

## Preface

Catastrophe theory, which treats the singularities of smooth real-valued functions, was introduced by René Thom in the late 1960's. It gives a beautiful classification of smooth functions; together with the representative function of each class, the germ of that class, from the point of view of singularity. The nature of a singularity of a function, at a point of variable-space, is revealed by perturbing the function. The theory gives also the general form of a perturbation of the germ of a class; called the universal unfolding of that class. The universal unfolding of a class displays all of the qualitative properties of all of the functions belonging to the class.

From our standpoint, a *qualitative property*, or a *change in a qualitative property*, are essential features of a mathematical and physical system. The transition points, at 0°C and 100°C, of water under 1 atm pressure are singular points; and nothing else. The addition of a small quantity of impurity gives rise to shifts in the transition points to some extent, but there are still two of them. The addition of the impurity leads to a change in chemical potential. An invariant property, under small changes in the external conditions, *i.e.* a perturbation, is called a structurally stable property. It must be realised that structurally stable singular point sets provide the most important characterization of the system under consideration. Only the structurally stable property persists during repeated observations and can be the object of scientific investigations. That is, we do not recognise the result of an observation, as being scientific evidence, if it disappears as the result of a slight perturbation of the experimental conditions. Even if the freezing point of water is 1°C, we do not care; unless we are in a contest of measuring instruments. The important point is whether there exists a structurally stable singularity in that neighborhood.

Many scientists may feel that the use of the term, *qualitative*, in science and, above all, in physics has a pejorative ring; as Thom has warned. It was a physicist who reminded him, not without vehemence, of Rutherford's dictum, "*Qualitative is nothing but poor quantitative.*" [mT75] The application of catastrophe theory to the sciences leads to proper understanding of qualitative properties, and reveals the phenomena to be expected of systems belonging to a certain class; at the cost of numerical information.

According to Zeeman [mG79], *catastrophe* refers to the unexpec

tedness of discontinuous effects when they are produced by continuous causes; thus violating our intuition, which would normally lead us to expect continuous effects. Examples of catastrophes can be found in physics, chemistry, biology and sociology; as described in many of the books cited at the end of the present work. This book is devoted solely to applications which involve phase transitions and critical phenomena in material systems. These are the most striking catastrophic phenomena in the physical sciences. The resultant discussions are classical and intrinsically phenomenological. The theory can also unify concepts and nomenclatures which have been in some disarray because of their more or less independent development in different scientific fields. Another advantage of the theory is that it emphasizes the metastable states, whereas "rigorous" statistical mechanics does not, and even denies their existence. [Z88] Indeed, if one heats water very carefully in a clean test-tube one can raise the temperature to about 170°C before it explodes.

Most of the theoretical and experimental work of the present author has concerned phase transitions and critical phenomena in ferroelectric systems. Therefore, the material systems which are treated in this book are mainly ferroelectric systems, which can be described fairly well by classical means. The reader who is interested in other systems can translate concepts into his own language. Many of the systems which are dealt with are one-order parameter systems, while some are multi-order parameter systems. The mathematics and physics of multi-order parameter systems are still developing, and successful investigation will be rewarding.

Most of the illustrations are in the form of computer graphics. Programs written in BASIC are available upon request (write to the author's private address).

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