right\}, \tag{15}$$

which follows from Eqs. 8.

(3) From initial stresses measured by the X-ray diffraction technique on the moving surface depending on thickness h. In this case the derivatives of curvature and strain changes are calculated from the expressions:

$$\left\{\frac{\mathrm{d}\tilde{æ}}{\mathrm{d}h}\right\}=\frac{[C]-h[B]}{[B][D]-[C]^2}\{\bar{\sigma}\}; \qquad \left\{\frac{\mathrm{d}\tilde{\varepsilon}}{\mathrm{d}h}\right\}=\frac{[D]-h[C]}{[B][D]-[C]^2}\{\bar{\sigma}\}, \tag{16}$$

which are obtained by solving Eqs. 8.

Note that commonly used sign conventions are assumed in the presented algorithm. In particular, the curvature is considered positive, if layer growing/removing produces a deflection of the substrate which is concave upwards.

Some Extensions of Stoney's Equations

Some extensions of Stoney's equations follow from the presented common algorithm of the layer growing/removing methods (Sect. 3). In particular, by solving Eqs. 16 with respect to the vector of initial stresses one obtains the expressions

$$\{\bar{\sigma}\}=\frac{[B][D]-[C]^2}{[C]-h[B]}\left\{\frac{\mathrm{d}\tilde{æ}}{\mathrm{d}h}\right\}=\frac{[B][D]-[C]^2}{[D]-h[C]}\left\{\frac{\mathrm{d}\tilde{\varepsilon}}{\mathrm{d}h}\right\}, \tag{17}$$

which present the extensions of Stoney's Eqs. 5 and 6 in a differential form for a biaxial stress state and for an inhomogeneous substrate-coating system.

For a multilayer coating consisting of n thin layers, each with a thickness of Δh_i, and with a small total thickness of $z_2 - z_1 = \sum_1^n \Delta h_i << z_1$, assuming $h = z_1 \equiv h_1$ from Eqs. 17 for initial stresses within the i-th layer, it follows

$$\begin{Bmatrix}\bar{\sigma}_{xi}\\ \bar{\sigma}_{yi}\end{Bmatrix}=\pm\frac{E}{1-\mu^2}\frac{h_1^2}{6}\begin{bmatrix}1 & \mu\\ \mu & 1\end{bmatrix}\begin{Bmatrix}\frac{\Delta æ_i}{\Delta h_i}\end{Bmatrix}=\pm\frac{E}{1-\mu^2}\frac{h_1}{2}\begin{bmatrix}1 & \mu\\ \mu & 1\end{bmatrix}\begin{Bmatrix}\frac{\Delta\varepsilon_i}{\Delta h_i}\end{Bmatrix}, \tag{18}$$

where E and μ are the modulus of elasticity and Poisson's ratio of substrate and $\Delta æ_i$, $\Delta\varepsilon_i$ are the variations in the curvature and strain measured at growing or removing of the i-th layer. The plus sign and the minus sign correspond to layer growing and layer removing, respectively.

For an equibiaxial state of stress ($\Delta æ_{xi} = \Delta æ_{yi} = \Delta æ_i$, $\Delta\varepsilon_{xi} = \Delta\varepsilon_{yi} = \Delta\varepsilon_i$) Eqs. 18 simplify:

$$\bar{\sigma}_{xi} = \bar{\sigma}_{yi} = \bar{\sigma}_i = \pm\frac{E}{1-\mu}\frac{h_1^2}{6}\frac{\Delta æ_i}{\Delta h_i} = \pm\frac{E}{1-\mu}\frac{h_1}{2}\frac{\Delta\varepsilon_i}{\Delta h_i}. \tag{19}$$

Eqs. 18 and 19 present the extensions of the first Stoney's formulae in Eqs. 5 and 6.

Computer program RS-PLATE and a computation example

On the basis of the presented algorithm (Sect. 3), a computer program RS-PLATE is developed, which enables calculation of residual stresses in isotropic inhomogeneous plates from strains, curvatures or initial stresses measured during the growing or removing process. According to the program, calculation of the derivatives of experimental data is carried out by a preliminary fitting with a polynomial.

Using the program RS-PLATE, a computational example is realized. In this equibiaxial residual stresses are computed in a galvanic steel coating from curvatures ($æ_x = æ_y = æ$) measured by a mirror instrument during the growing process of the coating on a copper plate substrate 80×80×1 mm. Fig. 2 shows the experimental data of curvature measurements and the distribution of initial and residual stresses ($\bar{\sigma}_x = \bar{\sigma}_y = \bar{\sigma}$, $\sigma_x = \sigma_y = \sigma$) in the coating ($z_2$ = 1.22 mm, E_2 = 200 GPa, μ_2 = = 0.30, $\alpha_2 = 14.3 \cdot 10^{-6}$ 1/°C) deposited on a plate substrate (z_1 = 1.00 mm, E_1 = 110 GPa, μ_1 = 0.34, $\alpha_1 = 17.5 \cdot 10^{-6}$ 1/°C). Calculated residual stresses at 95°C were reduced to temperature 20°C by equations for thermoelastic stresses in bimetallic plates [14].

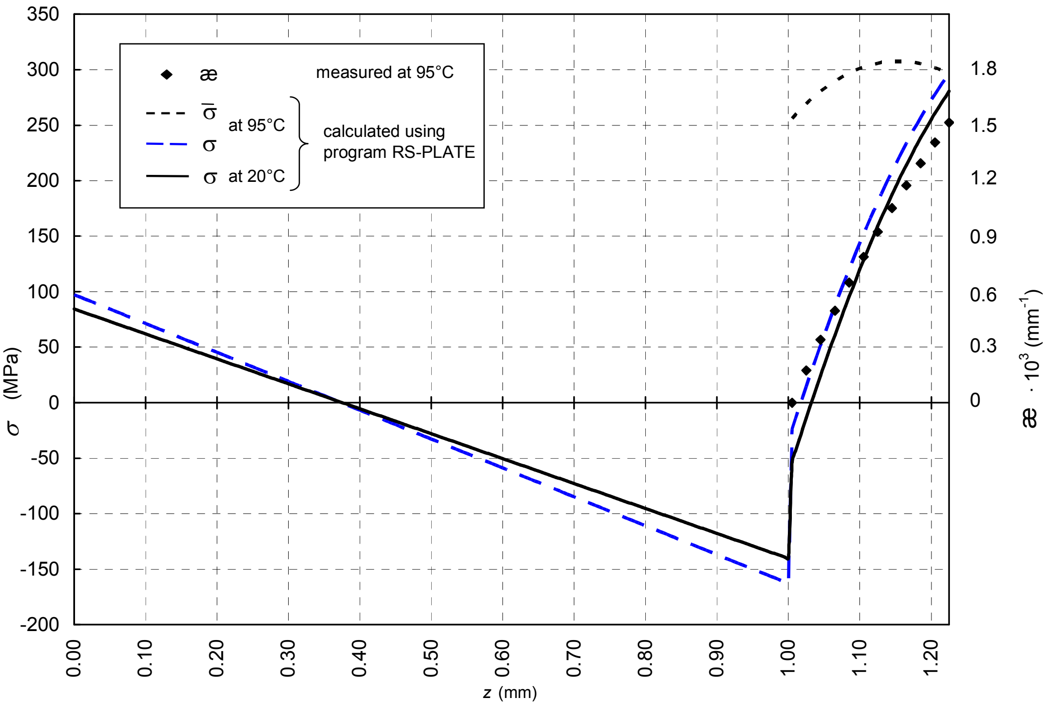

Figure 2. Measured curvatures and distribution of initial and residual stresses for a copper plate substrate with a galvanic steel coating

Summary

(1) In connection with the 100th anniversary of the publication of Stoney's paper, some historical remarks are made with respect to the development of these equations.

(2) An algorithm arising from a more common algorithm [11] is presented for calculation of residual stresses in a free rectangular isotropic plate inhomogeneous along thickness. The algorithm is universal and allows calculation of residual stresses at layer growing or layer removing from curva-

tures or strains measured on the stationary surface, or from initial stresses measured on the moving surface of the plate.
(3) Stoney's equations for a thick coating are extended for the biaxial stress state and for a substrate-coating system inhomogeneous along thickness.
(4) Stoney's equations for a thin coating (film) are extended for a multilayer coating with small total thickness.
(5) The computer program RS-PLATE, based on the presented algorithm, is introduced and, using the program, a computational example is realized.

Acknowledgement

This research was supported by the Estonian Ministry of Education and Science (SF Project – SF0140091s08).

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