

Evaluation and Comparison of a Few Methods for the Strain-Path Correction of FLCs

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Abstract. Forming limit curves are sensitive to the strain-path. Numerous methods were proposed in the literature since the 1970's to the last decades for taking into account this sensitivity. The aim of this work is to consistently compare these methods with practical application in mind. First, a literature review revealed that many available methods are different in form but all rely on the empirical assumption of iso-equivalent failure strain for all strain paths sharing the same final strain mode, independently of the strain path leading to it. The models relying on this hypothesis are summarized. A significantly different approach, called here "interpolation method", relies on different hypotheses – also empirical. The two approaches are further compared in order to identify the similarities and differences between the two. It appears that most stress-based approaches for the correction of FLC strain-path dependence rely on the same hypothesis as the iso-equivalent-strain method and they reduce to it under the assumption of isotropic hardening. The iso-strain method and the "interpolation" approach provide similar predictions in a series of configurations, while they significantly differ for other configurations. No theoretical or experimental proof is available to further discriminate the most accurate approach for strain-path correction of FLCs; the research opens new directions for the investigation of the topic.

Introduction

Forming Limit Curves (FLCs) remain the central tool for describing in-plane formability of sheet metals. Their classical experimental determination through the Nakazima or Marciniak tests has enabled engineers to identify the onset of localized necking across a variety of loading modes. However, their practical use in design and simulation has long been complicated by strain-path dependence. Even moderate prestrains may shift the forming limits significantly, affecting the accuracy of forming predictions in industrial operations where complex, nonlinear strain paths are common (see Fig. 1). The strain-path dependence of forming limits has been recognized since the earliest FLC studies. Classical tests showed that a sheet prestrained uniaxially could experience either an increase or decrease in its subsequent forming limit depending on the second strain path. Similar effects were later documented for plane strain and biaxial prestrains.

These observations revealed that forming limits could not be uniquely represented in principal strain space. Instead, the history of deformation affects the mechanical state of the material at necking. Research progressed along two main directions: i) Identifying state variables for which the forming limit would be path-independent (stress-based approaches). ii) Transforming the experimental strains to remove the influence of prior deformation through correction methods (equivalent-strain approaches). Both directions attempted to obtain a single, universal forming limit map.

Over the past decades, two main families of approaches have emerged to address strain-path dependence: strain-based forming limits with equivalent-strain correction [1-5], and stress-based forming limit representations [6-7], which propose that the state of stress at the onset of localization is insensitive to the deformation history. More recently, an interpolation-based method was proposed

[8], aiming to reconstruct non-linear forming limits without explicit correction through stress or equivalent strain.

The purpose of this contribution is to present a clarified comparison of the assumptions behind these families of approaches and a discussion of their mutual consistency, with emphasis on the underlying conditions (e.g., isotropic hardening) that allow equivalence or divergence.

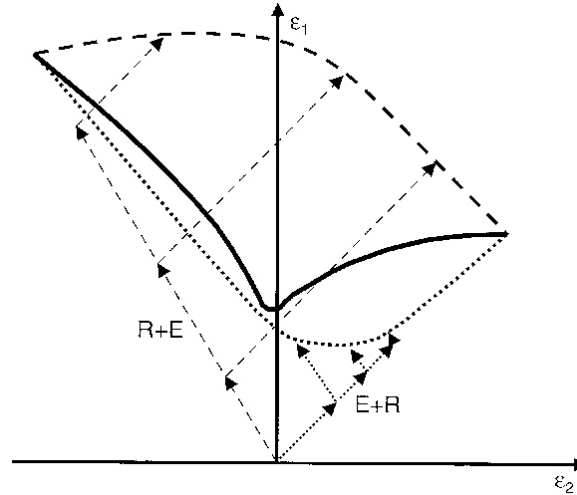


Fig. 1. Illustration of the impact of strain path on the forming limits of sheet metals, after [9].

Equivalent-Strain-Based Correction of Forming Limit Curves

The equivalent-strain approach assumes that the material state at the onset of localization is primarily controlled by the accumulated equivalent plastic strain. The correction method transforms strain paths so that prestrained states can be mapped back to an equivalent “virgin” state with the same hardening level (see Fig. 2).

This correction has several attractive properties:

- It is experimentally validated across a wide range of metals.
- It is easy to apply in practice.
- It remains compatible with standard strain-based forming limit diagrams used in industry.

Extensive experimental datasets show that for many steels and aluminum alloys, the method often provides a reliable prediction of forming limits under bilinear strain paths (see an example in Fig. 3). However, there is no proof for the good behavior of this approach, and several publications have shown that it sometimes also led to inaccurate predictions.

Stress-based forming limit diagrams (FLSDs) gained renewed prominence because they were thought to be inherently path-independent. Studies have shown that stresses at the onset of localization tend to collapse towards a relatively narrow locus that is less sensitive to strain-path changes than strain-based forming limits. It is noteworthy that under the isotropic hardening hypothesis, the stress-based approach fully derives from the equivalent-strain-based approach. If the yield surface expands uniformly with plastic strain, the forming limit associated with a given stress or strain state depends only on the current level of accumulated deformation. Therefore, removing the prestrain through an equivalent-strain transformation effectively restores the original forming limit., and stress-based criteria do not introduce independence beyond what is already implied by isotropic hardening. Their path independence stems from the same assumption that enables equivalent-strain correction and their predictions are expected to be similar in many practical situations. It was demonstrated in the literature that advanced hardening models lead to significant strain-path dependence of the stress-based forming limits, as illustrated in Fig. 4.

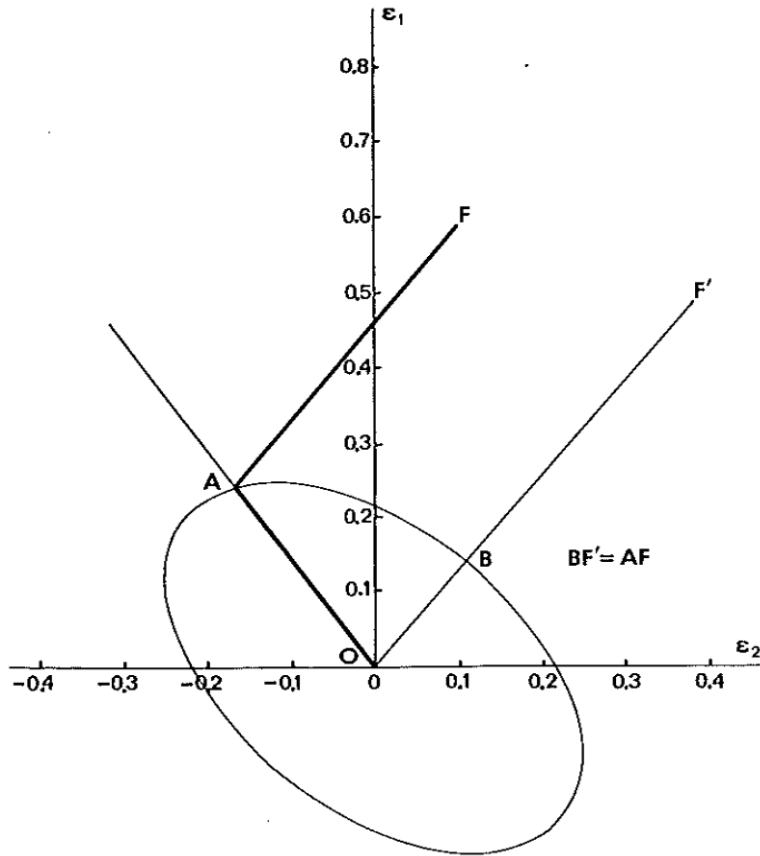


Fig. 2. Graphical construction of the equivalent strain concept proposed in [1] to solve for the strain-path dependence of FLCs. Strain state B has the same equivalent strain as strain state A, and the distances AF and BF' in this strain space are equal, which leads to identical equivalent strain values at F and F' – with OF' being linear.

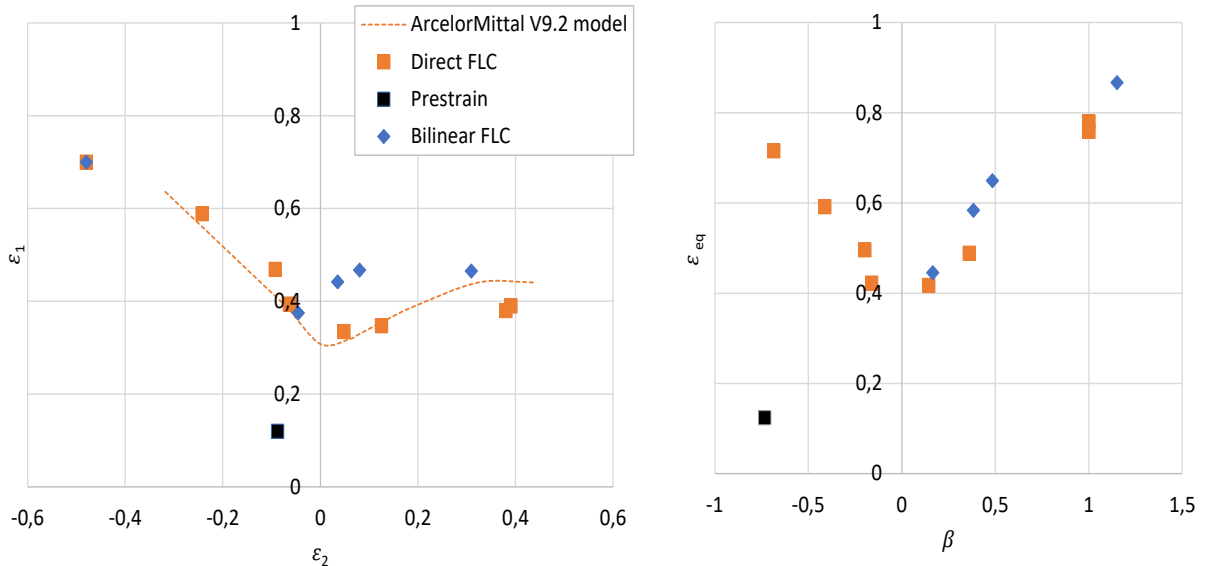


Fig. 3. Experimental forming limits of a mild steel sheet under proportional loading (orange symbols) and non-proportional loading (blue symbols). Left: classical FLD representation; right: (von Mises) equivalent strain vs strain ratio representation. The black square symbol designates the applied tensile pre-strain.

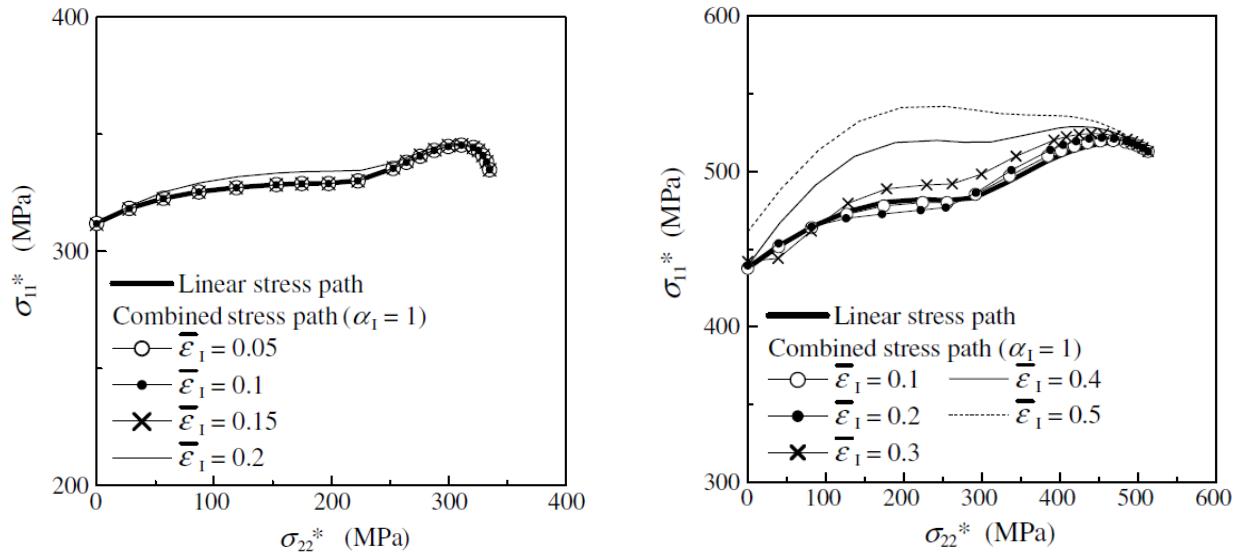


Fig. 4. Stress-based forming limits predicted with a quasi-isotropic hardening model (left) and microstructural-based hardening model with kinematic and latent hardening (right), from [10].

The Interpolation-Based Approach

An interpolation-based approach was proposed in [8] that seeks to reconstruct a forming limit surface by interpolating between several experimentally measured forming limits corresponding to different strain paths. The method has the advantage of avoiding explicit assumptions regarding hardening law or yield functions, providing smooth transitions between different strain paths. It allows for the reconstruction of forming limits under complex, potentially arbitrary, loading histories.

However, the method also comes with some limitations. Its accuracy potentially depends on the quantity and quality of available prestrain experimental data. The method assumes that the forming limit under a given bilinear path can be represented as an interpolation of limits from proportional loading. Potentially, when only a small number of experimental FLCs are available, the interpolation may produce less accurate predictions.

Comparisons with equivalent-strain correction reveal that while both can yield similar results when several experimental FLCs are used, differences emerge for large strain-path deviations or limited datasets (see Fig. 5.). These differences require further systematic investigation.

Conclusion

This study reviewed the main strategies developed to address strain-path dependence in forming limit curves and compared their assumptions, predictions and consistency. The principal conclusions are:

- Stress-based FLCs rely on the same hypothesis as the equivalent-strain-based correction under isotropic hardening and therefore provide similar levels of path independence.
- Experimental evidence supports the reliability of equivalent-strain correction for a wide range of sheet metals under bilinear strain paths.
- Interpolation-based methods can reproduce forming limits under complex loading paths but exhibit significant variability when limited experimental data are available.
- The combined experimental and theoretical results support the use of equivalent-strain correction as a reliable reference for both practical forming assessment and scientific analysis.
- Additional experimental investigations are needed to clarify the specific conditions under which interpolation methods provide advantages over equivalent-strain correction.

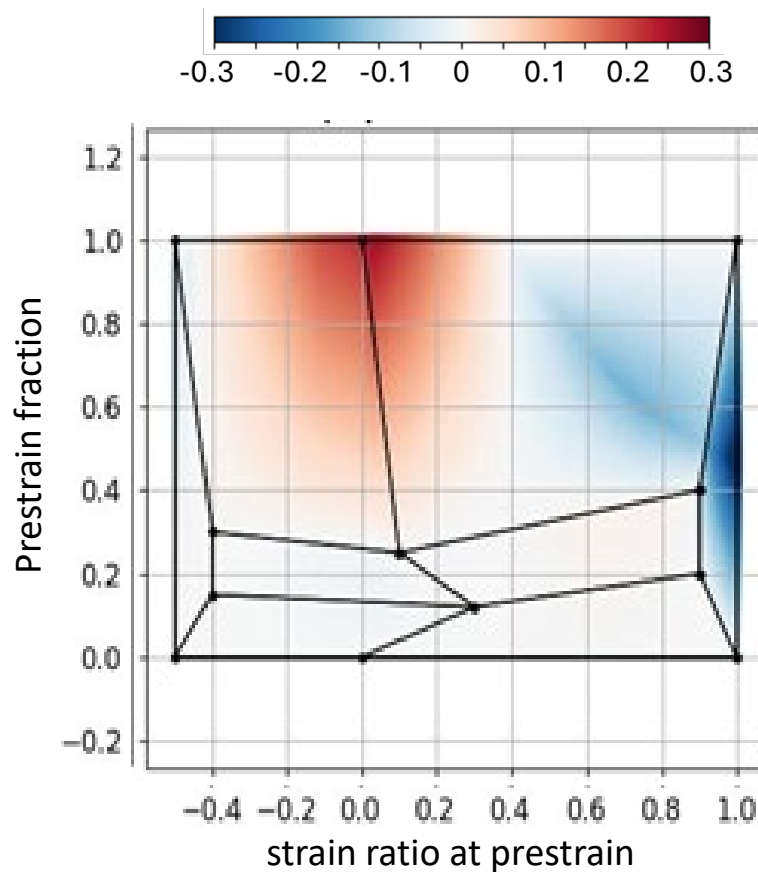


Fig. 5. Graphical representation of the difference between the equivalent strain at failure predicted by the equivalent-strain method and the interpolation method, for various prestrain fractions and strain ratios. The strain ratio of the second strain path was arbitrarily chosen as 0.4. Black dots (connected by black lines) show the experimentally explored pre-stains. It appears that for pre-strain fractions below 0.4, covered by experiments, the two methods deliver very similar results. Significant differences appear for very large pre-strains close to plane strain tension, and for moderate biaxial pre-strains.

Overall, this synthesis contributes to a clearer understanding of the compatibility and limitations of the main approaches used to correct strain-path effects in forming limit diagrams. It provides guidance for selecting appropriate methods in both research and industrial applications and highlights directions where further work is required.

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